The Cost of Information and Herd Behavior in Bank Runs

Wan-Ru Yang

Abstract

In the classic models of herding (Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992, 1998), etc), agents’ private signals are costless. This paper studies how costly private signals affect depositors’ herding behavior in bank runs. I assume that the quality of a depositor’s private signal depends on how much costs the depositor is willing to pay. More expensive signal means more precise information. The depositors are able to observe the predecessors’ actions. However, the cost on a private signal is each depositor’s private information. The model shows if some depositors choose to infer the quality of the bank’s investment by observing the predecessors’ withdrawal decisions rather than buying their own private signals, bank runs occur earlier. The depositors are able to anticipate whether the predecessors’ private signals are informative or not when their predecessors make different decisions. This paper demonstrates that the second patient depositor has the incentive to purchase more expensive information than the first patient depositor if the quality of his private signal is more improved and exceeds a critical level. When the second patient depositor’s private signal is more informative than the first patient depositor’s, the second patient depositor makes a withdrawal decision independent of the first patient depositor’s action. The third patient depositor does not have the incentive to purchase his private signal and a herd begins on the third agent. Therefore, the first two patient depositors’ decisions determine the existence of a bank runs equilibrium.

Keywords: Bank Runs. Imperfect Information. Herd Behavior. Costly Information
1 Introduction

In the classic herding models, agents make their decisions sequentially depending on their private signals and observable predecessors’ actions. When information cascades occur, agents rationally follow the predecessors’ behavior and ignore their own private signals. Many related literatures such as Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992, 1998), Bikhchandani and Sharma (2000) study herding behavior on social learning and in financial markets. In their models, agents’ signals are free. However, a more reasonable assumption is that agents pay costs to purchase their own information. I assume that the quality of an agents’ private signal is associated with the cost that the agent is willing to pay. This paper studies herding behavior in bank runs and shows whether costly information triggers information cascades earlier or not. If some depositors choose to infer the quality of the bank’s investment by observing the predecessors’ withdrawal decisions rather than buying their own private signals, the inefficient bank runs may be more likely to occur. The reason is that the qualities of the predecessors’ information determine the followers’ decisions.

This paper is closely related with the model developed by Cui (2007). Cui generalizes the information cascade model of Bikhchandani, Hirshleifer, and Welch (1992) to study herding behavior with costly private signals. Each agent chooses to acquire his private signal before deciding to invest a risky security. Cui demonstrates that the first agent has the incentive to acquire information but all of other agents are free riders. To maximize the expected payoff, the second agent and his followers are not willing to purchase their private signals. Therefore, an information cascade beginning on the second agent is earlier than that in the herding model with free private signals. Cui supposes that each agent’s optimal cost for a private signal is observable. I make a more reasonable assumption that is a significant difference between Cui and my model. In this paper, the agents infer whether the predecessors acquire private signals or not by observing the predecessors’ withdrawal decisions. All of agent’s optimal costs for the private signals are unobservable and each agent anticipates an interval of the cost that his predecessor is willing to pay. Therefore, this paper shows that the second agent still has the incentive to acquire information and the information cascades begin on the third agent. The third agent may uncertain about the quality of the second agent’s private signal; thus, the first two agents’ decisions determine the existence of a bank runs equilibrium.

Compared with Kultti and Miettinen (2003), this paper studies herding in bank runs; in other words, the predecessors’ withdrawal decisions are observable. Thus, in my model, the agents consider to purchase their private signals not the information about their predecessors’ actions. Kultti and Miettinen suppose that the agents have to pay costs for the information about their predecessors’ actions in a herding environment. All agents have their private signals. However, each agent is unable to observable the predecessors’ decisions unless he pays the observation cost. Kultti and Miettinen demonstrate if observation costs are moderate, only the third agent has the incentive to observe all of the predecessors’ actions and a herd begins after the third agent. Nevertheless, if the observation cost is expensive, the agent’s decision depends on the agent’s
private signal rather on other agents’ actions.

An interesting result of this paper also associated with the costs of information is the quality of more expensive information affects the agents’ incentive of purchasing private signals. If the quality of a private signal is more improved and exceeds a critical level, the second agent is willing to purchase more expensive information. When the second agent’s private signal is more informative than the first agent’s, the second agent makes a withdrawal decision independent of the first agent’s action. This paper obtains a similar conclusion as that in Kultti and Miettinen, but the explanation is different with Kultti and Miettinen. In this paper, the purchasing cost determines the quality of a private signal. The agent with an expensive and informative signal makes a decision only associated with his own information.

The rest of my paper is organized as follows. Section 2 introduces the model. In section 3, I study the condition that the agents are willing to acquire information and when a herd of bank runs occurs. Section 4 contains the conclusions.

2 The Model

2.1 Deposit Contract

The model has three dates, date t = 0, 1, and 2. The period 1 between date 1 and 2 is divided into N stages and N is finite.

There are N depositors. Each depositor has one unit of the consumption good and is identical to each other at date 0. A deposit contract is made at date 0. The deposit contract specifies that a depositor deposits one unit of his endowment in a bank at date 0 and then he is able to withdraw M1 units at date 1 or M2 units at date 2. Suppose that a proportion \( \alpha \) of depositors face liquidity shocks at date 1. The depositors who suffer a liquidity shock at date 1 are called impatient depositors and they have to make consumption and withdraw at date 1. The depositors who don’t experience the liquidity shock at date 1 are called patient depositors and they can wait to withdraw at date 2.

The bank invests the depositors’ deposits in a risky project at date 0. One unit of the consumption good invested at date 0 yields R units at date 2, and \( R = \bar{R} > 1 \) or \( R = \underline{R} = 0 \). The probability of \( \bar{R} \) is \( p \) and the probability of \( \underline{R} \) is \( 1 - p \), \( 0 < p < 1 \). If the project is liquidated at date 1, the bank obtains one unit of the consumption good for per unit of the investment. Suppose that the bank is able to reinvest at date 1 if bank runs do not occur. Assume that the fraction of depositors who withdraw at date 1 is \( \beta \) and \( \beta > \alpha \). The payment at date 2, \( M_2 \) is given by

\[
M_2 = \begin{cases} 
\frac{R(1 - \beta M_1)}{1 - \beta} & \text{if } \beta M_1 < 1 \\
0 & \text{if } \beta M_1 \geq 1 
\end{cases}
\]  

(1)

2.2 Costly Private Signals

The prior probability that a high (or bad) return of the investment at date 2 is \( P(\bar{R}) \) (or \( P(\underline{R}) \)) = 0.5. The patient depositor \( i \) has to pay a cost \( C_i \) to acquire the private signal \( S_i \) about the state of
the future return, where \(0 \leq C_i \leq 1\) and \(i = 1 \ldots N, S_i \in S\) and the set \(S = \{Sg, Sb\}\), where \(S_g\) denotes a good private signal and \(S_b\) denotes a bad private signal. The conditional probability of the signal \(S_i\) given the state of the future return is \(q_i\).

\[
P_i(S_i = S_g \mid R = \bar{R}) = P_i(S_i = S_b \mid R = R) = q_i(C_i)
\]

\[
P_i(S_i = S_b \mid R = \bar{R}) = P_i(S_i = S_g \mid R = R) = 1 - q_i(C_i)
\] (2)

\(q_i\) represents the patient depositor \(i\)’s signal precision and is an increasing function of the cost \(C_i\), which means that the private signal \(S_i\) is more informative when the patient depositor \(i\) pays more cost \(C_i\). If \(C_i = 0\), \(q_i(0) = 0.5\). Suppose that each patient depositor is unable to observe how much costs the predecessors pay. Because the patient depositors’ withdrawal decisions are observable, the patient depositor \(i\) infers the history of the costs \(\{C_1, C_2, \ldots, C_{i-1}\}\) from his predecessors’ actions.

The patient depositor \(i\)’s strategy \(X_i(S_i, H_i)\) is a function of the depositor’s private signal \(S_i\) and the history of withdrawal decisions before the patient depositor \(i\), \(H_i = (X_1, X_2, \ldots, X_{i-1})\). Let \(X_i(S_i, H_i) = 1\) represent that the patient depositor \(i\) withdraws at date 1, and \(X_i(S_i, H_i) = 0\) represents that the patient depositor \(i\) waits to withdraw at date 2.

### 2.3 Acquiring Information

Before making a withdrawal decision, the patient depositor \(i\) considers whether he purchases a private signal or not. The patient depositor \(i\) has the incentive to acquire private signal if he is better off than he anticipates the states of the return \(R\) from his predecessors’ actions. \(u_i(X_i, C_i)\) denotes the payoff for the patient depositor \(i\) after acquiring a private signal.

\[
u_i(X_i, C_i) = \begin{cases} M_1 - C_i & \text{if } X_i = 1 \\ M_2 - C_i & \text{if } X_i = 0 \end{cases}
\] (3)

\(\hat{D}_i\) denotes the expected payoff for the patient depositor \(i\) with a private signal.

\[
\hat{D}_i = P_i(g)EU_i(X_i, C_i \mid g_i) + P_i(b)EU_i(X_i, C_i \mid b_i)
\] (4)

The probability of obtaining a good signal to the depositor \(i\) is \(P_i(g)\). \(P_i(b)\) is the probability of obtaining a bad signal. Let \(EU_i(X_i, C_i \mid g_i)\) and \(EU_i(X_i, C_i \mid b_i)\) be the patient depositor \(i\)’s expected payoff given a good or bad signal.

\[
EU_i(X_i, C_i \mid g_i) = \left(\frac{1}{M_1}\right)M_1 - C_i
\] if \(X_i = 1\)

\[
P(\bar{R} \mid H_i, g_i)(\frac{\bar{R}(1 - \beta M_1)}{1 - \beta}) + P(R \mid H_i, g_i)(\frac{R(1 - \beta M_1)}{1 - \beta}) - C_i
\] if \(X_i = 0\) (5)

The representation of \(EU_i(X_i, C_i \mid b_i)\) is similar as (5). When the patient depositor \(i\) makes a withdrawal decision without his own private signal, his expected payoff is \(\hat{D}_i\).
\[ \tilde{D}_i = \left( \frac{1}{M_1} \right) M_1 \]

\[ P(R \mid H_i)(\frac{R(1-\beta M_1)}{1-\beta}) + P(R \mid H_i)(\frac{R(1-\beta M_1)}{1-\beta}) \]

if \( X_i = 1 \)

By backward induction, the patient depositor \( i \) decides to pay the cost of his private signal \( C_i \) or not. First, the patient depositor \( i \) chooses the optimal withdrawal time depends on his private signal or only the predecessors’ actions. Then, if the expected payoff \( \hat{D}_i \) is greater than \( \tilde{D}_i \), the patient depositor \( i \) has the incentive to acquire a private signal.

3 Depositors’ Herding

This section studies the patient depositors’ herding behavior on acquiring private signals and withdrawal times. The model shows the conditions on which the patient depositors are willing to purchase private signals and when an information cascade of bank runs begins.

3.1 The First Patient Depositor

A posterior probability of a high return \( \overline{R} \) that the first patient depositor depends on a good private signal \( g_1 \) is \( P(\overline{R} \mid g_1) = q_1(C_1) \). According to the assumption, \( q_1(C_1) > \frac{1}{2} \) if \( C_1 > 0 \).

Regardless of a good or bad signal, the first patient depositor will obtain \( 1-C_1 \) units when he withdraws at date 1. The expected payoffs of withdrawing at date 2 given good and bad signals are \( EU_1(X_1 = 0, C_1 \mid g_1) \) and \( EU_1(X_1 = 0, C_1 \mid b_1) \).

\[ EU_1(X_1 = 0, C_1 \mid g_1) = 2q_1(C_1) - C_1 \]

\[ EU_1(X_1 = 0, C_1 \mid b_1) = 2(1-q_1(C_1)) - C_1 \] \hspace{1cm} (7)

In (7), \( q_1(C_1) > \frac{1}{2} \) so \( EU_1(X_1 = 0, C_1 \mid g_1) > 1-C_1 \) and \( EU_1(X_1 = 0, C_1 \mid b_1) < 1-C_1 \).

Therefore, the first patient depositor will withdraw at date 2 if he obtains a good signal and he withdraws at date 1 if he obtains a bad signal.

Suppose that the first patient depositor does not acquire a private signal. The first patient depositor will have the payoff of 1 unit whether he withdraws at date 1 or date 2, thus \( \tilde{D}_1 = 1 \).

(8) represents \( \hat{D}_1 \) that is the expected payoff to the first patient depositor acquiring a private signal.

\[ \hat{D}_1 = P_1(g_1)EU_1(X_1 = 0, C_1 \mid g_1) + P_1(b_1)EU_1(X_1 = 1, C_1 \mid b_1) \]

\[ = \frac{1}{2} (2q_1(C_1) - C_1) + \frac{1}{2} (1-C_1) \]

\[ = q_1(C_1) + \frac{1}{2} - C_1 \] \hspace{1cm} (8)

Corollary 1 states when the first patient depositor has the incentive to acquire a private signal.

**Corollary 1** If \( q_1(C_1) > 1 \), the first patient depositor will acquire a private signal because \( \hat{D}_1 \)
is greater than $\tilde{D}_1$.

In Corollary 1, the condition $q'_1(C_1) > 1$ means that the percentage change in improvement of a private signal exceeds the percentage change in the cost of the private signal.

3.2 The Second Patient Depositor

The model considers when the second patient depositor infers that the first patient depositor purchases information about the state of the return $R$, how the second patient depositor decides to acquire a private signal and the time of withdrawing.

The analysis in this section is similarly as that in 3.1. Corollary 2 summarizes the second patient depositor’s herding behavior on acquiring a private signal and withdrawing the bank’s promised payment.

**Corollary 2** Suppose that the first patient depositor pays $C_1$ for a private signal.

(i) If $q_2(C_2) < q_1(C_1)$, the second patient depositor does not acquire a private signal. The second patient depositor will follow the first patient depositor’s withdrawal action.

(ii) If $q_2(C_2) > q_1(C_1)$ and $q'_2(C_2) > q'_1(C_1) \frac{C_1}{C_2} + 1$, the second patient depositor acquires a private signal. The second patient depositor decides the withdrawal time depending on his private signal, regardless of the first patient depositor’s action.

4. Collusions

The model shows that when the second patient depositor acquires a private signal, his actions will affect the followers’ decisions about the withdrawal time. Therefore, an information cascade begins on the third patient depositor. If the second patient depositor does not acquire a private signal and follows the first patient depositor’s action, an information cascade occurs earlier. The reason is that the third patient depositor and his followers do not have the incentive to acquire private signals, neither.

This paper demonstrates that the withdrawal action of the agent willing to acquire a more expensive signal influences other agents’. Therefore, the existence of bank runs is dominated by the decision of the patient depositor having the incentive to pay more for purchasing information.
Reference


