INCOME INEQUALITY AND SECURITY MARKET

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ABSTRACT

Witnessing the clear ascendancy of private enterprise alongside a tendency for its ownership benefits to flow into the hands of a few, we examine how this dramatic shift in ownership pattern relates to our security market. Drawing upon a two-period two-asset consumption investment model, we conclude that the existence of a security market will unambiguously reduce the variance of wealth distribution unless some credit constraint is imposed on poor investors while the wealthy can unlimitedly sell their securities short in the bear market. If a short sale is also banned on the wealthy investors, then we need to resort to investors’ heterogeneous beliefs of the prospect of risky investment as the legitimate reason for income inequality. To stem the deterioration of a widening wealth gap, a more direct and fair information disclosure mechanism has to be installed. An empirical study based on this model is thus conducted.

Keywords: Income inequality, Security market, Risk neutral probability, Credit constraint, Heterogeneous belief

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1. INTRODUCTION

Recent empirical studies (e.g., Juhn, Murphy and Pierce 1993; Machin 1996) have pointed to a substantial increase in wage and income inequality in several OECD countries during the past twenty years. The ratio of the 90th to the 10th percentile of the male wage distribution rose from 2.53 to 3.21 in the UK between 1980 and 1990 and from 4.76 to 5.63 in the U.S. over 1980–1989. Various explanations have been offered for this observed inequality upsurge in developed countries and conclude that a major cause of these changes has been a shift in the relative demands for skilled and unskilled labor. This study provides another explanation of the income inequality from the viewpoint of ownership change in our capitalistic society, particularly through security transaction.

We are now witnessing the most dramatic shift of ownership patterns globally in history: the clear ascendancy of private enterprise alongside a tendency for its ownership benefits to flow into the hands of a few. Jeff Gates (1998) claims that the central problem of modern capitalism is that it doesn’t create enough capitalists. In the U.S., for example, despite rising living standards, full employment, low inflation, and a bull market that cannot stop setting records, the gulf between the haves and the have-nots is widening. Concentration of wealth at the top results in what Gates calls ‘disconnected capitalism,’ and it causes politicians, corporate executives, investors, social activists, and common citizens to come up with creative strategies to make the system more inclusive.

The modern ownership system traces its origins to the seventeenth-century writings of English philosopher John Locke. Writing at a time when divine right was embodied in the king, he proposed the radical notion that rights to personal property arose whenever man mixed his labor with whatever he removed from nature. We have come a long way since Locke proposed personal property as a social innovation meant to help mankind evolve out of a state of nature.

As social institutions evolved to support personal contracts with the force of

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1 See Philippe Aghion and Jeffrey G. Williamson (1998).
law, contracts themselves became an ownable and exchangeable form of property. At first the law recognized letters of credit, promissory notes and such. Gradually, they were made out to whoever held them (the bearer). This innovation enhanced efficiency by enabling a creditor to sell to a third party an enforceable obligation even though the parties were unknown to each other.

In today's sophisticated capital markets, this securitization process is spreading rapidly. In 1985 just fifty-six nations had significant securities markets with total capitalization of $6.5 trillion. Ten years later, eighty-five nations had stock markets valued at nearly $18 trillion. Over time, the nature of the 'person' entering these contracts also has changed, becoming more 'institutional' as the corporation emerged as the most prevalent of these entities. This transformation parallels mankind's steady evolution away from descriptions denoting social status and toward a terminology where relationships are based more on contract. We have to acknowledge the modern reality that property is now more often accessed through corporate finance than through personal toil.

Karl Marx reveals in *The Communist Manifesto* three remedies to the two key concerns he identified: the concentration of wealth and the exploitation of working people by the force of finance capital. His first solution, abolishing private property, is best known. His second remedy is less well known, but more widely applied: the steady erosion of property rights by the ten 'erosive' measures he recommended, including the abolition of child factory labor, free primary school education, and progressive income taxes. The third and quite distinct remedy, not quite recognized by Marx, but equally appropriate to his purpose of supporting a broad-based personal ownership, is the extension of the ownership of capital from the few to the many. It is clear that Marx was not only a critic of capitalism, he was also a begrudging admirer, conceding that it 'has created more massive and more colossal productive forces than all preceding generations have together'.

This study presents another remedy to repair and promote a more equitable economic system by enhancing the genuineness and fairness of information disclosed to the public by the government (SEC), while preserving incentives for investment.

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2 see Jeff Gates (1998).
(i.e., retaining the existing property right system and capital market mechanism). If each individual develops a more homogeneous belief about the prospect of his investment due to the existence of a common source of reliable information, then free access to or exit from the capital market will enable him to diversify his investment portfolio and maximize his inter-temporal welfare level. The cross-time comparison of expected marginal utility will lead poor individuals to consume less proportion of their wealth than the wealthy and may gradually patch up the income inequality chasm. The primary factor contributing to the enlarged hiatus of wealth in our real world is the prevailing heterogeneity of conjectures about investment outcomes. What is worse is that wealthy people can usually access more rewarding inside information than others by exercising their ‘money power’ and amass a disproportionate investment return.

We are now in an age of information explosion particularly aided by the worldwide internet system. One must challenge the conventional wisdom that ownership is limited to tangible property and to those who merely invest financial capital. The dramatic changes under way tend to push authority, responsibility, risk, and reward—all the attributes we associate with ownership—outward and downward to intangible goods and to all employees (or laborers). Every individual whether he is a capitalist or a laborer should inherit a ‘property’ right of access to all the information needed to consume or invest. The ‘property’ conferred on either capitalists or laborers should include the fair playing field for information and knowledge acquirement. With this comprehensive ownership system we will then be able to entertain a more equalized and harmonious living environment.

Some may argue that inequality stimulates capital accumulation and growth. Consequently, there is a fundamental tradeoff between productive efficiency and social justice. However, the conventional microeconomic tradeoff between equity and incentives may not exist when we introduce capital market imperfections. For example, Aghion and Williamson (1998) list three reasons why redistribution to the less endowed can be growth enhancing: (a) redistribution creates opportunities, (b) redistribution improves borrowers’ incentives, and (c) redistribution reduces macroeconomic volatility. In particular, they show that redistributing wealth from the
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The empirical study presented by George R.G. Clarke (1995) supports the above negative relationship between inequality and growth. Robert J. Barro (1999) recently shows that, for growth, higher inequality tends to retard growth in poor countries and encourage growth in richer places. The same result was also derived by Deininger and Squire (1998). The Kuznets curve—whereby inequality first increases and later decreases during the economic development process—emerges as a clear empirical regularity. However, this relation does not explain the bulk of variations in inequality across countries or over time. Some variation of income may be attributed to or lessened by security transaction.

Each individual earns his (her) income either from his (her) labor contribution to production (i.e., wage income) or from his (her) return of capital investment. To pinpoint the role of a security market in the determination of income inequality among countries, this paper shies away from the issue of wage difference and attributes the income inequality of countries to the variation of return from either riskless investment (e.g., bank deposits) or risky security transactions. The latter will be solely determined by the variance of initial wealth. In other words, income inequality in this paper is virtually the change of wealth inequality in absence of identifying the role of labor in production. Strictly speaking, income inequality deals with the concept of “flow” distribution while wealth inequality considers the distribution of “stock”. Without explicitly specifying the production function, we cannot literally address the issue of income inequality. In the model constructed below we compare the wealth distribution in two periods. From the relative change (or ratio) of wealth distribution in these two periods we can capture how income inequality turns out from the investment behavior.

The following section sets up a two-period consumption and investment model with a risk-less bank account and a risky security. The optimal consumption and investment strategy is solved by the concept of a risk-neutral probability measure.
An example of a binomial model with CARA utility function is explored in section 3. The implications of a security market on the dynamics of income inequality are further examined in section 4 by imposing restraints on the borrowing of a bank account or prohibiting the short sale of a risky asset. We conclude that the existence of a security market will unambiguously reduce the variance of wealth distribution unless substantial short sales occur to wealthy investors in the 'bear' market while poor investors are refrained from borrowing and short selling. The gain thus realized by people who are capable of short selling in the bear market will lengthen the wealth gap from the poor who barely live from hand to mouth. If short selling is also banned or some upper limit on short sale volume is imposed on wealthy investors, we have to resort to investors' heterogeneous beliefs of the prospect of risky investment as the legitimate reason for income inequality. We conduct some empirical studies based on this model in section 5. Concluding remarks are made in section 6.

2. BASIC MODEL

This model is concerned with the problem of choosing the best consumption and investment strategy for each individual \(i(i = 1, 2, \cdots, L)\) in two periods, \(t = 0, 1\). At time \(t = 0\) a portion of the wealth for individual \(i\), \(v_i\), is consumed. The rest of the wealth is either invested in a risky asset \(S\) or deposited in a bank account \(B\). For simplicity, a sample space containing two uncertain states in period 1 is assumed, i.e., \(\Omega = \{\overline{w}|\overline{w} = \overline{w}_1, \overline{w}_2\}\). There is a probability measure \(P\) on \(\Omega\) with \(P(\omega)\) for all \(\omega \in \Omega\). Bank account \(B = \{B_t : t = 0, 1\}\) is a stochastic process with \(B_0 = 1\) and with \(B_1(\overline{w}_1) = B_1(\overline{w}_2) > 0\). Here, \(B_1\) can be thought of as the time 1 value of a savings account when $1 is deposited at time 0. The quantity \(r \equiv (B_1 - B_0)/B_0 \geq 0\) can be thought of as the risk-free interest rate pertaining to the time interval \((0, 1)\). The risky security process \(S = \{S_t : t = 0, 1\}\) is a non-negative stochastic process, and \(S_t\) should be thought of as the time \(t\) price of the risky security.\(^3\)

\(^3\) In addition to stocks, the securities addressed here can include any bond whose pricing behavior follows the same stochastic process. Although bonds are generally specified as an alternative stochastic
A trading strategy for individual $i$ is defined as a vector $H_i = (H_{i0}, H_{i1})$ with $H_{i0}$ and $H_{i1}$ denoting the amount of investment in $B$ and $S$, respectively. Therefore, the self-financing constraint for each individual implies $v_i - C_{i0} = H_{i0}B_0 + H_{i1}S_0$. The total value of investment or the net wealth $W_i(w)$ at time 0 will be entirely consumed at the end of time 1. In other words, $C_{i1}(w) = W_i(w) = H_{i0}B_1(w) + H_{i1}S_1(w) = H_{i0}(1 + r) + H_{i1}S_1(w)$, for all $w \in \Omega$. A consumption process $C_i = (C_{i0}, C_{i1})$ is said to be attainable if there exists a trading strategy $H_i$, satisfying the above self-financing constraint. The investor's consumption and investment problem is

$$\begin{align*}
\text{Maximize } & E \left[ \sum_{t=0}^{1} \alpha^t u(C_{it}) \right], \\
\text{Subject to : } & v_i = \text{initial wealth, and } C_i \text{ is attainable.}
\end{align*}$$

Here, $u$ is a specified concave increasing utility function and $\alpha$ is a specified scalar parameter representing the time preference of the individual and satisfying $0 < \alpha \leq 1$.

A gain process $G$ is a random variable that describes the total profit or loss generated by the portfolio between times 0 and 1, and can be represented by $G_i \equiv H_{i0}r + H_{i1}\Delta S$, where $\Delta S \equiv S_1 - S_0$. It is convenient to normalize the prices in such a way that the bank account becomes constant. In other words, we make the bank account the numeraire and define the discounted price process $S^* = \{S^*_t : t = 0, 1\}$ by setting $S^*_t \equiv S_t / B_t$, while the discounted gain process $G^*_t \equiv H_{i1}\Delta S^*$, where $\Delta S^* \equiv S_1^* - S_0^*$. With these notations we can rewrite $C_{i1} = B_1(v_i - C_{i0} + H_{i1}\Delta S^*) = B_1(v_i - C_{i0} + G^*)$, and the consumption and investment problem above is equivalent to

$$\begin{align*}
\text{Maximize } & E[u(C_{i0}) + \alpha u(B_1(v_i - C_{i0} + H_{i1}\Delta S^*))],
\end{align*}$$

The first-order conditions can be expressed as

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process with different risk characteristics, we lump stocks and bonds together as securities to simplify the problem of portfolio choice between them.
0 = \frac{\partial E[u(C_{i0}) + \alpha \cdot u(B_1(v_i - C_{i0} + H_{i1}\Delta S^*))]}{\partial C_{i0}} \\
= u'(C_{i0}) - \alpha \cdot \left( \sum_{\overline{w} \in \Omega} P(\overline{w})B_1(\overline{w})u'[B_1(\overline{w})(v_i - C_{i0} + H_{i1}\Delta S^*(\overline{w}))]\right)

\text{or } u'(C_{i0}) = \alpha B_1 E[u'(B_1(v_i - C_{i0} + H_{i1}\Delta S^*(\overline{w})))] \quad \text{(1)}

\text{and } 0 = \frac{\partial E[u(C_{i0}) + \alpha \cdot u(B_1(v_i - C_{i0} + H_{i1}\Delta S^*))]}{\partial H_{i1}} \\
= \sum_{\overline{w} \in \Omega} P(\overline{w})B_1(\overline{w})u'[B_1(\overline{w})(v_i - C_{i0} + H_{i1}\Delta S^*(\overline{w}))]\Delta S^*(\overline{w}) \\
= E[B_1u'(v_i - C_{i0} + H_{i1}\Delta S^*)\Delta S^*] \quad \text{(2)}

It can be easily shown that if there exists an optimal solution of the consumption and investment problem above, i.e., the first-order conditions are satisfied, then there are no arbitrage opportunities. A trading strategy \(H\) is an arbitrage opportunity if and only if (a) \(G^* \geq 0\) and (b) \(EG^* > 0\) It is known that there are no arbitrage opportunities if and only if there exists a risk-neutral probability measure \(Q\). Here, the risk-neutral probability measure \(Q\) is defined to be the probability measure that satisfies (a) \(Q(\overline{w}) > 0\), for all \(\overline{w} \in \Omega\), and (b) \(E_Q[\Delta S^*] = \sum_{\overline{w} \in \Omega} Q(\overline{w})\Delta S^*(\overline{w}) = 0\) \(\text{(3)}\)

Comparing equation (2) and equation (3), it is apparent that

\text{If } (C_{i0}, H_{i1}) \text{ is a solution of the optimal consumption and investment problem,}

then \(Q(\overline{w}) = \frac{p(\overline{w})B_1(\overline{w})u'[B_1(\overline{w})(v_i - C_{i0} + H_{i1}\Delta S^*(\overline{w}))]}{E[B_1u'(v_i - C_{i0} + H_{i1}\Delta S^*)]}, \overline{w} \in \Omega\) \(\text{(4)}\)

defines a risk-neutral probability measure.

Since \(v_i = C_{i0} + C_{i1}/B_1 - H_{i1}\Delta S^*\), we obtain the necessary and sufficient condition for the consumption process \(C_i = (C_{i0}, C_{i1})\) to be attainable as
Therefore, the optimal consumption and investment problem is equivalent to

\[
\begin{align*}
\max & \quad \mathbb{E} \left[ \sum_{t=0}^{1} \alpha^t u(C_{it}) \right], \\
\text{subject to} & \quad v_i = \mathbb{E}_Q [C_{i0} / B_0 + C_{i1} / B_1].
\end{align*}
\]

Define the state price density \(L\) as \(L(w) = Q(w) / P(w)\) and an adopted stochastic process \(N_t(w)\) as \(N_t = \mathbb{E}[L|\xi_t]/B_t\). Here, \(F = \{\xi_t : t = 0, 1\}\) is a filtration generated by the security process \(S = \{S_t : t = 0, 1\}\). Thus, \(E_Q[\sum_{t=0}^{1} C_{it} / B_t] = E[L \sum_{t=0}^{1} C_{it} / B_t] = E[\sum_{t=0}^{1} E[C_{it}L/B_t|\xi_t]] = E[\sum_{t=0}^{1} C_{it}N_t]\). As a consequence, our problem can be rewritten as

\[
\begin{align*}
\max & \quad \mathbb{E} \left[ \sum_{t=0}^{1} \sigma^t u(C_{it}) \right], \\
\text{subject to} & \quad v_i = E[\sum_{t=0}^{1} C_{it}N_t],
\end{align*}
\]

Introducing a Lagrange multiplier \(\lambda_i\) for each individual \(i\), we now want to solve

\[
\begin{align*}
\max & \quad E[\sum_{t=0}^{1} \sigma^t u(C_{it})] - \lambda_i \sum_{t=0}^{1} C_{it}N_t \text{ for all } i
\end{align*}
\]

The first-order conditions are

\[
\alpha^t u'(C_{it}) = \lambda_i N_t, \text{ for all } w \in \Omega, t = 0, 1 \tag{5}
\]

Equivalently, if \(I(.)\) is the inverse of the marginal utility function \(u'(.)\), then we must have
\[ C_{it} = I(\lambda_i N_t / \alpha^t), \] for all \( \omega \in \Omega, t = 0, 1 \) \hspace{1cm} (6)

Terms \( \lambda_i \) are simply the values such that the constraint in our problem is satisfied when equation (6) is substituted. In other words, \( \lambda_i \) are the solutions of

\[ E[\sum_{t=0}^{1} N_t I(\lambda_i N_t / \alpha^t)] = v_i \] \hspace{1cm} (7)

3. SOLUTION – THE BINOMIAL MODEL

The binomial model is simple, yet commonly used by practitioners for the price of a single risky security. In the following the binomial model will be assumed to describe the stock price. Each period has two possibilities: the security price either goes up by the factor \( u (u > 1) \) or it goes down by the factor \( d (0 < d < 1) \). The probability of an up move during a period is equal to the parameter \( p \).

The elements of underlying sample space \( \Omega = \{ \omega | \omega = \omega_1, \omega_2 \} \) can be interpreted as \( \omega_1 = \text{‘up’ move} \) and \( \omega_2 = \text{‘down’ move} \). Therefore, the time 1 price of the risky asset is given by \( S_1(\omega_1) = S_0u \) with probability \( p \), and \( S_1(\omega_2) = S_0d \) with probability \( 1 - p \). A desirable feature of the binomial security price model is that its return process is given simply by \( R(\omega) = \frac{S_1(\omega) - S_0}{S_0} \) which is equal to \( u - 1 \) when \( \omega = \omega_1 \), and equal to \( d - 1 \) when \( \omega = \omega_2 \). Because \( S_1 - S_0 = S_1 - B_1 S_0 = \frac{(1 + R)S_0 - (1 + r)S_0}{1 + r} = S_0 \left( \frac{R - r}{1 + r} \right) \), equation (3) implies

\[ E_Q \left( \frac{R - r}{1 + r} \right) = 0 \] \hspace{1cm} (3')

Let \( Q(\omega_1) = q \) and \( Q(\omega_2) = 1 - q \). Equation (3') can then be written as

\[ q \left[ \frac{u - 1 - r}{1 + r} \right] + (1 - q) \left[ \frac{d - 1 - r}{1 + r} \right] = 0. \] Hence, \( q = \frac{1 + r - d}{u - d} \). Moreover, the state price density \( L(\omega) = Q(\omega)/P(\omega) \) is equal to \( q/p \) when \( \omega = \omega_1 \) and equal
to \((1 - q)/(1 - p)\) when \(\bar{w} = \bar{w}_2\). Also, \(N_0 = 1\), \(N_1(\bar{w} = \bar{w}_1) = \frac{q}{p(1 + r)}\), and 
\(N_1(\bar{w} = \bar{w}_2) = \frac{1 - q}{(1 - p)(1 + r)}\) by definition.

To solve our problem explicitly by using the risk-neutral probability measure \(Q\), we need to define our utility function. Consider the following utility functional form\(^4\): 
\[u(c) = -e^{-\gamma c},\]
where \(\gamma\) is a constant measuring the degree of the individual's risk aversion. His marginal utility equals \(u'(c) = \gamma e^{-\gamma c}\). As a result, the inverse function of \(u'\) becomes 
\[I(\eta) = -\frac{\ln \eta}{\gamma} + \frac{\ln \gamma}{\gamma}.
\]

According to equation (6) we can derive the optimal time 0 and time 1 consumption as

\[C_{i0} = I(\lambda_i N_0) = -\frac{\ln \lambda_i}{\gamma} + \frac{\ln \gamma}{\gamma}\]

\[C_{i1}(\bar{w} = \bar{w}_1) = I(\lambda_i N_1(\bar{w} = \bar{w}_1)/\alpha) = I\left(\frac{\lambda_i q}{p(1 + r)\alpha}\right)\]

\[= -\frac{\ln \lambda_i}{\gamma} - \frac{\ln q}{\gamma} + \frac{\ln \gamma}{\gamma}\]

\[C_{i1}(\bar{w} = \bar{w}_2) = I(\lambda_i N_1(\bar{w} = \bar{w}_2)/\alpha) = I\left(\frac{\lambda_i (1 - q)}{(1 - p)(1 + r)\alpha}\right)\]

\[= -\frac{\ln \lambda_i}{\gamma} - \frac{\ln (1 - q)}{(1 - p)(1 + r)\alpha} + \frac{\ln \gamma}{\gamma}\]

As for \(\lambda_i\), it can be solved from equation (7) by substituting the \(C_{i0}\) \& \(C_{i1}\) above into it. It turns out

\[\ln \lambda_i = \ln \gamma - \frac{(1 + r)r \nu_i}{2 + r} - \frac{q}{2 + r} \ln \left[\frac{q}{p(1 + r)\alpha}\right] - \frac{1 - q}{2 + r} \ln \left[\frac{1 - q}{(1 - p)(1 + r)\alpha}\right]\]

Therefore, the optimal consumption can be solved as follows:

\(^4\) We adopt the binomial model with CARA preference simply for easy interpretation. Other specifications of the model may not necessarily have an explicit solution form, compounding the difficulty of analysis.
The optimal trading strategy for each individual can be seen from the following definition of time 1 wealth for individual $i$:

$$W_i(\bar{w}_1) = C_i(\bar{w}_1) = H_{i0}(1+r) + H_{i1}S_0u, \quad \text{and} \quad W_i(\bar{w}_2) = C_i(\bar{w}_2) = H_{i0}(1+r) + H_{i1}S_0d.$$ 

The amount of money deposited in the bank account, $H_{i0}$, and invested in the risky asset, $H_{i1}S_0$, by individual $i$ at time 0 can be solved as

$$H_{i0} = \frac{v_i}{2r} + \frac{q-2r}{(1+r)(2r)\gamma} \ln \left( \frac{q}{p(1+r)\alpha} \right) + \frac{1-q}{(1-p)(1+r)\gamma}$$

$$H_{i1}S_0 = \frac{\ln(p - \ln q + \ln(1-q) - \ln(1-p))}{\gamma(u-d)}$$
From equations (11) and (12) we can readily derive the following proposition:

**PROPOSITION 1:** Assume that an individual can short sell a risky asset or borrow from the bank account without any limit. We can obtain the following results based on the binomial model with CARA (constant absolute risk averse) utility function:

(a) Unless \( q < p \), the individual will short sell the risky asset, i.e., \( H_{0,0} S_0 < 0 \).

(b) The amount of money invested in the risky asset (long or short) is independent of the individual’s initial wealth.

(c) The greater his initial wealth is, the greater amount of money that individual \( i \) will deposit in the bank account, i.e., \( \frac{\partial H_{0,0}}{\partial v_i} > 0 \). The optimal amount of money deposited in the bank account will become negative (\( H_{0,0} < 0 \), or say individual \( i \) borrows from bank) unless initial wealth \( v_i \) is greater than or equal to

\[
\hat{v} = \frac{1 + d}{\gamma(u - d)} \ln \frac{p}{q} + \frac{1 + u}{\gamma(u - d)} \ln \frac{1 - q}{1 - p} - \frac{\ln[(1 + r)\alpha]}{\gamma}
\]

(d) The greater \( p \) is, the greater the amount of money is that will be invested in the risky asset (i.e., \( \frac{\partial (H_{1,0})}{\partial p} > 0 \)), and the less money that will be saved in the bank account (i.e., \( \frac{\partial H_{0,0}}{\partial p} < 0 \)). Moreover, a greater degree of risk averse (i.e., greater \( \gamma \)) will result in a smaller position in the risky asset investment (whether long or short).

In the absence of capital market imperfection, all individuals choose to invest the same amount of risky assets \( H_{1,0} S_0 = \frac{1}{\gamma(u - d)} \ln \frac{p(1 - q)}{q(1 - p)} \), no matter what the distribution of initial wealth will be. The reason is that the opportunity cost of investing in risky assets can be gauged by the comparison of the actual probability of good event \( p \) with the one occurring in a fair game as measured by the risk neutral probability \( q \). If \( p \) is relatively greater than \( q \), then the individuals will allocate a greater amount of investment in the risky assets. This amount of investment is a function of \( p, q \) and the degree of individuals’ risk aversion \( \gamma \), but has nothing to do with initial wealth. All individuals wish to invest up to this optimal point and those who have net savings above this level lend (i.e., deposit in the bank), while those who save below it borrow (i.e., have a negative bank account).

Since the accumulated wealth equals the consumption at the end of time 1
(i.e., $W_i(\overline{w}_1) = C_{i0}(\overline{w}_1), W_i(\overline{w}_2) = C_{i1}(\overline{w}_2)$), we can examine the impact of a security market on the amount of consumption in each period and the final wealth distribution among individuals based on equations (8), (9) and (10). It can be shown that:

**PROPOSITION 2:** Assume that an individual can short sell a risky asset or borrow from the bank account without any limit.

(a) The existence of a security market will narrow the variance of wealth distribution by a factor of $(1 + r)^2$. In other words, $rac{\text{Var}(W_i)}{\text{Var}(v_i)} = \left(\frac{1 + r}{2 + r}\right)^2$ no matter whether $\overline{w} = \overline{w}_1$ or $\overline{w} = \overline{w}_2$. Note that the variances above are taken with respect to the initial wealth of the whole population by assuming all the other parameters, i.e., $p, q, u, d, \alpha, \gamma$ & $r$, to be given.\(^5\)

(b) A greater initial wealth ($v_i$) will bring about greater consumption in either period. Usually an individual $i$ will have a positive net savings in period 0 (i.e., $C_{i0} \leq v_i$), unless his initial wealth is sufficiently small so that $v_i < \tilde{v} \equiv 1 + (1 - q)\ln \frac{1 - q}{1 - p} + q\ln \frac{p}{q} - \ln[(1 + r)\alpha]]$, and will result in negative savings.

(c) The impact of $q$ on consumption can be shown as follows: If $q < p$ (a long position in the risky asset will be taken), then a greater $q$ (that is, $u$ is closer to $1 + r$ or $d$ is relatively smaller than $1 + r$) will imply a smaller $C_{i0}$ and $C_{i1}(\overline{w}_1)(= W_i(\overline{w}_1))(i.e., \frac{\partial C_{i0}}{\partial q} < 0 \& \frac{\partial C_{i1}(\overline{w}_1)}{\partial q} < 0)$. If $q > p$ (a short position in the risky asset will be taken), then a greater $q$ will always imply a greater $C_{i0}(i.e., \frac{\partial C_{i0}}{\partial q} > 0)$, but lead to a smaller $C_{i1}(\overline{w}_1)(= W_i(\overline{w}_1))$ except when $\ln \left[\frac{q(1 - p)}{p(1 - q)}\right] > 1 - p + \frac{2 + r - q}{pq}$, then $\frac{\partial C_{i1}(\overline{w}_1)}{\partial q} > 0$. As for the impact of $q$ on $C_{i1}(\overline{w}_2)(= W(\overline{w}_2))$, a greater $q$ will imply a greater $C_{i1}(\overline{w}_2)$ except when $q$ is so small that $\frac{2 + r}{1 - q} < \ln \frac{p}{q} + \ln \frac{1 - q}{1 - p}$, then $\frac{\partial C_{i1}(\overline{w}_2)}{\partial q} < 0$.

The amount of an optimal bank account deposit is determined according to the inter-temporal comparison of the marginal utility of consumption between period 0

\(^5\) As mentioned in the introduction, we use the relative change (or ratio) of wealth distribution to measure the income distribution resulting solely from capital investment. Therefore, the term $\frac{\text{Var}(W_i)}{\text{Var}(v_i)}$ can be visualized as an alternative measurement of income inequality.
and period 1. Ignoring the complication of a risky asset whose amount has been derived in equation (12) from the no-arbitrage condition, we are virtually dealing with the following problem:

\[ \text{Maximize } u(C_{t0}) + au[(v_i - C_{t0}) \cdot (1 + r)] \]

The first-order condition becomes \( u'(C_{t0}) = au'[(v_i - C_{t0}) \cdot (1 + r)] \cdot (1 + r) \). Therefore,

\[ -\gamma \cdot e^{-\gamma C_{t0}} = \alpha(-\gamma)e^{-\gamma(v_i - C_{t0})(1 + r)}(1 + r) \]

which implies

\[ C_{t0} = \frac{v_i(1 + r)}{2 + r} - \frac{\ln[\alpha(1 + r)]}{\gamma(2 + r)} \]

The net savings after period 0 consumption is thus

\[ v_i - C_{t0} = \frac{v_i}{2 + r} + \frac{\ln[\alpha(1 + r)]}{\gamma(2 + r)} \]

Basically, \( \ln[\alpha(1 + r)]/\gamma \) in the second term above reflects the time value of a one-dollar risk-free investment in the second period. After correcting the time value difference in two periods, the amount of period 0 consumption \( C_{t0} \) should equate with period 1 consumption \( C_{t1} = (v_i - C_{t0})(1 + r) \). In other words, if one dollar's value in period 1 were equal to one dollar's value in period 0, or \( \alpha(1 + r) = 1 \), then the initial wealth should have been allocated into period 0 consumption and net savings by the weights of \( \frac{1 + r}{2 + r} \) and \( \frac{1}{2 + r} \), respectively. Once the factor of time value difference is taken into account, the net savings in period 0 (i.e., \( v_i - C_{t0} \)) should be added (while the period 0 consumption should be subtracted) by the amount \( \frac{\ln[\alpha(1 + r)]}{\gamma(2 + r)} \). In sum, the optimal size of a bank account \( H_{t0} \) as shown in equation (11)) can be derived by subtracting the investment in the risky asset \( H_{t1}S_0 \) as shown in equation (12)) from the net savings above \( (v_i - C_{t0}) \).

The time 1 wealth \( W_i(w_1) \) or \( W_i(w_2) \) for individual \( i \) consists of two parts: bank account investment \( H_{t0} \) times \( (1 + r) \), and security investment \( H_{t1}S_0 \) times \( u \).
(where \( \bar{w} = \bar{w}_1 \) or \( d \) (where \( \bar{w} = \bar{w}_2 \)). Therefore, the change in time 1 wealth among individuals is only a fraction \( \frac{1+r}{2+r} \) of the change in initial wealth (see equations (9) and (10)). The resulting reduction of wealth disparity from security transactions is attributed to the independence of \( H_{t1}S_0 \) from an individual's initial wealth. When \( q < P \) (bull market), \( H_{t1}S_0 > 0 \). The investment in risky assets will account for a greater proportion of initial wealth for the poor than for the rich. When \( q > p \) (bear market), \( H_{t1}S_0 < 0 \). Poor investors can shy away from the bad investment by a greater proportion of wealth than do the rich ones.

4. IMPERFECT SECURITY TRANSACTION AND INCOME INEQUALITY

If each individual can freely access or exit from the security market or borrow from a bank without any credit or income constraint (i.e., if capital markets are perfect), then he will eventually acquire or sell short a fixed amount of risky assets regardless of his initial wealth. The size of his initial wealth will matter only in the determination of initial consumption and the amount of the bank account. Over time the absolute level of an individual's wealth may differ depending on whether a 'good' or 'bad' return occurs to the security investment in the next period. However, the income inequality among individuals in the second period is solely determined by the difference in their bank account balances. Since the latter will not vary as much as initial wealth across the population, the variation of wealth will shrink in the end as shown in proposition 2.

We wonder if the imposition of any restriction on bank loan borrowing or on a security's short sale (i.e., if capital markets are imperfect) will invalidate the above conclusion. Since this paper focuses on the role of a security market in the determination of income inequality, we skip the discussion of any security market with no transactions after imposing a specific restriction (for instance, when \( q > p \) is anticipated by every investor, but a short sale is precluded by the government). In the beginning we examine what will happen if a negative bank account is prohibited while a short sale of securities is allowable.
As shown in proposition 1, an investor will have a negative bank account when
\[ v_i < \hat{v} \equiv \frac{1 + d}{\gamma(u - d) - \ln \frac{p}{q} + \frac{1 + u}{\gamma(u - d)}} \ln \frac{1 - q}{1 - p} - \frac{\ln((1 + r)\alpha)}{\gamma}. \]
Hence, we assume that \( H_{0,i} = 0 \) for investors with \( v_i \) less than \( \hat{v} \) (while \( H_{0,i} \) will remain the same as equation (11) when \( v_i \geq \hat{v} \)) in the consequence of the borrowing restriction. For those low-income individuals whose initial wealth is less than \( \hat{v} \), they face the following problem:

**Maximize**

\[ E\left[u(C_{0}) + au((v_i - C_{0}) \cdot (1 + R))\right] \]

We have here substituted the self-financing constraint that \( W_i = C_{i1} = (v_i - C_{0}) \cdot (1 + R) \) into the argument of the time 1 utility above. The first-order condition for the problem becomes

\[ E[u'(C_{0}) - \alpha \cdot u'((v_i - C_{0}) \cdot (1 + R)) \cdot (1 + R)] = 0 \]

or

\[ 1 = E\left(\frac{au'((v_i - C_{0}) \cdot (1 + R))(1 + R)}{u'(C_{0})}\right) \quad (13) \]

Because \( 1 = E_Q \left(\frac{1 + R}{1 + r}\right) \) according to equation (3'), comparing with equation (13) will yield the risk-neutral probability measure in this problem as

\[ Q(\bar{w}) = \frac{P(\bar{w}) \cdot (1 + r) \cdot \alpha \cdot u'((v_i - C_{0}) \cdot (1 + R(\bar{w}))]}{u'(C_{0})} \]

which implies

\[ q = \frac{p(1 + r)\alpha e^{-\gamma C_0 u}}{\gamma e^{-\gamma C_0 u}}, \quad C_{0} = \frac{v_i u + \frac{1}{1 + u} \cdot \ln \left[\frac{q}{p(1 + r)\alpha}\right]}{1 + u}. \]

Therefore, the investment in a risky asset equals

\[ H_{1,i} = v_i - C_{0,i} = C_{0,i} = \frac{v_i}{1 + u} - \frac{1}{(1 + u)\gamma} \cdot \ln \left[\frac{q}{p(1 + r)\alpha}\right], \quad \text{and time 1 wealth becomes} \]

\[ W_i(\bar{w}_1) = (v_i - C_{0,i})u = \frac{v_i u}{1 + u} - \frac{u}{(1 + u)\gamma} \cdot \ln \left[\frac{q}{p(1 + r)\alpha}\right]. \]
or \( W_i(w_2) = (v_i - C_i) d = \frac{v_i d}{1 + d} - \frac{d}{(1 + d) \gamma} \cdot \ln \left( \frac{1 - q}{(1 - p)(1 + r) \alpha} \right) \).

Depending on the relationship of \( p \) and \( q \) and whether a bear or bull market occurs, we can depict the relationship between period 1 wealth \( W_i(w_1), W_i(w_2) \) and initial wealth \( v_i \) as follows:

**Figure 1** Wealth Change with Borrowing Constraint When \( q < p \) and \( w = w_1 \)

**Figure 2** Wealth Change with Borrowing Constraint When \( q < p \) and \( w = w_2 \)

**Figure 3** Wealth Change with Borrowing Constraint When \( q > p \) and \( w = w_1 \)

**Figure 4** Wealth Change with Borrowing Constraint When \( q > p \) and \( w = w_2 \)
Note that $\hat{W}(\overline{w}_1) = \frac{u(1-q)}{\gamma(1+u)} \ln \left( \frac{p(1-q)}{q(1-p)} \right)$, which is greater than zero when $q < p$ and less than zero when $q > p$. Moreover, $\hat{W}(\overline{w}_2) = -\frac{dq}{\gamma(1+d)} \ln \left( \frac{p(1-q)}{q(1-p)} \right)$, which is less than zero when $q < p$ and greater than zero when $q > p$. The slope of the straight line connecting any two points in the figures above is always less than one. Hence, the variance of wealth among individuals will decrease.

Confronting the borrowing constraint, poor investors are forced to readjust their consumption investment choices. They wish to invest in the stock market up to the point where the marginal utility from a one-dollar consumption in period 0 equals the expected marginal utility from a one-dollar security investment. The criterion is determined by the comparison of the expected time value of a one-dollar risk-neutral investment, i.e., $p(1+r)\alpha$, with the fair market value for the security, i.e., $q$. If the expectation of a security market is bullish (i.e., $u$ or $d$ is large relative to $1+r$, or equivalently, $q$ is relatively small), then the investors are willing to allocate a larger amount of wealth in the security investment. The excessive time value of a one-dollar security investment in period 1 over a one-dollar value in period 0 can be measured by $\frac{1}{\gamma} \ln \left( \frac{p(1+r)\alpha}{q} \right)$, which is independent of investors’ initial wealth.

Analogous to the discussion of the consumption and bank account investment problem in page 14, the remaining wealth after correcting for the time value difference from the security investment will be allocated into period 0 consumption and security investment by the weights of $\frac{u}{1+u}$ and $\frac{1}{1+u}$, respectively. It is the constant term $\frac{1}{\gamma} \ln \left( \frac{p(1+r)\alpha}{q} \right)$ that induces the poor investors to allocate a greater proportion of their wealth in security investment when the market is bullish and a smaller proportion when the market is bearish than for rich investors. As a result, the wealth disparity will improve.

If all investors are prohibited from short selling, then it suffices to analyze the case of $q > p$. (When $q < p$, no short sale position will be taken. The restriction on a short sale will thus become redundant.) However, the restriction on a short sale at the time when every investor is pessimistic about the market (i.e., when $q > p$) will eventually evacuate the security transaction no matter whether the bank loan is
available or not. Therefore, if we would ever consider the short sale restriction that had a real impact on income inequality, it must have been the one that is confined only to a portion of investors.

To limit any default risk, a financial intermediary (bank or security brokerage) will usually grant a bank loan or short sale transaction on the base of an investor's credit. Accordingly, poor investors will be more likely subject to the bank loan or short sale restriction than rich investors. In the following we assume that an individual who cannot afford positive savings at the end of period 0 (and therefore no credit is available) will be deprived of the right of a bank loan or short sale. In the case of $q < p$, no short selling will be taken. The variance of wealth will diminish from the security transaction even if we impose the bank loan restriction as explained in the beginning of this section.

When $q > p$, a short sale is desirable. According to proposition 2, an individual whose initial wealth $v_i < \tilde{v} \equiv \frac{1}{\gamma} \{ (1 - q) \ln \frac{1 - q}{1 - p} + q \ln \frac{q}{p} - \ln [(1 + r)\alpha] \}$ will have negative savings in period 0. Hence, the ban on a bank loan and short selling owing to a credit constraint will induce those individuals whose initial wealth is below $\tilde{v}$ to consume up all their wealth in period 0 and leave nothing for investment in either the bank account or security. The relationship between initial wealth and period 1 wealth can be described in the following figures:

**Figure 5**  Credit constraint on bank loan & short sale with $q > p$

& \( \varpi = \varpi_1 \)

**Figure 6**  Credit Constraint on Bank Loan & Short Sale with $q > p$

& \( \varpi = \varpi_2 \)
Here, $\tilde{W}(\bar{w}_1) = -\frac{1-q}{\gamma} \ln \left[ \frac{q(1-p)}{p(1-q)} \right] < 0$, and $\tilde{W}(\bar{w}_2) = \frac{q}{\gamma} \ln \left[ \frac{q(1-p)}{p(1-q)} \right] > 0$.

$$v^* = \frac{1}{\gamma} \left\{ (3+r)q \ln \frac{q}{p} + [(3+r)q - 1] \ln \frac{1-p}{1-q} - \ln((1+r)\alpha) \right\}$$

It should be noted that for those individuals whose initial wealth is below $v^*$, their final wealth may differ in a greater degree than their initial wealth as illustrated by the steep straight line connecting points $A$ and $B$ with the slope greater than one.

We can summarize the discussion above in the following proposition:

**PROPOSITION 3:** A security transaction will smooth income inequality if no credit constraint is imposed on investors. Neither the prohibition on bank loans nor a short sale restriction can enlarge wealth disparity as long as the restriction is faithfully imposed on every investor. However, if an individual is not allowed to borrow from a bank or short sell his securities due to his credit constraint, then we may observe an increase in wealth disparity among those individuals with initial wealth less than $v^* \equiv \frac{1}{\gamma} \left\{ (3+r)q \ln \frac{q}{p} + [(3+r)q - 1] \ln \frac{1-p}{1-q} - \ln((1+r)\alpha) \right\}$. The latter will occur when $q > p$ is anticipated (so a short sale is desirable, but not available, for those credit-shortage investors with $v_i < \tilde{v}$) and the windfall of a short sale occurs to wealthy investors in the bear market (i.e., $\bar{w} = \bar{w}_2$).

The proposition that the creation of a security market would diminish income equality among investors as long as one could forestall the partial execution of a short sale restriction on poor investors in a bear market sounds not quite congruous with our actual observation. The benign impact of a security market on the income equality above is essentially built on the assumption that all investors possess the same estimation of the parameters that characterize their risky investments. In particular, the probability of a bull market ($p$) and the investment return in a risky asset relative to a bank account as represented by the risky neutral probability ($q$) are assumed identical for all investors. In reality, investors have diverse opinions about $p$ and $q$ (or $u$ and $d$ relative to $1+r$). Once we take into account this information asymmetry about $p$ or $q$, the total variance of wealth among investors will substantially increase.
If we assume that the means and variances of \( p \) and \( q \) are \( \mu_p, \mu_q \) and \( \sigma_{p}^{2}, \sigma_{q}^{2} \) respectively, then we can derive the total variance of wealth (denoted by \( \sigma_{W}^{2} \)) from equations (9) and (10) as follows:

(1) When \( \bar{w} = \bar{w}_1 \),

\[
\sigma_{W}^{2} = \left( \frac{1 + r}{2 + r} \right)^{2} \sigma_{v}^{2} + \left[ \frac{2 + r - q}{(2 + r)\gamma} \right]^{2} \cdot [\text{var}(\ln p) + \text{var}(\ln q)] + \left[ \frac{1 - q}{(2 + r)\gamma} \right]^{2} \cdot [\text{var}(\ln(1 - p)) + \text{var}(\ln(1 - q))] + \frac{1}{[(2 + r)\gamma]^{2}} \left[ \ln \frac{p(1 - q)}{q(1 - p)} \right]^{2} \cdot \sigma_{q}^{2}
\]

(14)

(2) When \( \bar{w} = \bar{w}_2 \),

\[
\sigma_{W}^{2} = \left( \frac{1 + r}{2 + r} \right)^{2} \sigma_{v}^{2} + \left[ \frac{q}{(2 + r)\gamma} \right]^{2} \cdot [\text{var}(\ln p) + \text{var}(\ln q)] + \left[ \frac{1 + r + q}{(2 + r)\gamma} \right]^{2} \cdot [\text{var}(\ln(1 - p)) + \text{var}(\ln(1 - q))] + \frac{1}{[(2 + r)\gamma]^{2}} \left[ \ln \frac{p(1 - q)}{q(1 - p)} \right]^{2} \cdot \sigma_{q}^{2}
\]

(15)

Here, \( \sigma_{v}^{2} \) stands for the variance of initial wealth among individuals. We assume that \( v, p \) and \( q \) are mutually independent in the above derivation. As for the variances of \( \ln p, \ln q, \ln(1 - p) \) and \( \ln(1 - q) \), they can be calculated by applying the following ‘certainty equivalence’ property:

\[
E[G(x)] = G[E(x) - \pi], \text{ where the certainty equivalent } \pi \text{ can be approximated as } \pi \approx -\frac{\sigma_{x}^{2} \cdot G''[E(x)]}{2G'[E(x)]}
\]

Therefore,
\[
\text{var}(\ln p) = E[(\ln p)^2] - [E(\ln p)]^2
\]

\[
\approx \left[ \ln(p) + \frac{\sigma_p^2(1 - \ln(p))}{2\ln p} \right]^2 - \left[ \ln(p) - \frac{\sigma_p^2}{2p} \right]^2
\]

\[
\approx \frac{\sigma_p^2}{p - \sigma_p^2/2} \left[ \frac{\ln(p^2 - \sigma_p^2/2)}{\ln p} - 1 \right]
\]

Likewise, \( \text{var}(\ln q) \approx \left[ \ln(q) + \frac{\sigma_q^2(1 - \ln(q))}{2\ln q} \right]^2 - \left[ \ln(q) - \frac{\sigma_q^2}{2q} \right]^2
\]

\[
\approx \frac{\sigma_q^2}{q - \sigma_q^2/2} \left[ \frac{\ln(q^2 - \sigma_q^2/2)}{\ln q} - 1 \right]
\]

\[
\text{var}[\ln(1 - p)] \approx \left[ \ln(1 - p) + \frac{\sigma_p^2(1 - \ln(1 - p))}{2(1 - p)\ln(1 - p)} \right]^2 - \left[ \ln(1 - p) - \frac{\sigma_p^2}{2(1 - p)} \right]^2
\]

\[
\approx \frac{\sigma_p^2}{(1 - p)^2 - \sigma_p^2/2} \left[ \frac{\ln((1 - p)^2 - \sigma_p^2/2)}{\ln(1 - p)} - 1 \right]
\]

\[
\text{var}[\ln(1 - q)] \approx \left[ \ln(1 - q) + \frac{\sigma_q^2(1 - \ln(1 - q))}{2(1 - q)\ln(1 - q)} \right]^2 - \left[ \ln(1 - q) - \frac{\sigma_q^2}{2(1 - q)} \right]^2
\]

\[
\approx \frac{\sigma_q^2}{(1 - q)^2 - \sigma_q^2/2} \left[ \frac{\ln((1 - q)^2 - \sigma_q^2/2)}{\ln(1 - q)} - 1 \right]
\]

An increase in \( \sigma_p^2 \) or \( \sigma_q^2 \) will raise \( \text{var}(\ln p) \) and \( \text{var}[\ln(1 - p)] \) or \( \text{var}(\ln q) \) and \( \text{var}[\ln(1 - q)] \), adding the variance to \( \sigma^2_W \). Accordingly, wealth inequality may deteriorate (i.e., \( \sigma^2_W > \sigma^2_W \)) in spite of the diminishing variance contribution from initial wealth (the first term in equation (14) or (15)). The policy implication from these derivations is obvious: a government should do its best to combat the roots of wealth variance, i.e., \( \sigma^2_p \) & \( \sigma^2_q \). It should cultivate an investment environment that is congenial for investors to fathom a homogeneous expectation about the investment outcome. For example, all financial statements and important news about companies have to be properly audited or monitored and disclosed from a credible source (e.g., SEC). Regulations against insider trading should also be strictly enforced.
Somewhere between unbridled capitalism and the welfare state there must be a more just and equitable economic system which provides genuine opportunities for all individuals, while preserving incentives for investment and growth. Widespread personal ownership of capital assets facilitated by the blossoming of such financial intermediaries as pension or mutual funds provides a way for the general public to partake in the fruitful investment return in the security market. In particular, those who own securities benefit disproportionately from a record-breaking inflow of funds (dominantly baby-boomer retirement savings) into the stock market. This has fueled a steady increase in share value far higher that what can be justified by underlying economic activity. It is doubtless that the rapid development of these institutional investors would be very constructive to disseminate truthful financial information to the public and harness the widening wealth gap. Nevertheless, we should be extremely cautious in preventing any unfair information disclosure among diverse economic groups from unraveling the system that has held our democracy and capitalism together for the last century.

5. EMPIRICAL STUDY

According to the model above, we conduct the following empirical analysis of how a security transaction affects the change of wealth distribution for each country. Since data is not available for wealth distribution across countries, we use the income inequality of each country as measured by the Gini coefficient to denote the change in wealth inequality. The Gini index measures the area between the Lorenz curve and the diagonal line in the figure with the vertical axis representing the proportion of total income and the horizontal axis representing the proportion of the country's population. Thus, a Gini index of zero represents perfect equality, while an index of 100 implies absolute inequality. As for the activity of security transactions for each country, it is measured by the turnover rate of the security transactions which is defined to be the total value of shares traded during the period divided by the average market capitalization for the period. A higher turnover rate reflects a deeper involvement of security transactions for the people in that specific
Data on income inequality comes from extensive compilation of a large panel of countries in Deininger and Squire (1996). These data are also available on the NBER web sites. Deininger and Squire provided both Gini coefficients and quintile shares. For the purpose of our study, we select the Gini coefficient data in Deininger and Squire (1996) for those countries that conducted a nationwide household survey from 1986 till 1990. In addition to wage income these datasets also include non-wage earnings — e.g., pensions and income from self-employment. Because we want to know the relationship between inequality and security market, we have to omit the countries whose security transaction data are absent during this period. The security transaction data comes from Emerging Stock Markets Factbook 1995 issued by the International Finance Corporation.

Our sample covers the period from 1986 to 1990 for which we have the most number of countries with their Gini coefficient data available. There are a total of 52 countries with some estimate of Gini coefficients in at least one of the five years in the period. Since not every country prepares an annual Gini coefficient estimate, we calculate the average Gini coefficients over the years when their Gini data are available. The average of the security turnover rate over the 1986–1990 period for these 52 countries is calculated as the explanatory variable (denoted as TUROV). We show the cross-country regression of Gini with respect to TUROV in Table 1 (equation 1). The resulting significant negative coefficient estimation agrees with our model (Proposition 2), where an active security transaction lessens the degree of income inequality.

In addition to TUROV, we consider several explanatory variables. In order to test the inverse-U hypothesis of Kuznets, we add two variables: LRGDP and LRGDP². The former is the logarithm of real GDP per capita and the latter is the square of LRGDP. These two variables can be calculated based on the World Bank Global Development Network Growth Database which is available in World Bank's Economic Growth Research Homepage. All of the equations in Table 1 confirm Kuznets' inverse-U hypothesis whereby economic development worsens income distribution at a decreasing rate.
Havighurst & Neugarten (1975) concluded that education is the primary force that affects the mobility of a society after examining the relationship of education level and social mobility for United States, Great Britain, Australia and Brazil. The study of Ward (1978) further showed that the more mobile a society is, the more even its income distribution will become. We cannot deny the role of education in the determination of wage difference among countries. Thereby, we choose the secondary school enrollment ratio (SSER), i.e., the ratio of total enrollment regardless of age to the population of the age group that officially corresponds to the level of secondary education, to reflect a country’s effort in enhancing its human resource. Secondary education completes the provision of basic education that began at the primary level and aims at laying the foundations for lifelong learning and human development, by offering more subject- or skilled-oriented instruction using more specialized teachers. SSER is available in World Bank’s Global Development Network Growth Database. The regression result in Table 3 conforms with our conjecture that education development contributes to income equality.

In Table 3 we also include the explanatory variables LEN-DEP, LEN-LIB and INF to account for the degree of imperfection in the capital market. According to Galor and Zeira (1993), a more perfect capital market leads to a lesser difference between the lending rate and deposit rate. When the gap of interest rates becomes wider as a result of imperfect information, the credit constraint on either loan borrowing or a security short sale will become more stringent, reducing the opportunity of realizing a short sale investment gain in bad time for the less wealthy people according to Proposition 3. Therefore, we would expect an opposite relationship between GINI and market imperfection. The terms LEN-DEP (lending rate minus deposit rate), LEN-LIB (lending rate minus LIBOR) and INF (percentage change of consumer price index) are three alternatives of measuring market imperfection. A country with higher inflation will suffer from greater instability in investment and result in a higher interest gap. The regressions in Table 3 (equations 5, 6 & 7) show that all the three variables have a positive impact on income inequality, congruous with Proposition 3. Since these three variables are highly correlated as seen in Table 2, only one of the three variables is added in the regression (equations 5, 6 & 7)
Table 3  Regression Analysis of Income Inequality and Security Transaction

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Notes:
GINI: Adjusted Gini Coefficient.
TUROV: The total value of shares traded during the year divided by the average market capitalization.
LRGDP: Logarithm of real GDP per capita.
LRGDP,2: Square of Logarithm of real GDP per capita.
SSER: Secondary school enrollment rate.
LEN-DEP: Lending rate minus deposit rate.
LEN-LIB: Lending rate minus 3-month LIBOR.
INF: Inflation rate (measured by the consumer price index).
FARM: Agricultural value added divided by GDP.
GPOP: Growth rate of population.
TRADE: Openness to trade. It is the sum of imports and exports divided by GDP.
at a time.

Term TUROV is negatively related to GINI at the 1% significance level whether LRGDP, LRGDP_2, SSER, LEN-DEP, LEN-LIB or INF are added or not. The latter contributes to the improvement of an adjusted R-square from 0.08 to around 0.4. The major improvement of an adjusted R-square stems from LRGDP and LRGDP_2. Since SSER highly correlates with LRGDP (0.78 as shown in Table 2), SSER has a nominal contribution of an adjusted R-square (from 0.44 to 0.46) once LRGDP and LRGDP_2 are included.

To see how robust the regression analysis in Table 3 is, we add three more explanatory variables to accommodate the impact of agricultural development (FARM), population growth (GPOP) and openness to trade (TRADE) in Table 4. We expect that an agricultural society has a more even income distribution than an industrial society. We can ascertain this relation from the negative coefficient estimate of FARM (as measured by the proportion of real GDP accounted for by agricultural value added) in the regression of GINI. Kremer and Chen (1999) argue that a country with a high population growth rate tends to have low education development (the correlation coefficient of SSER and GPOP is $-0.71$ shown in Table 2) and hence suffers from a deteriorating income distribution. The position impact of GPOP on GINI is assured from the regressions in Table 4 (significant at 5% level). Finally, as a result of trade's factor price equalization effect, we expect that a country more open to international trade (as measured by the proportion of total trade in GDP) will have its domestic wage and capital return closer to the world level. Hence, a great TRADE should lead to a reduction in GINI, which is demonstrated in Table 4 as well.\(^6\)

The regression equations 1, 2 & 3 of Table 4 have results significantly congruous with our model above. Since FARM and GPOP are highly correlated with LRGDP and LRGDP_2, some coefficient estimates in equations 4, 5 & 6 are no longer as assignificant as before. Nevertheless, the negative impact of TUROV on GINI

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\(^6\) All of the variables LEN-DEP, LEN-LIB, INF, FARM, GPOP and TRADE are available from World Bank's World Development Indicators CD-ROM and Global Development Network Growth Database. The five-year average (from 1986 to 1990) of each variable is used to run the regression in Table 3 and 4.
### Table 4 Robust Analysis

**Dependent variable: GINI**

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<tr>
<th>Equation</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
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<td>(0.0395)</td>
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**Notes:**

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FARM: Agricultural value added/GDP.
LEN-LIB: Lending rate minus 3-month LIBOR.
GPOP: Growth rate of population.
INF: inflation rate (% change of consumer price index).
TRADE: (imports + exports)/GDP.

INCOME INEQUALITY AND SECURITY MARKET
always holds (significant at the 1% confidence level). In sum, we can robustly conclude that a country's income distribution will be more even as its security transaction becomes more prevalent.

6. CONCLUSION

The security market provides an individual with an opportunity to enrich his inter-temporal portfolio. When an investor, whether rich or poor, is sanguine about the investment prospect (more specifically, the odds of an up-movement relative to risk-free rate are greater than the ones of a down-movement, or $q < p$), he will abate his consumption and save some fraction of his initial wealth for investment. If we assume no subsistence level of consumption (as the consequence of the CARA utility function) and no credit constraint imposed on the poor, the average saving propensity for the poor will be greater than the rich, resulting in an improvement in the wealth inequality.\(^7\) If the investor is pessimistic about the security return, then he would rather increase his current consumption than save it. For those individuals whose wealth is less than a certain baseline, they virtually live from hand to mouth. The curb on bank loans and short sales due to their credit shortage will deprive them of the opportunity of wealth accumulation in the next period. However, individuals whose wealth exceeds the baseline will be eligible for short selling and accumulate wealth once the security market happens to be bearish (of course, their wealth will shrink if the market turns up bullish instead). Therefore, we may observe an enlarged income gap in the bear market when a bank loan and a short sale are prohibited for poor investors while short selling in the security market is permissible for the rich.

We cannot simply count on the argument that the forbearance from borrowing and short selling for the poor, but the allowance of a short sale windfall for the

\(^7\) The optimal amount of security investment will be the same under the CARA preference framework. When the market is sanguine, the poor will reduce his (or her) consumption even to a trivial amount. If necessary, he (or she) will borrow on credit to grasp the window of opportunity. However, if we impose the assumption of a subsistence level of consumption and a credit constraint on the borrowing, then the average savings rate for the poor may be less than the rich.
wealthy in the bear market, is the unique window of a widening wealth gap from the security transaction. It is too strong an assumption that all individuals maintain an identical conjecture on all the parameters of investment in order to reach the above conclusion. This study raises the concern that the heterogeneity of investors’ beliefs about investment performance is the culprit for income inequality engendered by the security market. The problem is aggravated by the looming conflict of interests between shareholders and managers or between shareholders and bondholders. One should be alarmed when the rich are more likely to maintain the information advantage over the poor, in particular when the market for information acquisition is not perfect.

The empirical study across countries corroborates the constructive role played by the security market in mitigating the aggravation of income disparity between rich and poor countries. The percentage of secondary school enrollment is another significant factor in narrowing the income inequality among countries. Analogous to the advocation of free primary school education (by Marx for example), we proclaim that every human being should inherit the basic right of being informed either through a formal education system or a public disclosure mechanism with appropriate government supervision.

There should be no center or periphery in the process of knowledge learning or information gathering. The system would be deeply flawed as long as capitalists remain triumphant in their pursuit of a better investment opportunity. It seems that our current system is very favorable to financial capital, which is free to go where it is best rewarded, which in turn has led to the rapid growth of global financial markets. However, the development of a global economy has not been matched by the development of a global society. Economic and political arrangements are out of kilter. Unless every person in every nation is placed on an equal footing for the right of being informed, then capitalism cannot avoid periodic turmoil and instability brought up by rampant income inequality.
REFERENCES


47–64.


所得分配與證券市場的關係研究

胡聯國*

摘要

目睹私人企業的快速興起以及私人財富所有權的大量集中，我們亟思此一所有權結構之變化是否與我們的證券市場有著因果之關係。建構在兩期、兩投資標的投資消費模型上，我們得出以下的結論：一般而言，證券市場的存在會減少貧富的差距。可是當市場看空時，如果較窮的投資者由於其信用之限制無法賣空，而較富有的投資者可以不受限制地恣意放空，則此時證券的買賣有可能助長貧富的差距。另一種例外的現象是值因於嚴重的資訊不對稱，分歧的市場榮枯看法亦會加深所得分配之不均。因此政府有必要規劃推動一個更為公平、公正、公開的市場交易環境。最後跨國的實證亦支持本研究之結論。

關鍵詞：所得分配不均、證券市場、風險中立之機率測度、信用限制、看法歧異性

* 國立政治大學國貿系教授及國立高雄應用科技大學教授。