INFLATION AND ECONOMIC GROWTH IN FINANCIAL MARKETS WITH ADVERSE SELECTION AND COSTLY STATE VERIFICATION

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ABSTRACT

The relationship between inflation and economic growth is examined in a model with adverse selection and costly state verification problems. The model shows that the interaction between both problems plays a crucial role in determining this relationship. Results are consistent with recent empirical findings whereby an increase in inflation lowers economic growth if initial inflation rates are high, but such an increase raises economic growth if initial inflation rates are low.

Keywords: Inflation, Growth, Adverse selection, Costly state verification, Externality

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1. INTRODUCTION

The relationship between inflation and economic growth found in the literature is not always in agreement. More than three decades ago, theorist economists [for example, Mundell (1965) and Tobin (1965)] postulated that the output level and capital labor ratio are positively correlated with the inflation rate in the steady state. Nonetheless, Sidrauski (1967) and Brock (1974) suggested that inflation has no effect on the steady state output level and capital labor ratio. To clarify this theoretical ambiguity, one needs to resort to an empirical investigation. Unfortunately, early empirical studies on this regard are as ambiguous as the theory. For instance, Johnson (1967) found that there is no conclusive relationship between inflation and economic growth.

The empirical findings of this relationship have dramatically changed in recent years. This change comes from the fact that many countries experienced severe episodes of high inflation in the 1970s and 1980s, which seldom existed in the 1950s and 1960s. By including those data, recent empirical studies [as in De Gregorio (1992, 1993) and Barro (1995)] suggested a negative relationship between inflation and economic growth. High inflation apparently raises a possibility that economic growth could be negatively correlated with inflation. In particular, Bullard and Keating (1995) further found a nonlinear relationship between inflation and economic growth, namely that this relationship is positive for countries with low initial inflation rates, but negative for countries with high initial inflation rates. As a result, recent empirical evidence indicates that high (low) growth is associated with low (high) inflation. Obviously, these empirical findings are not captured by most existing literature and, as pointed by Azariadis and Smith (1996), searching for theoretical explanations for these empirical findings is a real challenge.

This paper's purpose is to construct a model that may account for these empirical findings. The model is a simplified version from Boyd and Smith (1993).
who propose a framework with adverse selection and costly state verification problems and examine how both problems interact and their effects on capital investment. Economists have long recognized that financial markets are characterized with a wide variety of informational imperfections and have realized that such imperfections may give rise to adverse selection and costly state verification problems. Nonetheless, to date scant attention has been paid to demonstrate that both problems could be related and that this relation may have significant effects on capital investment.

To explore these issues in a monetary growth model, this paper adds a costly state verification problem and fiat money into the adverse selection model of Bencivenga and Smith (1993). In this framework, fiat money, introduced by government transfers to all old agents, is one of the portfolio choices of lenders. Alternatively, each lender can costlessly establish a bank and offer contracts to borrowers and, of course, lending to borrowers is subject to both adverse selection and costly state verification. A standard condition of no arbitrage and competition among lenders ensure that the rate of return from lending is equal to that from holding money.

As in Bencivenga and Smith (1993), separating the equilibrium in the adverse selection problem has a feature that a borrower's loan application will be rejected with non-negative probability; in other words, the number of loans made to the borrower in financial markets as a whole is subject to an adverse selection problem. Furthermore, following the strategy of Becsi et al. (1999) I propose a general structure of banking verification costs in which verification costs per loan is a function of the number of total loans made to the borrower. This assumption is justified by the fact that there may exist externalities in verifying the outcome of borrowers' investment projects. Given this assumption, there is an interaction between adverse selection and costly state verification problems as the number of loans made to the borrower and the cost of verification per loan are related and jointly determined. This paper's objective is to examine how the money growth rate influences this interaction and then economic growth.

Under this framework, a unique equilibrium exists along a balanced growth path if banking verification costs display no externality. In this case, the Mundell–Tobin effect holds for the relationship between inflation and economic growth. However,
if banking verification costs display economies of scale or positive externalities, then there are two equilibria in which one is characterized with low economic growth and high inflation, while the other has high economic growth and low inflation. Furthermore, comparative-static analyses show that an increase in the money growth rate will lower economic growth under low growth and high inflation equilibrium, whereas such an increase will raise economic growth under high growth and low inflation equilibrium. These results confirm recent aforementioned empirical findings. Moreover, effects from changing some parameters on the number of projects financed and economic growth are examined.

The rest of this paper proceeds as follows. Section 2 presents the model's environment and Section 3 describes the operations of financial markets and then derives equilibrium contracts. Section 4 characterizes equilibrium consequences. Comparative-static analyses are performed in Section 5 and Section 6 concludes the paper.

2. MODEL

The model's environment is modified from Bencivenga and Smith (1993). The economy's population is an infinite sequence of two-period lived overlapping generations. Time is discrete and indexed by $t = 0, 1, 2 \cdots$. Each generation is identical in composition and size, and contains two kinds of agents in equal size: lenders and borrowers. Each young agent is endowed with a unit of labor. For simplicity, we normalize the population of lenders and borrowers in each generation to one, respectively.

Lenders only value their old-period consumption. A young lender at $t$ can sell his labor to firms and earn a real wage rate $w_t$, which can be lent to the young borrower. Alternatively, each lender can sell his wage rate to the old forfait money, which is introduced by way of government transfers to all old agents at each period's beginning. To lend to the young borrower, we assume that any lender can costlessly establish a financial intermediary (or in short, a bank) and offer contracts to the intended borrower. We also follow Bencivenga and Smith (1993) by assuming that
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competition among lenders ensures that all gains from trade accrue to borrowers. Given this, it should be clear that the return from lending to the young borrower must equal the return from holding money.

Borrowers are classified as two types: Type L and Type H. A $\lambda$ fraction of borrowers is Type H. With probability $p_i, i = H, L$, the investment project operated by a Type $i$ borrower can convert $x$ units of time $t$ consumption good (with his own labor) into $Qx$ units of time $t + 1$ capital. With probability $1 - p_i$, the project fails and produces nothing. In this event, the borrower claims bankruptcy at the time of repayment. Assuming that $1 \geq p_L > p_H \geq 0$, Type L borrowers are low-risk. The type of borrowers are private information though all agents know the distribution. Furthermore, as borrowers' capital technology is a linear one, a maximal scale is needed to limit the loan's size. Following Bencivenga and Smith (1993), we assume that this maximal scale at time $t$ is equal to the wage rate (that is, $w_t$).

Borrowers only care about consumption in the old period. Thus, if a young borrower sells his labor to firms and earns $w_t$ (when he is rejected with credit), then he will store this wage rate for old-age consumption. One unit of time $t$ output stored by a Type $i$ borrower yields $\beta_i \leq 1$ units of time $t + 1$ output. For simplicity, I assume that $\beta_L > \beta_H = 0$.

Any borrower whose project succeeds operates a firm in the last period of his life. The firm operator can rent capital in positive or negative amounts and hire labor to produce output according to the following technology

$$y_t = \psi_t^\eta k_t^\sigma L_t^{1-\sigma}$$

where $k_t$ and $L_t$ are respectively the amount of capital and labor employed by each firm and $\psi_t$ is the average per firm capital stock. Capital depreciates fully after production, and since each firm will employ the same amount of capital in

2 In fact, all we need is that this maximal scale has to tie the economy's current capital stock. We may also assume a capital production technology with decreasing returns to scale and allow the borrower to choose the size of loans. However, as in Bencivenga and Smith (1993), this paper focuses on adverse selection problems in which the amount of credit rationing is defined by the number of loans made to the borrowers (not the size). As a result, I maintain this assumption.
equilibrium, $\psi_{t+1} = k_{t+1}$. For simplicity, $\eta = 1 - \sigma$. Therefore, the production technology is linear as in the AK model. Since labor and capital markets are competitive, the rental rates of labor and capital are given as

$$w_t = (1 - \sigma)k_t^{\eta+\sigma}L^{-\sigma} = (1 - \sigma)k_tL^{-\sigma}$$

and

$$\rho_t = \sigma k_t^{\eta+\sigma-1}L^{-\sigma} = \sigma L^{-\sigma}$$

I also assume that a project's outcome is costlessly observable only to the borrower who operates it. This creates incentives for a borrower to claim bankruptcy, independent of his project's true realization. To learn the project's true outcome, any lender or bank can utilize costly-state-verification technology. It is well known that the optimal financial contracts in this context are characterized by verification in the event of default. The verification technology will absorb $\delta^b$ units of output per unit of loan made. As will be clear, each project utilizes the same maximal scale as input so that $\delta^b$ can be viewed as the verification cost per loan (per project). One may think that there may exist externalities if a bank monitors many borrowers. In other words, per loan verification cost could be related to the number of total loans made to borrowers. This paper's purpose is to consider a generalized structure of banking verification costs and examine how inflation influences financial market activities and banking verification costs so as to affect economic growth. The specific functional form of $\delta^b$ will be made explicitly later.

Money is issued by the government and distributed to the old agents as lump-sum transfers. Let $M_t$ denote the outstanding stock of fiat money at $t$ and $P_t$ denote the corresponding price level. The money supply evolves according to the following

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3 Each firm is symmetric.
4 It is assumed that any agent cannot enter the capital market and deduce who is operating a firm. Otherwise, any agent can know who is cheating without monitoring.
5 Thus, only non-stochastic verification is allowed. As claimed by Boyd and Smith (1994), the gains from stochastic verification are small.
rule

\[ M_{t+1} = \mu M_t, \mu > 0 \]  \hspace{1cm} (4)

In other words, money supply grows at an exogenous constant rate \( \mu - 1 \). Finally, at the first period’s beginning, that is \( t = 0 \), the government gives \( M_0 \) units of money to each initial old agent. Moreover, at the initial period, each old agent who operates a firm (consisting of a fraction \( f \in (0, 1] \) of all old agents) is endowed with \( k_0 \) units of capital.

3. FINANCIAL MARKETS AND EQUILIBRIUM CONTRACTS

Financial markets operate in the way described in Bencivenga and Smith (1993). At the beginning of each period, every bank announces contracts intended for each type of borrowers and if a bank’s offer is not dominated by others, then he is approached by borrowers.\(^6\) The equilibrium contract is defined as such that there is no incentive for any bank to offer alternative contracts, taking the rate of inflation, \( \rho_{t+1} \), and other banks’ offers as given.

The terms of contracts are similar to those derived in Bencivenga and Smith (1993), except that the contract will specify a non-stochastic verification when the borrower claims bankruptcy. As stated, competition among lenders ensures that all gains from trade accrue to borrowers and each lender can costlessly establish a bank. These two assumptions imply that each bank will earn zero profit. Thus, the loan rate to a Type \( i \) borrower is given as

\[ R_{i,t} = \frac{R_t^m + (1 - p_i)\delta^b}{p_i}, i = H, L, t \geq 1, \]  \hspace{1cm} (5)

where \( R_t^m \) is the return from holding money (the reverse of the inflation rate). With

\(^6\) Financial markets are closed after loan transactions are complete. Therefore, any Type L borrower who is rejected for credit and must work for the wage rate cannot lend that wage to the borrower. See Bencivenga and Smith (1993) for a discussion.
probability $1 - p_i$, the project fails, the borrower claims bankruptcy, and verification takes places. Moreover, competition also ensures that banks design contracts under which borrowers’ expected payoffs are maximized.

In separating equilibrium, banks offer incentive-compatibility contracts to separate borrowers as to their types. To this end, the bank offers a contract intended for Type H borrowers (denoted as $C^H$) and another one for Type L ($C^L$). To derive a self-selection, we need the situation where different types of borrowers have a different opportunity cost when they are being denied loans. As stated, if a young borrower at $t$ is rejected for credit (credit rationed), he can utilize his labor to work and store his wage at the old-period consumption. Since $\beta_L > \beta_H = 0$, this ensures that the Type L borrower has a lower opportunity cost. Given this, incentive-compatibility constraints imply that each borrower will prefer the contract corresponding to his type.

As in Benci venga and Smith (1993), the separation can be achieved by distorting the first best contract for Type L borrowers while offering Type H with their corresponding first best contract such that Type H borrowers are indifferent between accepting $C^H$ and $C^L$. Given the characterizations of Type H borrowers and linear technology in capital production, the first best contract for Type H borrowers will specify that the loan quantity equal $w_t$ and that the loan rate is given as in (5). The distortion of the contract intended for the Type L borrower is derived by rationing a fraction of borrowers who apply for this contract. As a result, the loan quantity in $C^L$ is $w_t$ and the loan rate is given in (5).

Denoting the probability a young Type L borrower derives credit in time $t$ as $\pi_t$, one sees that the expected payoff of a low-risk borrower is given as

$$\pi_t P_L q_L (Q p_{t+1} - R_{L,t}) + (1 - \pi_t) \beta_L w_t$$

(6)

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7 We assume that $\lambda$ is sufficiently large to ensure that a pooling equilibrium is not possible. See Benci venga and Smith (1993).
8 Conditions for this are similar to those specified in Benci venga and Smith (1993) and are ignored here.
9 As the first best contract, a Type H borrower always receives credit if he applies for $C^H$. 
where \( q_L \) is the size of loan in \( C^L \) and equal to \( w_t \). We further assume that \( \beta_L \) is sufficiently small (smaller than \( P_L(Q\rho_{t+1} - R_{L,t}) \)); thus, this expected payoff is an increasing function of \( \pi_t \). The bank maximizes (6) subject to (5) and the incentive-compatibility constraint of self-selection given as

\[
P_{HqH}(Q\rho_{t+1} - R_{H,t}) \geq P_{HqL}\pi_t(Q\rho_{t+1} - R_{L,t})
\]

(7)

where \( q_H \), the loan quantity in \( C^H \), is \( w_t \). The left-hand side of (7) is the expected payoff of a Type H borrower in revealing his true type while the right-hand side is the expected payoff of a Type H borrower when he pretends to be a Type L.

The bank obviously can always raise the expected payoff of Type L borrowers by increasing \( \pi_t \). Thus, (7) should hold as an equality. As a result, we have that

\[
\pi_t = \frac{p_Hw_t(Q\rho_{t+1} - R_{H,t})}{p_Hw_t(Q\rho_{t+1} - R_{L,t})} = \frac{p_L(p_HQ\rho_{t+1} - R_t^m - (1 - p_H)\delta^b)}{p_H(p_LQ\rho_{t+1} - R_t^m - (1 - p_L)\delta^b)}
\]

(8)

Note that \( \pi_t \) is the probability that a Type L borrower’s project is financed. As each borrower has the same characterizations and the population is normalized to one, \( \pi_t \) can be thought of as the total number of Type L projects financed in the financial markets as a whole. Thus, in aggregate one can think that each bank finances a total number of \( \pi_t \) Type L projects in the financial markets.

We are now in a position to specify the functional form of the verification cost per project (\( \delta^b \)). As stated, we consider the case where banking verification may display externalities. Specifically, the verification cost per project relates to the number of total projects financed by the bank, \( \pi_t \). Without loss of any generality, we assume that \( \delta^b \) is given as

\[
\delta^b = \delta(\pi_t) = \alpha\pi_t^\theta, \quad 0 \leq \alpha \leq 1, \quad -1 \leq \theta \leq 1
\]

(9)

where \( \alpha \) represents the technology level in verification.\(^{10}\) Equation (9) implies that

\(^{10}\) This functional form of verification cost is similar to the one proposed by Becsi et al. (1999). In their paper, Becsi et al. (1999) proposed a model in which the deposit in the banking system is
the verification cost per loan decreases as the total number of loans financed by the bank, \( \pi_t \), increases if \( \theta \in [-1, 0) \). The reverse relation holds if \( \theta \in (0, 1] \). In other words, the verification technology exhibits economies of scale if \( \theta \in [-1, 0) \) and diseconomies of scale if \( \theta \in (0, 1] \). No externality exists if \( \theta = 0 \).

The rationales for the assumption in (9) are given as follows. While we assume that the bank is costlessly established, the verification activities do incur costs and it is likely, though we do not assume explicitly, that a bank’s verification costs have significant fixed components such as office space, computers, and trained manpower. Given this, the bank’s verification costs per loan (per project) could decrease as the number of financed loans increase. This displays economies of scale and is captured by our assumption of \( \theta \in [-1, 0) \). It is also worth noting that the assumption in (9) for \( \theta \in [-1, 0) \) coincides with the argument of recent literature in finance-development nexus. Many studies of this literature [as in Saint–Paul (1992), Levine (1992), Bose and Cothren (1996)] have argued that the operations of financial markets incur fixed costs. Given this argument, these studies demonstrate that economic growth raises a need for firms to participate in financial markets and, given the fixed costs in maintaining financial markets, more firms entering the financial markets will reduce the participation cost (per firm) in these markets.

The case of economies of scale analyzed above gives us a clue to consider whether verification activities should display diseconomies of scale. Obviously, for the case of diseconomies of scale to arise the verification activities must not only contain a large part of variable costs (that is, the spending cost of verification in one project has nothing to do with the other), but the verification activity of an

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11 Note that \( \pi_t \in [0, 1] \).
12 Given the constant probability of success of projects, more projects financed means that more projects need to be monitored.
13 I am grateful to an anonymous referee for raising this point.
additional project also adds more burden to the bank on verification of the current project. Under our assumptions that each lender can costlessly establish a bank\textsuperscript{14} and each lender can at most finance one project, it is obvious that diseconomies of scale should not exist, because in this case every lender will launch a bank and monitor the borrower by himself. By doing so, each lender at most only needs to monitor a project and thus no externality exists.

In a real world where establishing a bank is not costless, it is also obvious that financial intermediaries will not exist if their cost structure displays diseconomies of scale. Note that the literature [as in Diamond (1984) and Williamson (1987)] has argued that financial intermediaries exist, because they are more efficient in monitoring borrowers than an individual lender. Consequently, in the following analysis we rule out the case where banking verification displays negative externalities or diseconomies of scale.

While it is plausible to assume that the verification cost per loan is a function of the total number of loans made to borrowers, one question may still arise as to why the costs per loan depend only on the total number of financed projects for Type L borrowers ($\pi_t$)? This is because the important function of financial markets in this framework is to distinguish Type L borrowers from Type H. Thus, $\pi_t$ being equal to zero means that the function of financial markets is disrupted; in other words, financial markets should be closed. Equation (9) implies this result for $\theta \in (-1,0]$ (the case for which multiple equilibria arise), because the verification cost per loan and the loan rate given in (5) are infinity when $\pi_t = 0$.

Note that $\pi_t$ can be viewed as a measure of financial market activities as it can represent the total number of loans made. Equation (8) then implies that higher rates of money growth will induce more resources to channel towards capital investment and thus raise financial market activities. As will be seen below, relating banking verification costs per loan to the levels of financial market activity, as assumed in (9), raises a possibility that effects from an increase in the money growth rate (and thus the inflation rate) on financial market activities as well as economic

\textsuperscript{14} This assumption is simplifies our analysis. For a discussion on banks that are costly to establish, see Saint-Paul (1992).
growth depend on the initial equilibrium status. Furthermore, this may give rise to the multiple relationships between inflation and economic growth as obtained recently by Bullard and Keating (1995) and Bruno and Easterly (1998).

4. CHARACTERIZATIONS OF EQUILIBRIA

Since each borrower who produces a positive amount of capital becomes a firm operator, the number of firms at $t$ is $[P_H + (1 - \lambda)p_L \pi_{t-1}]$. The total labor amount includes all young lenders and borrowers who are credit-rationed (equal to $[1 + (1 - \lambda)(1 - \pi_t)]$). Thus, per firm labor employment in equilibrium is given as

$$L_t = \frac{1 + (1 - \lambda)(1 - \pi_t)}{\lambda p_H + (1 - \lambda)p_L \pi_{t-1}}, \quad t \geq 1$$  \hspace{1cm} (10)

Using (3), we derive the rate of return on capital at time $t$ as

$$\rho_t = \sigma \left[ \frac{1 + (1 - \lambda)(1 - \pi_t)}{\lambda p_H + (1 - \lambda)p_L \pi_{t-1}} \right]^{1-\sigma}, \quad t \geq 1$$  \hspace{1cm} (11)

Furthermore, the capital stock of each firm at $t + 1$ is given as

$$k_{t+1} = Qw_t = Q(1 - \sigma)k_t L_t^{-\sigma}$$  \hspace{1cm} (12)

From (12), the growth rate between time $t$ and $t + 1$ is derived as

$$g_t = \frac{k_{t+1}}{k_t} = Q(1 - \sigma)L_t^{-\sigma}$$  \hspace{1cm} (13)

To examine the long-run properties of this economy, we define a balanced growth equilibrium as follows.

**DEFINITION.** Given $M_0, k_0$ and $f$, a balanced-growth equilibrium comprises a set of non-negative sequences \{\{k_t, L_t, M_t, P_t, \rho_t, w_t, R_t^{eq}, R_{t,t}, \text{and } \pi_t\}, \quad t \geq 1\} satisfying equations (2), (3), (4), (5), and (8). In addition, along with a balanced growth
path, $y_t, k_t, w_t, M_t$, and $P_t$ all grow at constant rates, whereas $\rho_t, R_t^m, R_{t,t}, \pi_t$, and $L_t$ remain unchanged.

Before characterizing the relationship between inflation and economic growth under a balanced growth path, notice that per lender real balances at $t$ (denoted as $m_t$) are given as

$$m_t = \frac{M_t}{P_t} = w_t - [\lambda + (1 - \lambda)\pi_t]w_t = (1 - \lambda)(1 - \pi_t)w_t$$

Note that along a balanced growth path, the rate of return from holding money (the inverse of inflation) between time $t$ and $t + 1$ is derived as

$$P_t^m = \frac{P_t}{P_{t+1}} = \frac{g_t}{\mu}$$

where $g_t$ is the growth rate of income. Substituting (15) into (8) and after some manipulations yields

$$g_t = \frac{\mu [Q_{t+1}p_Lp_H(1 - \pi_t) + p_H(1 - p_L)\alpha\pi_t^{1+\theta} - p_L(1 - p_H)\alpha\pi_t^\theta]}{p_L - p_H\pi_t}$$

where $\rho_{t+1}$ is a function of $\pi_t$ and given in (11) after updating one period. It becomes clear that $(8')$ and (13) jointly determine $\{g_t, \pi_t\}$. Once $g_t$ and $\pi_t$ are determined, all other variables can be derived. Notice further that any feasible balanced growth path displays that $g_t = g$ and $\pi_t = \pi$. As a result, all equilibrium quantities under a balanced growth path can reduce to determine the equilibrium values of $g$ and $\pi$. Figure 1 depicts the determination of $g$ and $\pi$.

For illustrative purposes, we call the locus defined by $(8')$ as 'FM' (financial market activities) and by (13) as 'KL' (the relationship of the marginal product of capital and labor employment). Since $\partial L/\partial \pi < 0$, the KL locus has a positive slope and intersects the vertical axis at $g = Q(1 - \sigma)(2 - \lambda/p_H)^{-\sigma}$. Intuitively, an

\[15\] Obviously, no one wants money if $\pi_t = 1$. Though the equilibrium values of $\pi_t$ in the following analysis are less than one, we could assume that the population of lenders is greater than the population of borrowers. In this case, money is always needed even if there is no credit rationing.
increase in $\pi$ means that more borrowers are able to implement their projects and, under our assumption, the number of firms will increase. This implies that per firm labor employment will decrease and, given competitive labor and capital markets, the level of the wage rate increases. As the maximal scale of an investment project is equal to the wage rate, this says that the size of each loan (and thus the amount of resources) allocated to capital investment increases. Apparently, this will raise economic growth. The locus of KL captures this fact.

To find the shape of the FM locus, we calculate the following:

$$\frac{\partial g}{\partial \pi} \bigg|_{(8')} = \mu \left\{ \frac{Q p_{LP} p_{PH} (p_L - p_H \pi) (1 - \pi) \partial \rho/\partial \pi - (p_L - p_H)}{(p_L - p_H \pi)^2} + A - B \right\}$$

where

$$A = p_H (1 - p_L) \alpha \pi^\theta [(p_L - p_H \pi) (1 + \theta) + p_H \pi]$$

and
$B = p_L(1 - p_H)\alpha p_t^{\theta - 1} \left( [(p_L - p_H)\pi + p_H\pi] \right)$

Since $\partial L/\partial \pi < 0$, $\partial \rho/\partial \pi < 0$. Therefore, the square brackets in the numerator of (16) are always negative. For $\theta \in [-1, 0)$, $A$ goes to positive infinity and $B$ goes to negative infinity when $\pi$ approaches zero. As a result, when $\pi$ is nearly equal to zero, the slope of the FM locus is positive infinity. Moreover, when $\pi$ approaches zero, the value of $g$ derived from (8') is also negative infinity.\(^\text{16}\) However, when $\pi$ approaches one, $A$ is greater than $B$, but the margin is small so that if $Q$ is sufficiently large, then the slope of the FM locus is negative.\(^\text{17}\) Furthermore, in this case the value of $g$ derived from (8') is negative. Consequently, the shape of the FM locus in the case of $\theta \in [-1, 0)$ is like the locus labeled as FM' in Figure 1. For $\theta = 0$, the slope of the FM locus is always negative so that the FM locus in this case is like the FM'' locus in Figure 1.\(^\text{18}\)

To understand economic intuitions of the FM locus, rewrite (8') as

$$R^m = \frac{g_t}{\mu_t} = \frac{Q\rho_{t+1}p_Lp_H(1 - \pi_t) + p_H(1 - p_L)\alpha\pi_t^{1+\theta} - p_L(1 - p_H)\alpha\pi_t^{\theta}}{p_L - p_H\pi_t} \quad (8'')$$

The FM locus then states the equilibrium condition of the financial markets. To see this, there are two alternative assets: money and capital loans. Given the risk-neutral agents, in equilibrium the rate of return from money $R^m$ should be equal to that from capital loans. Note also that $\pi_t$ has two opposite effects in determining the rate of return from capital loans. First, an increase in $\pi_t$ will lower the rental rate of capital since $\partial \rho/\partial \pi < 0$ (see (11)). Second, a higher value of $\pi_t$ implies a lower verification cost per project if $\theta \in [-1, 0)$ and thus will raise the rate of return from capital loans. It can then be inferred from Figure 1 that the first effect dominates the second in the smaller values of $\pi$ while the reverse is true for larger values of $\pi$. If there is no externality, then only the first effect shows up.

\(^\text{16}\) Thus, when $\pi$ approaches 0, the FM locus asymptotes in this case.
\(^\text{17}\) $A - B = \alpha(p_L - p_H)[p_H(1 - p_L) - \theta(p_L - p_H)]$ when $\pi = 1$.
\(^\text{18}\) The following example produces KL, FM', and FM'' in Figure 1 (using Mathematica): $p_L = 0.8$, $p_H = 0.4$, $\sigma = 0.5$, $\lambda = 0.5$, $\mu = 1.1$, $\alpha = 0.2$, $Q = 4$, and $\theta = -0.8$ for the FM' locus, and $\theta = 0$ for FM'.

Under some parameter conditions, there obviously exists a unique equilibrium (labeled "U" in Figure 1) if there is no externality. From Figure 1, there are generally two equilibria or none for $\theta \in [-1,0)$. The key factor driving the FM locus to intersect the KL locus twice is that, at the domain of $\pi$, the maximal value of $g$ derived from (8') is greater than that derived from (13). Comparing (8') with (13), this can be ensured if $\mu$ is not too small. We summarize these results in the following proposition.

**Proposition 1. (Existence of Unique Equilibrium)** If parameters satisfy that

$$
\frac{\mu[(Qp_pL_pH + \alpha(p_L - p_H))]}{p_L} > Q(1 - \sigma)(\frac{2 - \lambda}{\lambda p_H})^{-\sigma}
$$

then there exists a unique equilibrium if $\theta = 0$; that is, a unique equilibrium exists when banking verification costs exhibit no externality.

**Proposition 2. (Possibility of Multiple Equilibria)** When banking verification costs display economies of scale or participation externalities, there are multiple equilibria, if $\mu$ is not too small.

Note that

$$
\frac{\partial g}{\partial \pi} \bigg|^{13} > 0 > \frac{\partial g}{\partial \pi} \bigg|^{8'}
$$

for the case of $\theta = 0$. Furthermore, when $\pi = 1$, the value of $g$ is negative from (8') and positive from (13). Therefore, the unique equilibrium can be ensured if the value of $g$ derived from (8') is greater than that from (13) when $\pi = 0$. This is the parameter condition specified in Proposition 1.

If banking verification costs display participation externalities, high (low) financial activities are associated with a low (high) average verification cost. Therefore, in Figure 1 there are two equilibria. The first one (labeled as "L" in the figure) is the low finance-growth equilibrium, representing low financial activities and economic growth (lower values of $\pi$ and $g$). The second one (labeled as "H") is the high finance-growth equilibrium due to high financial activities and economic growth.

From (15), the inflation rate along a balanced growth path is equal to $\mu/g$. Thus, the associated inflation rate is high (low) under a low (high) finance-growth equilibrium. This is consistent with recent empirical results. Moreover, recent aforementioned empirical studies also suggest that an increase in the inflation rate tends to lower the growth rate for economies with high initial inflation rates.
Nonetheless, such an increase within low-initial-inflation rate economies will raise economic growth. The next section will show this result.

5. COMPARATIVE-STATIC ANALYSIS

This section considers the effects from changing some parameters on financial activities and economic growth. The parameters we focus on include the rate of money growth, technology progresses in capital production (that is, increases in $Q, p_L$, and $p_H$), and technology progress in verification technology (a decrease of $\alpha$). Note that an increase in $Q$ represents technology progress in both types of investment projects while an increase in $p_L$ and $p_H$ captures technology progress only to the low- and high-risk project, respectively. To proceed, we first calculate the vertical shifts of the loci defined by (8') and (13) due to these changes.

An increase in the money growth rate in general shifts the FM locus down, but has no effect on the KL locus. Technology progresses in both types of investment projects (an increase in $Q$) shift both loci up. An increase in the probability of success in the low-risk project (that is, an increase in $p_L$) shifts the FM locus down, but the KL locus up, while such an increase in the high-risk project shifts both loci up. Finally, technology progress in banking verification technology (a decrease of $\alpha$) will shift the FM locus down, but has no effect on the KL locus.

Given these results, we first examine the case for $\theta = 0$ and the results are summarized in Table 1 and Proposition 3.

<table>
<thead>
<tr>
<th>An increase in Effect on</th>
<th>$\mu$</th>
<th>$Q$</th>
<th>$p_L$</th>
<th>$p_H$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>$+$</td>
<td>$+$</td>
<td>$?$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>$+$</td>
<td>$?$</td>
<td>$-$</td>
<td>$?$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

**Proposition 3.** (*Characterizations of Equilibrium under No Externalities*) When banking verification costs exhibit no externality, the balanced-growth equilibrium has the following properties:
(1) *An increase in the money growth rate raises financial activities and economic growth.*

(2) *Technology progresses in both types of investment projects increase economic growth, but has an ambiguous effect on financial activities. An increase in the probability of success in a low-risk type project lowers financial activities and has an ambiguous effect on economic growth; however, such an increase in high-risk projects raises economic growth, but has an ambiguous effect on financial activities.*

(3) *Technology progress in banking verification technology raises financial activities and economic growth.*

For the case of positive externalities, comparative-static results of a low finance-growth equilibrium differ from that of a high finance-growth equilibrium. In particular, one can easily verify that a high finance-growth equilibrium possesses properties analogous to the equilibrium when banking verification costs exhibit no externality. As a result, Table 1 and Proposition 3 can be applied to the high finance-growth equilibrium. Table 2 and Proposition 4 summarize the properties of the low finance-growth equilibrium.

**Table 2  Results of Comparative-static Analysis:**

**Low Finance–growth Equilibrium for \( \theta \in [-1, 0) \)**

<table>
<thead>
<tr>
<th>Effect on</th>
<th>( \mu )</th>
<th>( Q )</th>
<th>( p_L )</th>
<th>( p_H )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>-</td>
<td>?</td>
<td>+</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

**Proposition 4.** *(Characterizations of Participation Externalities Equilibria)* When banking verification costs exhibit participation externalities, the high finance-growth equilibrium has properties analogous to the (unique) equilibrium when there is no externality on banking verification costs. For the low finance-growth equilibrium, it has the following features.

(1) *An increase in the money growth rate will reduce financial activities and economic growth.*
(2) Technology progresses in both types of investment project lower financial activities, but has an ambiguous effect on economic growth. An increase in the success probability of a low-risk investment project raises financial activities and growth; nonetheless, such an increase in a high-risk project has an ambiguous effect on economic growth, but reduces financial activities.

(3) Technology progress in banking verification technology reduces financial activities and economic growth.

From comparative-static analyses, one sees that an increase in the money growth rate facilitates economic growth when banking verification costs exhibit no externality. It can be shown that the slope of the KL locus is less than one; thus, the amount of increase in the money growth rate is greater than the amount of a corresponding increase in the economic growth rate. As a result, the inflation rate (given as the inverse of $R_t^{in}$ in (15)) increases along with an increase in the money growth rate. Therefore, the Mundell-Tobin effect holds when there is no externality on banking verification costs. However, if the costs display positive externalities, the effect of an increase in the money growth rate on economic growth depends on the initial equilibrium. Under a high finance-growth equilibrium associated with low initial inflation rates, an increase in the money growth rate (that is, an increase in the inflation rate) raises financial activities and economic growth. Nonetheless, the reverse is true for a low finance-growth equilibrium associated with high initial inflation rates. These results are consistent with recent empirical findings that an increase in inflation raises economic growth with low initial inflation rates, but lowers economic growth if initial inflation rates are high.

As the interaction of adverse selection and costly state verification problems raises a possibility of multiple equilibria, comparative-static analyses also reveal that this interaction plays a crucial role in determining effects from changing parameters on the relationship between inflation and economic growth. For example, an increase in $Q$ under the high finance-growth equilibrium raises economic growth and inflation. However, such an increase under a low finance-growth equilibrium has an ambiguous effect on economic growth, but nevertheless raises the inflation rate.
6. CONCLUSION

This paper studies the relationship between inflation and economic growth under a model with adverse selection and costly state verification problems. I show that a unique equilibrium exists if banking verification costs exhibit no externality. In this case, the relationship between inflation and economic growth follows the Mundell–Tobin effect. When banking verification costs exhibit economies of scale, multiple equilibria emerge and the relationship between inflation and economic growth depends on initial inflation rates. If initial inflation rates are high (low), an increase in the inflation rate lowers (raises) economic growth. These results are consistent with recent empirical findings. Moreover, comparative-static analyses show that the interaction of adverse selection and costly state verification problems is important in determining the effects from changing parameters on inflation, financial activities, and economic growth.

REFERENCES


通貨膨脹與經濟成長之關係探討：
當金融市場面臨逆向選擇與監督成本

洪福聲

摘 要

本文建立一個訊息不對稱模型，允許金融市場存在逆向選擇問題及投資計劃之成效需要成本加以確認（監督成本）。在此架構之下，我們發現逆向選擇問題與監督成本的存在，對於決定一國之通貨膨脹與經濟成長的長期關係，扮演著重要的角色。模型的結果顯示，一國之通貨膨脹率與經濟成長率的關係視該國原始通貨膨脹率的大小而定。假若該國之原始通貨膨脹率低，則通貨膨脹率的增加可以促進長期的經濟成長；然而，如果原始通貨膨脹率較高，則通貨膨脹率的繼續增加會降低該國的長期經濟成長。此一結論與親近的一些實證文獻相一致。

關鍵詞：通貨膨脹、經濟成長、逆向選擇、監督成本、外部性

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