Trade and Entrepreneurship with Heterogeneous Workers*

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Abstract

This paper investigates the impacts of progressive trade openness, technological externalities, and heterogeneity of individuals on the formation of entrepreneurship in a two-country occupation choice model. We show that trade opening gives rise to a non-monotonic process of international specialization, in which the share of entrepreneurial firms in the large (small) country first increases (decreases) and then decreases (increases), with the global economy exhibiting first de-industrialization and then re-industrialization. When countries have the same size, we also show that strong technological externalities make the symmetric equilibrium unstable, generating equilibrium multiplicity, while sufficient heterogeneity of individuals leads to the stability and uniqueness of the symmetric equilibrium.

Keywords: entrepreneurship, trade liberalization, externality, heterogeneity, stability

JEL Classification: F12, F16, J24, O14, R12
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1 Introduction

It is now widely recognized that entrepreneurial capacity is a key-determinant of productivity improvements, thus affecting the ability of a country to compete on the international marketplace (Baumol, 1968). Simultaneously, a large empirical body of literature shows that the rate of new firm formation significantly varies with the level of economic development (Harada, 2005) as well as across regions and countries (Armington and Acs, 2002; Audretsch et al., 2002; Reynolds et al., 1994). In such a context, it is reasonable to expect a progressive liberalization of international trade to change the incentives for individuals to become entrepreneurs, thereby affecting the countries’ industrial structure. Indeed, by lowering trade costs, on the one hand, a deeper economic integration fosters more competition from abroad, which tends to lower prices and profits. This in turn should reduce the incentives for individuals to start a new business, a point that has been emphasized by Hirschman (1968) in the debate on import substituting industrialization policies. On the other hand, lower trade costs also make the market size bigger by facilitating exports, a fact that might compensate firms for their lower mark-ups. The outcome of this trade-off is, therefore, a priori undetermined. Furthermore, as the incentives individuals face also change with the size and economic structure of their country, it seems natural to expect the impact of trade liberalization to vary across countries.

In order to develop some ideas about the interaction between entrepreneurship and international trade, we exploit the Employment Status Survey of the Ministry of International Affairs and Communication of Japan. This survey provides data about the numbers of employed people who want to change careers and people not engaged in work who want to find a job. It also reports the numbers of people who wish to start a business in each of these two groups. This enables us to compute the share $S_b$ of people who wish to start up a business. This magnitude can be interpreted as an index
of potentially new entrepreneurs. The survey goes one step further and asks those people whether they are already involved in activities to find a job or to create their own business. This yields the share $S_{bp}$ of people who are ready to start up a new business. We then plot $S_b$ and $S_{bp}$ against the share $S_t$ of the value traded (imports plus exports) in the Japanese GDP, which we may consider as an index of trade openness. Figure 1 shows that these two magnitudes are positively correlated from the end of 1950’s until the mid 1980’s. After that, the correlation becomes negative. Hence, as the Japanese economy was getting open, the share of new entrepreneurs first rose and then declined. Although other factors are likely to be involved, this suggests the existence of a U-shaped relationship between the level of industrialization and the degree of trade openness.

The purpose of this paper is to show how progressive trade liberalization affects countries’ industrial structure through the channel of entrepreneur-
ship and creation of new firms. In order to achieve our goal, we develop a new model that combines (i) a two-country trade setting in which the manufacturing sector operates under monopolistic competition and increasing returns, and (ii) an occupational choice approach in which heterogeneous individuals are entitled to be either a worker in an existing firm or an entrepreneur producing a new variety. The monopolistic competition setting appears to be especially well suited to analyze the creation of small businesses that have a limited market power, while product differentiation allows us to capture the fundamental idea that entrepreneurs are often market-makers. Furthermore, assuming heterogeneous individuals means that they have idiosyncratic and subjective attitudes toward entrepreneurship, as stressed in the literature (Casson, 2005).

Our main results are summarized as follows. First of all, we find that the large country always retains a more than proportional share of firms. This does not mean, however, that this country always benefits from lower trade costs. Indeed, trade liberalization does not translate into a simple and monotonic process of international specialization. Specifically, we will see that the whole process of economic integration is to be split into two contrasting phases. In the first one, which occurs when trade costs remain relatively high, the industrial basis of the large country grows whereas that of the small country shrinks. Because consumers living in the small country have access to a much wider range of varieties, the local firms lose a substantial market share in their home market, thus reducing the incentives for people to become entrepreneurs. On the contrary, the large country firms benefit from a market expansion effect that leads more people to become entrepreneurs. Consequently, during the first phase of the integration process, countries become more dissimilar and inequality rises.

In the second phase, which is reached when trade costs are low enough, we observe a complete reversal in the foregoing tendencies. On the one hand,
trade costs are now sufficiently low for the small country firms to benefit from a much larger market, thus inducing more individuals to become entrepreneurs. On the other hand, because foreign competition is exacerbated by lower trade costs, business is less profitable in the large country. Hence, during the second phase, economic integration fosters convergence between countries. In our setting, the total number of varieties in the economy also varies with the degree of trade openness. In particular, when trade costs remain high, the global economy experiences de-industrialization, while it faces re-industrialization when international integration becomes sufficiently deep. All those results reveal that trade liberalization has contrasted effects on countries through the creation and destruction of local firms. This pattern provides a rationale for the U-shaped curve observed in Figure 1. Although Japan is involved in a multilateral network of trade with countries of different sizes, we may view it as being the “large” country in our setting. Indeed, in 2005, more than half of its trade (both exports and imports in value) is done with developing countries while its trade with the U.S. stands only for twenty-five percent of its exports and fifteen percent of its imports. Note also that Japan is the second richest country in the world and that size is expressed here in terms of purchasing power.

This is not the end of the story, however. According to business analysts, entrepreneurship cannot be explained solely by individual characteristics of people, as assumed in the occupational choice model (Jovanovic, 1982; Holmes and Schmitz, 1990). It must also account for the social and institutional environment in which they find themselves (Shane and Venkataraman, 2000). Specifically, business analysts argue that individuals are influenced by what others do, especially when facing fuzzy market conditions. Such influences, which are reminiscent of bandwagon effects, may be subsumed in a network externality (Minniti, 2005). In the same vein, we know from economic geography that the clustering of a growing number of firms gives
rise to agglomeration economies, which lower the production costs of firms located in the cluster. Such economies stem from a wider array of intermediate inputs and a larger labor pool available to firms, as well as from information spillovers that develop within the cluster (Duranton and Puga, 2004). Both agglomeration and network economies generate increasing returns with respect to size and may, therefore, be captured in the same way. More precisely, these two strands of literature are reconciled in our framework by assuming that potential entrepreneurs are positively affected by an externality whose intensity rises with the number of local entrepreneurs. Though somewhat ad hoc, such a modeling strategy is consistent with various underlying microeconomic mechanisms. It is worth stressing that it entails no loss of generality as long as we confine ourselves to a positive analysis.

In order to uncover the pure impact of agglomeration externalities on the nature of trade, we consider the case in which countries are identical. In such a case, there always exists a symmetric equilibrium where their industrial structures are identical, and when the externality is weak, it is unique and globally stable. However, when the externality is sufficiently strong, the symmetric equilibrium ceases to be stable. To put it differently, once it is recognized that the entrepreneurship process is subject to a sufficient amount of agglomeration/network externalities, it appears that *trade liberalization favors the emergence of inequalities between countries that are otherwise identical*. Such a result points to the exacerbation of the tendencies toward inequality identified in the foregoing.

We further uncover a new force that tends to smooth out the destabilizing effects of the agglomeration externality, namely the heterogeneity of the population of individuals. By increasing the degree of heterogeneity within the population according to specifications that will be explained below, we are able to establish the following two results. First, we show
that a sufficient amount of “local” heterogeneity in the vicinity of the symmetric equilibrium overcomes the impact of the agglomeration externality and restores the local stability of this equilibrium. Second, allowing for a large amount of “global” heterogeneity suffices to guarantee the uniqueness, hence the global stability, of the symmetric equilibrium. Because there is ample evidence that individuals have very contrasted attitudes toward entrepreneurship, these two results suggest that the emphasis put on the network externality in the business literature may not be warranted. They also shed light on the macroscopic consequences of microeconomic attitudes: 

more heterogeneity at the individual level would be accompanied by less disparity at the global level. Conversely, more homogeneous individuals could strengthen the impact of the network externality, thereby exacerbating international inequalities. Thus, as shown by Herrendorf et al. (2000) in a different context, heterogeneity has a stabilizing effect. Finally, we compare our two results to those derived in other models where the idea of heterogeneity has been applied. As will be seen, the similarity between results obtained in settings that otherwise vastly differ suggests that there is probably a general principle at work.

A few papers have studied the impact of trade on occupational choices. Unlike us, they focus on skill formation and tackle different, but related, issues. The following papers are worth mentioning. First, Acemoglu (2003) shows in the HOS model that international trade, via the Stolper-Samuelson effect, enhances skill-biased technical change and raises the demand for skilled labor, thus leading to a larger skilled labor share and a wider wage gap between the skilled and unskilled. Ishikawa (1996) develops an HOS model in which scale economies in human capital enable the large country to specialize in the production of a skilled-labor-intensive good, whereas the small country specializes in producing an unskilled-labor-intensive good. Last, Amiti and Pissarides (2005) build a trade model with monopolistic
competition, horizontally heterogeneous workers, and skill formation. They study the relationship between skill mismatch and firm agglomeration and show that decreasing trade costs and better skill match induce both skill formation and the agglomeration of firms.

The remaining of the paper is organized as follows. The model and some preliminary results are presented in Section 2. Section 3 studies the impact of trade liberalization on countries’ industrial structure. The role of network externalities and the stabilizing effect of individuals’ heterogeneity are dealt with in Section 4. Section 5 concludes.

2 The model and intermediate results

2.1 The economy

The economy involves a total mass of individuals equal to one, two countries \( i = 1, 2 \) with a population of size \( m_1 \) and \( m_2 \), respectively, and two production sectors; without loss of generality, we assume that \( m_1 \geq m_2 \).

Individuals are heterogeneous. The type of individuals living in country \( i \) is represented by \( \alpha_i \in [\underline{\alpha}, \bar{\alpha}] \) with \( 0 \leq \underline{\alpha} < \bar{\alpha} \). Types are distributed according to the distribution function \( F_i: [\underline{\alpha}, \bar{\alpha}] \rightarrow [0, m_i] \), which has a differentiable density function \( f_i \) such that \( f_i(\alpha) > 0 \) for all \( \alpha \in [\underline{\alpha}, \bar{\alpha}] \). As in most microeconomic studies of the formation of entrepreneurship, individuals are entitled to be either a worker in an existing firm or an entrepreneur producing a new variety. More precisely, individuals of type \( \alpha \) are endowed with \( \alpha \) units of labor and 1 unit of entrepreneurship.

All individuals have the same quasi-linear log utility with respect to a homogeneous good \( (A) \) and a continuum \( N \) of varieties of a horizontally

\(^1\)Since the industrial density, population, and income of countries are among the main explanatory variables in new firm creation (Armington and Acs, 2002), we often work with countries having different sizes.
differentiated good \((M)\):

\[
U = \mu \ln M + A \quad \mu > 0
\]

(2.1)

where the subutility over the varieties is of the CES-type:

\[
M = \left[ \int_0^N q(x)^{\sigma-1} \sigma \, dx \right]^{\frac{\sigma}{\sigma-1}}
\]

with \(\sigma > 1\) being the elasticity of substitution between any two varieties.\(^2\)

Although quasi-linear preferences rank far behind homothetic preferences in general equilibrium models of trade, Dinopoulos et al. (2006) show that “quasi-linear preferences behave reasonably well in general-equilibrium settings”. All individuals are spatially immobile and endowed with \(A > 0\) units of the homogeneous good. The initial endowment \(A\) is supposed to be larger than \(\mu\) for the consumption of this good to be strictly positive at the market outcome. Consequently, our setting involves no income effect.

The traditional sector supplies the homogeneous good under perfect competition using labor as the only input of a constant-returns technology. The unit input requirement is set to one by choice of units. Shipping the homogeneous good is assumed to be costless, thus implying that its price is equalized across countries. This makes that good the natural choice for the numéraire. Consequently, in equilibrium, wages are the same in both countries and equal to 1.

The industrial sector produces the differentiated good under increasing returns and monopolistic competition, using both entrepreneurs and workers. Each firm produces a differentiated variety and the total mass \(N\) of firms is endogenous.

As discussed in the introduction, entrepreneurship often flourishes in countries in which agglomeration and network externalities are at work as

\(^2\)Such preferences are also used by Martin and Rogers (1995) and Pflüger (2004) in trade and geography models.
they increase the productivity of newly created firms. They take the form of technological spillovers in start-up operations due to learning effects in the industrial sector of each country (Duranton and Puga, 2004; Minniti, 2005). This means that, as an entrepreneur, an individual runs a number \( h_i(n_i) > 0 \) of firms that increases with the number \( n_i \) of entrepreneurs living in her country. Producing \( q \) units of a variety in country \( i \) requires \( 1/h_i(n_i) \) units of entrepreneurship and \( cq \) units of labor, where, without much loss of generality, we set \( c = 1 \). When \( h'(n_i) > 0 \), increasing the number of entrepreneurs thus amounts to reducing the fixed requirement of entrepreneurial units because \( h(n_i) \) is the inverse of this fixed requirement. Externalities are confined to each country in order to capture the well-documented fact that the scope of spillovers is limited in space (Audretsch and Feldman, 2004). We assume that \( h_i \) is continuously differentiable and increasing in \( n_i \in [0, m_i] \). This implies that, in equilibrium, the total mass of firms operating—or of varieties produced—in country \( i \) is equal to

\[
N_i = n_i h_i(n_i)
\]

so that \( N = n_1 h_1(n_1) + n_2 h_2(n_2) \).

Last, shipping one unit of a differentiated good requires \( \tau \geq 1 \) units of this good (the iceberg trade cost). Thus, zero trade costs means \( \tau = 1 \), whereas countries are autarkic when \( \tau \to \infty \).

Since the workers’ wage equals 1, the entrepreneurship decision may be viewed as an effort made by an individual of type \( \alpha \), who pays an opportunity cost equal to \( \alpha \) when she chooses to become an entrepreneur. In this context, individuals are heterogeneous in the effort level required to be entrepreneurs. In an entrepreneurship equilibrium to be defined below, an individual of type \( \alpha \) makes this effort if and only if she makes an income \( y_i = h_i w_i \) higher than \( \alpha \), where \( w_i \) is the salary earned when she starts a new business. Clearly, our two interpretations of an heterogeneous population of individuals are formally equivalent.
2.2 The market equilibrium

Fixing the number of entrepreneurs $n_1$ and $n_2$ in each country, we now determine the market equilibrium and the corresponding entrepreneurs’ salaries as functions of $n_1$ and $n_2$. Let $p_{ij}$ denote the mill price charged by a firm located in country $i = 1, 2$ to its customers living in country $j = 1, 2$. The individual demands in countries $i$ and $j \neq i$ for variety $x \in [0, N]$ produced in country $i$ are respectively given by

$$q_{ii} = \frac{\mu P_i^{\sigma-1}}{p_{ii}^\sigma} \quad q_{ij} = \frac{\mu P_j^{\sigma-1}}{(\tau p_{ij})^\sigma}$$  \hspace{1cm} (2.2)

where $P_i$ is the price index of the differentiated good in country $i$. Since all firms located in country $i$ charge the same mill price in country $j$, the price index in country $i$ is given by

$$P_i = \left[ N_i p_{ii}^{-(\sigma-1)} + N_j (\tau p_{ji})^{-(\sigma-1)} \right]^{-\frac{1}{\sigma-1}}.$$

Note that (2.1) implies that a country $i$ individual consumes $\mu/P_i$ units of the manufactured goods, thus implying that her expenditure on this good is equal to $\mu$. Therefore, country $i$’s expenditure on good $M$ is constant and equal to $\mu m_i$.

The profit of a country $i$ firm is as follows:

$$\pi_i = (p_{ii} - 1) q_{ii} m_i + (p_{ij} - 1) \tau q_{ij} m_j - w_i$$

thus implying that its equilibrium mill prices are given by

$$p_{ii}^* = p_{ij}^* = \frac{\sigma}{\sigma - 1}.$$

Let $\phi := \tau^{1-\sigma} \in [0, 1]$ be the degree of trade openness: a larger value of $\phi$ means lower trade costs, with $\phi = 0$ when $\tau \to \infty$ and $\phi = 1$ when $\tau = 1$. Then, the equilibrium price index is expressed as

$$P_i = \frac{\sigma}{\sigma - 1} (n_i h_i(n_i) + \phi n_j h_j(n_j))^{-\frac{1}{\sigma-1}}.$$
Under free entry and exit, whence zero profits, the equilibrium salary in country $i$ is given by a firm’s gross profits:

$$w_i(n_i, n_j) = \frac{1}{\sigma} \left[ \frac{\mu m_i}{n_i h_i(n_i)} + \phi \frac{\mu m_j}{n_j h_j(n_j)} + \phi \frac{\mu m_i}{n_i h_i(n_i)} + \phi \frac{\mu m_j}{n_j h_j(n_j)} \right].$$  \hspace{1cm} (2.3)

Thus, the equilibrium salary prevailing in country $i$ decreases with the number of entrepreneurs in this country. It also decreases with the number of entrepreneurs in country $j$ because trade makes the two national labor markets interdependent through the mass of varieties they trade. Last, since gross profits are higher, a stronger preference for the industrial good (i.e., larger $\mu$) and/or more differentiated varieties (i.e., lower $\sigma$) leads to higher equilibrium salaries.

The expression (2.3) may be given a very intuitive interpretation. The first bracketed term is the revenue gleaned by a country $i$ firm in its domestic market, whereas the second stands for the revenue gleaned in the foreign market, which is “discounted” by $\phi \in [0, 1]$ on account of the resources needed to sell abroad. All firms compete for the total expenditure on the manufactured good spent by country $i$’s residents, which is equal to $\mu m_i$. This “pie” is equally divided among country $i$ firms, but not between country $i$ firms and country $j$ firms because of the existence of trade costs ($0 < \phi < 1$). Furthermore, the pie accruing to a country $i$ firm is distributed between the entrepreneur and the workers according to the shares $(p^* - 1)/p^*$ and $1/p^*$, respectively, where $p^* = \sigma/(\sigma - 1)$ is the equilibrium mill price. This implies that an entrepreneur receives a fraction $1/\sigma$ of the pie.

The equilibrium income $y_i$ of an entrepreneur living in country $i$ is defined as follows:

$$y_i(n_i, n_j) = h_i(n_i) w_i(n_i, n_j)$$

$$= \frac{1}{\sigma} \left[ \frac{\mu m_i h_i(n_i)}{n_i h_i(n_i)} + \phi \frac{\mu m_j h_i(n_i)}{n_j h_j(n_j)} + \phi \frac{\mu m_i}{n_i h_i(n_i)} + \phi \frac{\mu m_j}{n_j h_j(n_j)} \right].$$  \hspace{1cm} (2.4)

Then, for any given $n_1$ and $n_2$, the following equality must hold at the
market equilibrium:

\[ \sum_{i=1,2} y_i(n_1, n_2) n_i = \sum_{i=1,2} (p^* - 1) (m_i q_{ii} + \tau m_j q_{ij}) N_i 
= \frac{\mu}{\sigma} (m_1 + m_2). \tag{2.5} \]

### 2.3 The entrepreneurship equilibrium

We now describe the equilibrium occupational choices. An \( \alpha \)-type individual living in country \( i \) earns \( \alpha \) as a worker and chooses to become an entrepreneur if and only if her worker income is less than the earnings she makes as an entrepreneur:

\[ \alpha \leq y_i(n_i, n_j) \]

so that the mass of entrepreneurs in this country is given by \( F_i[y_i(n_i, n_j)] \).\(^3\)

A pair \( (n_1^*, n_2^*) \) is an \emph{entrepreneurship equilibrium} if and only if

\[ n_1^* = F_1[y_1(n_1^*, n_2^*)] \quad n_2^* = F_2[y_2(n_1^*, n_2^*)]. \tag{2.6} \]

Observe that \( \frac{\partial F_i[y_i(n_i, n_j)]}{\partial n_j} < 0 \), so that \emph{entrepreneurship decisions are strategic substitutes between countries}.

Since \( y_i(n_1^*, n_2^*) = F_i^{-1}(n_i^*) \) holds whenever \( (n_1^*, n_2^*) \) is an entrepreneurship equilibrium for some \( \phi \), by equation (2.5) the equilibrium must always lie on the locus of

\[ E(n_1, n_2) := F_1^{-1}(n_1) n_1 + F_2^{-1}(n_2) n_2 - \frac{\mu}{\sigma} (m_1 + m_2) = 0 \tag{2.7} \]

regardless of the value of \( \phi \) and the functional form of \( h_i(\cdot) \). Observe that this locus is downward sloping in the \( n_1-n_2 \) plane because of strategic substitutability of entrepreneurship decisions between countries.

**Observation 2.1.** The locus of \( E(n_1, n_2) = 0 \) is downward sloping.

---

\(^3\)An individual being negligible, her occupational choice has no impact on the mass of available varieties. Thus, maximizing income amounts here to maximizing utility.
We now have to find conditions for an entrepreneurship equilibrium to exist. First, note that (2.3) implies that $w_i$ has a minimizer $w_i^\text{min} > 0$ independent of $\sigma$. This in turn implies that $y_i$ has a minimizer $y_i^\text{min} \geq h_i(0)w_i^\text{min} > 0$ as long as $h_i(0) > 0$. The continuous function $y_i(n_1, n_2)$ also has a maximizer $y_i^\text{max}$ in the compact set $[F(y_i^\text{min}), m_1] \times [F(y_i^\text{min}), m_2]$. Assume that the interval $(\alpha_i, \overline{\alpha}_i)$ is wide enough to include $[y_i^\text{min}, y_i^\text{max}]$, which implies that there are always some individuals with sufficiently low $\alpha$ who choose to become entrepreneurs and some with sufficiently high $\alpha$ who choose to become workers, whatever the others’ choice. Under these conditions, we may restrict the domain of $F \circ y_i$ over the compact and convex set $[F_1(y_i^\text{min}), F_1(y_i^\text{max})] \times [F_2(y_i^\text{min}), F_2(y_i^\text{max})]$. Furthermore, the continuous function $F_i \circ y_i$ takes its value in $[F_1(y_i^\text{min}), F_1(y_i^\text{max})]$ because $F_i$ is increasing. Hence, Brouwer’s fixed point theorem implies that the mapping $(F_1 \circ y_1, F_2 \circ y_2)$ has a fixed point in the restricted domain of $(n_1, n_2)$, and this point is an entrepreneurship equilibrium. We will assume throughout the rest of the paper that $[y_i^\text{min}, y_i^\text{max}] \subset (\alpha_i, \overline{\alpha}_i)$ for $i = 1, 2$.

In the next two sections, we study how the shares of entrepreneurs in both countries, whence the size and the international distribution of the industrial sector, react to trade opening in the absence or presence of externalities.

3 The impact of trade opening

Throughout this section, we assume that there are no agglomeration externalities, i.e., $h_i(n_i) \equiv 1$ for $i = 1, 2$, implying that $N_i = n_i$ while an entrepreneur’s income is equal to the salary she earns in operating a single firm so that $y_i(n_i, n_j) = w_i(n_i, n_j)$. In order to focus on the interactions between country size and trade openness, we consider the case in which the type distributions in the two countries are identical up to a scale parameter that reflects the country size, i.e., $[\alpha_i, \overline{\alpha}_i] = [\alpha, \overline{\alpha}]$, and $F_i(\alpha) = m_iG(\alpha)$ for
a common distribution function $G: [\underline{\alpha}, \overline{\alpha}] \rightarrow [0, 1]$. We assume that $G$ has a differentiable density $g$ and $g(\alpha) > 0$ for all $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. Accordingly, we focus on $s_i := n_i/m_i$, the fraction of entrepreneurs in each country $i$. So, from now on, we refer to $(s_1^*(\phi), s_2^*(\phi))$ as being the entrepreneurship equilibrium.

### 3.1 National industrialization

It follows directly from (2.6) that $(s_1^*(\phi), s_2^*(\phi))$ is an entrepreneurship equilibrium if and only if $s_1^*(\phi)$ and $s_2^*(\phi)$ satisfy the following two conditions:

$$
\bar{D}_1(s_1, s_2; \phi) := G(\bar{w}_1(s_1, s_2; \phi)) - s_1 = 0
$$

$$
\bar{D}_2(s_1, s_2; \phi) := G(\bar{w}_2(s_1, s_2; \phi)) - s_2 = 0
$$

where $\bar{w}_i: [0, 1]^2 \times [0, 1] \rightarrow \mathbb{R}$ is the salary of an entrepreneur in country $i$, which is defined by

$$
\bar{w}_i(s_1, s_2; \phi) = \frac{1}{\sigma} \left( \frac{\mu m_i}{m_i s_i + \phi m_j s_j} + \phi \frac{\mu m_j}{m_j s_j + \phi m_i s_i} \right).
$$

(3.1)

It is readily verified that, for any $(s_1, s_2)$, $\partial \bar{D}_i/\partial s_j < 0$ for $i, j = 1, 2$. Furthermore, we have

$$
\left| \frac{\partial \bar{D}_1}{\partial s_1} / \frac{\partial \bar{D}_1}{\partial s_2} \right| > \left| \frac{\partial \bar{D}_2}{\partial s_1} / \frac{\partial \bar{D}_2}{\partial s_2} \right|.
$$

To show it, observe that

$$
\left| \frac{\partial \bar{D}_1}{\partial s_1} / \frac{\partial \bar{D}_1}{\partial s_2} \right| > \left| \frac{\partial G(\bar{w}_1)}{\partial s_1} / \frac{\partial G(\bar{w}_1)}{\partial s_2} \right| = \left| \frac{\partial \bar{w}_1}{\partial s_1} / \frac{\partial \bar{w}_1}{\partial s_2} \right|
$$

while

$$
\left| \frac{\partial \bar{D}_2}{\partial s_1} / \frac{\partial \bar{D}_2}{\partial s_2} \right| < \left| \frac{\partial G(\bar{w}_2)}{\partial s_1} / \frac{\partial G(\bar{w}_2)}{\partial s_2} \right| = \left| \frac{\partial \bar{w}_2}{\partial s_1} / \frac{\partial \bar{w}_2}{\partial s_2} \right|.
$$

The desired result then follows from

$$
\left| \frac{\partial \bar{w}_1}{\partial s_1} \cdot \frac{\partial \bar{w}_2}{\partial s_2} \right| > \left| \frac{\partial \bar{w}_1}{\partial s_2} \cdot \frac{\partial \bar{w}_2}{\partial s_1} \right|
$$

which always holds.

**Observation 3.1.** In the $s_1$-$s_2$ plane,
1. the locus of $\bar{D}_i(s_1, s_2) = 0$ is downward sloping;

2. $\bar{D}_i(t_1, t_2) > 0$ if $(t_1, t_2)$ belongs to the south-west domain delineated by $\bar{D}_i(s_1, s_2) = 0$, while $\bar{D}_i(t_1, t_2) < 0$ if $(t_1, t_2)$ belongs to the north-east domain; and

3. at any entrepreneurship equilibrium $(s_1^*, s_2^*)$, $\bar{D}_i(s_1, s_2) = 0$ is steeper than $\bar{D}_2(s_1, s_2) = 0$.

By continuity, the third statement implies that the equilibrium $(s_1^*(\phi), s_2^*(\phi))$ is unique for any $\phi$.

When each country is in autarky ($\phi = 0$) or when the two countries are fully integrated ($\phi = 1$), the entrepreneurs’ salary is independent of the country size because

$$\bar{w}_i(s, s; 0) = \bar{w}_i(s, s; 1) = \frac{\mu}{\sigma s}$$

for any $s$ and $m_1, m_2$. Since

$$\bar{D}_i(s, s; 0) = \bar{D}_i(s, s; 1) = G\left(\frac{\mu}{\sigma s}\right) - s$$

holds regardless of the value of $m_1$ and $m_2$, it must be that $\bar{D}_i(\bar{s}, \bar{s}; 0) = \bar{D}_i(\bar{s}, \bar{s}; 1) = 0$, where $\bar{s} \in (0, 1)$ is the unique solution to the equation

$$G\left(\frac{\mu}{\sigma s}\right) - s = 0.$$ 

Thus,

$$s_1^*(0) = s_2^*(0) = s_1^*(1) = s_2^*(1) = \bar{s}.$$ 

In other words, size does not matter for the share of entrepreneurs in the two polar cases in which trading is either prohibitively expensive or costless. Note, however, that all individuals are better off in the latter than in the former case because they have access to a wider array of varieties.

Let us now come to the more interesting case in which $0 < \phi < 1$. We then have:
Observation 3.2. (i) If $m_1 = m_2$, then $s_1^*(\phi) = s_2^*(\phi) = \bar{s}$ for all $\phi \in (0, 1)$.

(ii) If $m_1 > m_2$, then $0 < s_2^*(\phi) < s_1^*(\phi) < 1$ for all $\phi \in (0, 1)$.

Proof. (i) If $m_1 = m_2$, then (3.1) implies that $\bar{w}_1(s, s; \phi)$ is independent of $m_1$ and $m_2$. Hence, we have $s_1^*(\phi) = s_2^*(\phi) = \bar{s}$ for all $\phi \in (0, 1)$.

(ii) If $m_1 > m_2$, then we have $\bar{D}_1(\bar{s}, \bar{s}; \phi) > 0$ and $\bar{D}_2(\bar{s}, \bar{s}; \phi) < 0$ for all $\phi \in (0, 1)$. By Observation 3.1, this implies that $(s_1^*(\phi), s_2^*(\phi))$ lies below the bisector, i.e., $s_1^*(\phi) > s_2^*(\phi)$ for all $\phi \in (0, 1)$.

Since $y_2^\text{min} > \alpha$ by assumption, it must be that $\bar{D}_2(s_1, 0; \phi) > 0$ for any $s_1$ and $\phi$. This implies that $\bar{D}_2(s_1, 0; \phi) = 0$ never intersects the $s_1$-axis, which in turn implies that $(s_1^*(\phi), s_2^*(\phi))$ never lies on the $s_1$-axis, i.e., $s_2^*(\phi) > 0$ for all $\phi \in (0, 1)$. Likewise, $y_1^\text{max} < \bar{\sigma}$ implies $s_1^*(\phi) < 1$ for all $\phi \in (0, 1)$.

From (2.7), observe that the equilibrium $(s_1^*(\phi), s_2^*(\phi))$ always lies on the locus of

$$E(s_1, s_2) := m_1 G^{-1}(s_1) s_1 + m_2 G^{-1}(s_2) s_2 - \frac{\mu}{\sigma} (m_1 + m_2) = 0. \quad (3.2)$$

As in Observation 2.1, this locus is downward sloping in the $s_1$-$s_2$ plane. This implies that, as $\phi$ increases from 0 to 1, $s_1^*(\phi)$ and $s_2^*(\phi)$ always move in opposite directions. Specifically, the equilibrium $(s_1^*(\phi), s_2^*(\phi))$ starts from $(\bar{s}, \bar{s})$, moves continuously along the locus $E(s_1, s_2)$, and ends up at $(\bar{s}, \bar{s})$. We will show below that $(s_1^*(\phi), s_2^*(\phi))$ changes its direction only once. Since $(s_1^*(\phi), s_2^*(\phi))$ is given by the intersection point of $E(s_1, s_2) = 0$ and $\bar{D}_1(s_1, s_2; \phi) = 0$, it suffices to show that, as $\phi$ increases from 0 to 1, $\bar{D}_1(s_1, s_2; \phi) = 0$ changes its direction only once.

Lemma 3.3. For any given $(s_1, s_2)$, the equation $\bar{D}_1(s_1, s_2; \phi) = 0$ has at most two solutions in terms of $\phi$.

Proof. Fix any point $(s_1, s_2)$. We claim that

$$\frac{\partial \bar{D}_1}{\partial \phi} = g(\bar{w}_1) \frac{\partial \bar{w}_1}{\partial \phi}$$
changes its sign at most once. Since \( g(\cdot) > 0 \), it is sufficient to show that \( \partial \bar{w}_1 / \partial \phi = 0 \) has at most one solution, which can be established by direct computation. Hence, \( \bar{D}_1(s_1, s_2; \phi) \) changes its slope at most once, which in turn implies that \( \bar{D}_1(s_1, s_2; \phi) = 0 \) has at most two solutions in \( \phi \).

This has the following implication.

**Observation 3.4.** As \( \phi \) increases from 0 to 1, \( (s_1^*(\phi), s_2^*(\phi)) \) changes its direction only once.

**Proof.** Fix any point \((t_1, t_2)\) on the locus of \( \bar{E}(s_1, s_2) = 0 \). By continuity of the equilibrium with respect to \( \phi \), it is sufficient to show that \((t_1, t_2)\) becomes an equilibrium for at most two distinct values of \( \phi \). Furthermore, \((t_1, t_2)\) is an equilibrium only if it satisfies \( \bar{D}_1(t_1, t_2; \phi) = 0 \) for some \( \phi \). Finally, Lemma 3.3 implies that there exist at most two such \( \phi \)'s.

To sum up, we have:

**Proposition 3.5.** For all \( \phi \in [0, 1] \), there exists a unique entrepreneurship equilibrium \( (s_1^*(\phi), s_2^*(\phi)) \).
(i) For any \( m_1 \) and \( m_2 \), \( s_1^*(0) = s_2^*(0) = s_1^*(1) = s_2^*(1) = \bar{s} \).

(ii) If \( m_1 > m_2 \), then \( s_1^*(\phi) > s_2^*(\phi) > 0 \) for all \( \phi \in (0,1) \). Furthermore, there exists a unique \( \hat{\phi} \in (0,1) \) such that \( \bar{s} < s_1^*(\phi) < s_1^*(\phi') \) and \( \bar{s} > s_2^*(\phi) > s_2^*(\phi') \) when \( 0 < \phi < \phi' \leq \hat{\phi} \), while \( s_1^*(\phi) > s_1^*(\phi') > \bar{s} \) and \( s_2^*(\phi) < s_2^*(\phi') < \bar{s} \) when \( \hat{\phi} \leq \phi < \phi' < 1 \).

(iii) If \( m_1 = m_2 \), then \( s_1^*(\phi) = s_2^*(\phi) = \bar{s} \) for all \( \phi \in (0,1) \).

Figure 2 depicts the entrepreneurship equilibrium as well as the relevant loci. Since both countries exhibit a reversal in the evolution of their industrial structure at \( \hat{\phi} \), we refer to the interval \((0, \hat{\phi})\) as describing the first phase of the integration process, while \((\hat{\phi}, 1)\) corresponds to the second one.

Proposition 3.5 has several important implications. First, the share of entrepreneurs is always larger in the large country than in the small country. This implies that the large country is relatively more specialized in the industrial sector than the small country. This in turn means that the salary of an entrepreneur is higher in the large country than in the small one. Therefore, because \( P_1^* < P_2^* \), both entrepreneurs and workers in country 1 are better off than their counterpart in country 2. Accordingly, we may safely conclude that, once countries have different sizes, spatial frictions in trade generate asymmetries in the international distribution of income and welfare.

Second, the global economy displays a home market effect. Recall that such an effect arises when the large country accommodates a more than proportional share of firms (Helpman and Krugman, 1985). The share of country 1’s industrial firms in the global economy is such that

\[
\frac{n_1^*(\phi)}{n_1^*(\phi) + n_2^*(\phi)} = \frac{m_1 s_1^*(\phi)}{m_1 s_1^*(\phi) + m_2 s_2^*(\phi)} > \frac{m_1}{m_1 + m_2} \tag{3.3}
\]

because \( s_1^*(\phi) > s_2^*(\phi) \) once \( 0 < \phi < 1 \) and \( m_1 > m_2 \). In other words, the share of firms in the large country always exceeds its relative size.

Third, trade liberalization has a dramatic impact on each country’s de-
gree of industrialization. Indeed, trade links the two countries in a way such that one country always develops its industry at the expense of the other. More precisely, during the first phase of integration the number of entrepreneurs increases in the large country but decreases in the small one. This means that the large country gets more industrialized, whereas the small one experiences de-industrialization. During the first phase, the per capita income increases in the large country and decreases in the small one because both $s_1^*$ and $w_1^*$ rise, whereas $s_2^*$ and $w_2^*$ fall. The first phase thus agrees with the prediction made in new economic geography models in which, as trade costs go down, the share of firms grows in the large country, but decreases in the small one (Ottaviano and Thisse, 2004). A major difference is that further economic integration no longer exacerbates international inequalities. Quite the opposite, during the second phase, the small country gradually recoups its industrial basis and the two industrial structures converge. At first glance, the creation of new firms through the development of entrepreneurship might be viewed as a substitute to the international mobility of capital and firms in the global economy. Results are different, however. There are two reasons for that. First, individuals choose to become entrepreneurs, instead of being endowed with units of capital. Second, entrepreneurs invest in their own country, while capital-owners seek the country with the highest rental rate of capital. Thus, individuals face different incentive structures in the two settings.

The above pattern may be explained as follows. As discussed in the introduction, trade liberalization gives rise to two conflicting effects, which shape the global economy. The former, called the market expansion effect, finds its origin in the fact that exporting becomes easier, thus strengthening the incentives to become an entrepreneur. The latter, which we refer to as the competition effect, is the mirror image of the former: as it becomes easier for each country to import new varieties, the incentives to
become an entrepreneur are weaker. Because of strategic substitutability of entrepreneurship decisions between countries, if one effect dominates the other in one country, the reverse must hold in the other one. In order to study the behavior of these two effects as $\phi$ varies from 0 to 1, we take the partial derivative of (3.1) with respect to $\phi$:

$$
\frac{\partial \bar{w}_i}{\partial \phi}(s_1, s_2; \phi) = \frac{m_j s_j}{\sigma} \left[ \frac{\mu m_j}{(m_j s_j + \phi m_i s_i)^2} - \frac{\mu m_i}{(m_i s_i + \phi m_j s_j)^2} \right].
$$

The first bracketed term corresponds to the market expansion effect and the second to the competition effect. Clearly, when $\phi = 0$, we have

$$
\frac{\partial \bar{w}_i}{\partial \phi}(s, s, 0) = \frac{\mu m_j}{\sigma} \left( \frac{1}{m_j} - \frac{1}{m_i} \right).
$$

Hence, the market expansion effect dominates the competition effect for the large country, whereas the opposite holds for the small country. When $\phi = 1$, we have

$$
\frac{\partial \bar{w}_i}{\partial \phi}(s, s, 1) = \frac{\mu m_j}{\sigma s (m_1 + m_2)^2} (m_j - m_i)
$$

so that, for the large country, the competition effect dominates the market expansion effect, whereas the opposite holds for the small country.

Assume now that $\phi \in (0, 1)$. First, recall that $\mu m_i$ is the total expenditure of country $i$’s residents on the industrial good, which is constant regardless of the trade openness $\phi$. Assuming that $m_1 > m_2$, a marginal increase in the access to the foreign market is always larger for the small country firms than for the large country firms. Second, the equilibrium value of the price index being given by

$$
P^*_i = \frac{\sigma}{\sigma - 1} (m_i s_i + \phi m_j s_j)^{\frac{1}{\sigma - 1}}
$$

the term $m_i s_i + \phi m_j s_j$ may be interpreted as the competition index in country $i$. Since $m_1 s^*_1 > m_2 s^*_2$, the competition index in the large country is larger than that in the small country, but the difference between those indices becomes smaller as $\phi$ rises. Consequently, as integration proceeds, the
competition effect becomes stronger relative to the market expansion effect for the large country firms, and vice versa for the small country firms. What Proposition 3.5 says is that, during the first phase of integration, the market expansion effect dominates the competition effect for the large country as it does when $\phi = 0$, so that the number of firms in the large country increases while that of the small one decreases. Conversely, during the second phase, the competition effect dominates the market expansion effect for the large country as it does when $\phi = 1$, so that the large country workers face weaker incentives to get skilled, which in turn implies that some of the small country workers choose to become entrepreneurs.

Two final remarks are in order. First, it is worth stressing that all the properties derived above hold for any distribution of types. Our main assumption is the quasi-linearity of preferences, which allows us to abstract from the income effect and to isolate the competition effect which goes only through the decrease in the price index. Second, when both countries have the same size, all results boil down to a trivial outcome in which the two countries keep the same industrial structure, showing once more that the standard assumption of identical countries conceals several important effects about the impact of trade openness.

### 3.2 Global industrialization

It remains to study the trajectory of the equilibrium as a function of the degree of openness. As $\phi$ changes, the entrepreneurial income changes, thus inducing some individuals to modify their occupational choice. Clearly, the mass of the individuals who switch occupation depends on the shape of the density function $g$. If this function has a complex form, then so may be the equilibrium trajectory. The following assumption imposes a standard regularity condition on the density $g$ that allows us to obtain a simple and neat characterization of the equilibrium trajectory.
Assumption 3.1. The density function $g$ is $\rho$-concave for some $\rho > -1/2$.

Such an assumption is far from being new in the economics literature. It has been introduced by Caplin and Nalebuff (1991a, 1991b) and used extensively in differentiated oligopoly models (see, e.g. Anderson et al. 1992). Note that $\rho$-concavity with $\rho < 0$ is a weaker requirement than log-concavity (0-concavity is equivalent to log-concavity). Hence, our assumption covers the class of log-concave densities, which include many probability distributions such as the beta, Dirichlet, exponential, gamma, Laplace, normal, uniform, and Gumbel distributions.

Thus, we find it fair to say that our $\rho$-concavity assumption imposes a relatively mild restriction on the density function $g$.

Under this regularity condition, we can show the following.

Lemma 3.6. Under Assumption 3.1, the locus of $\bar{E}(s_1, s_2) = 0$ is strictly concave.

Proof. Because $g$ is $-1/2$-concave, the Prékopa-Borell theorem implies that $G$ is $\rho'$-concave for some $\rho' > -1$. This in turn implies that $1/G$ is strictly convex (Caplin and Nalebuff, 1991b). Given (2.7), it then suffices to show that $d(G^{-1}(x)x)/dx = (G^{-1})'(x)x + G^{-1}(x)$ is increasing in $x$ for the statement to hold. Since $(G^{-1})'(x) = 1/G'(G^{-1}(x))$ and since $G^{-1}$ is increasing, this is amount to saying that

$$\frac{G(\alpha)}{G'(\alpha)} + \alpha$$

is increasing in $\alpha$. Taking the derivative of this expression, it is readily verified that this holds if and only if

$$2(G'(\alpha))^2 - G(\alpha)G''(\alpha) > 0$$

for all $\alpha$, which means that $1/G$ is strictly convex. ||

As shown by Prékopa (1971), log-concavity may require restrictions on the parameter values for some of these distributions.
The slope of the locus of \( E(s_1, s_2) = 0 \) is \(-m_1/m_2\) at \((\bar{s}, \bar{s})\). Provided \(m_1 > m_2\), Lemma 3.6 and Observation 2.1 imply that

\[
\left| \frac{dn_1^* (\phi)}{d\phi} \right| < \left| \frac{dn_2^* (\phi)}{d\phi} \right|
\]

for all \(\phi \neq 0, 1, \hat{\phi}\). Therefore, the size of the industrial sector in the small country is more sensitive to variations in trade obstacles than in the large one.

Denote by \(N^*(\phi)\) the total number of firms in the whole economy:

\[
N^*(\phi) = m_1 s_1^*(\phi) + m_2 s_2^*(\phi).
\]

Since \(n_2^*\) decreases (respectively, increases) faster than \(n_1^*\) (respectively, decreases) over the interval \((0, \hat{\phi})\) (resp., \((\hat{\phi}, 1))\), we have:

**Proposition 3.7.** Suppose that \(m_1 \neq m_2\). Under Assumption 3.1, \(N^*(\phi)\) decreases over \((0, \hat{\phi})\) but increases over \((\hat{\phi}, 1)\).

In other words, *as trade barriers are gradually removed, the global economy experiences de-industrialization, but faces re-industrialization when international integration gets sufficiently deep.* By making the whole array of varieties available in the global economy accessible to all consumers, the first integration phase induces less individuals to become entrepreneurs. On the other hand, during the second phase, the global market is sufficiently integrated to make the incentives to get entrepreneurs stronger and to bring the level of industrialization back to its initial level.

At this stage, it is worth stressing the analogy between the foregoing proposition and the bell-shaped curve of spatial development obtained in economic geography (Ottaviano and Thisse, 2004). In the former, there is no labor mobility between countries, but occupational choice makes endogenous the industrial structure of each country. In the latter, there is no sectoral mobility of labor, but the migration of workers between countries or regions permits the emergence of economic agglomerations. The analogy lies in the
fact that, during the first phase of integration, the two countries become more dissimilar, while their industrial structure converges during the second one.

4 The impact of externality and heterogeneity

We now assume that there are agglomeration externalities in entrepreneurship and study how such externalities affect the formation of the trading partners’ industrial structure. To achieve our goal, we use a general reduced-form expression $h_i$ and do not describe the various processes that stand behind such externalities. This does not entail any significant loss of generality since our analysis is positive. In order to focus on the pure impact of these external economies, we assume that the two countries are identical. This means that they have the same size, i.e., $m_1 = m_2$, which is normalized to 1 so that $n_i = s_i$, the same density of types, i.e., $F_1 = F_2 = F$ (with a density $f$ and a common support $[\alpha, \bar{\alpha}]$), and the same externality function, i.e., $h_1 = h_2 = h$ with $h(\cdot) > 0$ and $h'(\cdot) \geq 0$. In such a context, there is always a symmetric entrepreneurship equilibrium. Yet, in the presence of externalities, equilibria involving very uneven factor distributions typically exist as well (Cooper and John, 1988; Matsuyama, 1991). We show that the same holds in our setting once externalities are sufficiently strong.

We then revisit these issues by stressing the stabilizing effect of the heterogeneity of individuals. To this end, we distinguish between local and global heterogeneity. We demonstrate that sufficient local heterogeneity makes the symmetric equilibrium locally stable, while a sufficient amount of global heterogeneity leads to the uniqueness, hence the global stability, of this equilibrium, thus showing how restrictive is the assumption of identical individuals in models with externalities.
4.1 Externality and multiplicity

As a preliminary step, we identify a sufficient condition under which our setting displays multiple equilibria. Recall that all equilibria are given by the intersection points of the following two loci:

\[ D_1(n_1, n_2) = F[y_1(n_1, n_2)] - n_1 = 0 \]
\[ D_2(n_1, n_2) = F[y_2(n_1, n_2)] - n_2 = 0. \] (4.1)

Let \( \bar{n} \in [0, 1] \) be the unique solution to

\[ F\left(\frac{\mu}{\sigma n}\right) = n. \] (4.2)

Observe that the symmetric state \((\bar{n}, \bar{n})\) is always an equilibrium irrespective of \(h(\cdot)\) and \(\phi\). Let

\[ \bar{y} = y_i(\bar{n}, \bar{n}) = \frac{\mu}{\sigma \bar{n}}. \] (4.3)

In order to check whether there exist other equilibria, we study the shape of the two loci \(D_1(n_1, n_2) = 0\) and \(D_2(n_1, n_2) = 0\) in the \(n_1-n_2\) plane. First, the locus \(D_i(n_1, n_2) = 0\) intersects the \(n_j\)-axis with \(j \neq i\) at \(n_j > 1\) because \(\alpha < y_i^{\text{min}}\). Likewise, \(D_i(n_1, n_2) = 0\) intersects the \(n_i\)-axis at \(n_i < 1\) because \(y_i^{\text{max}} < \alpha\). Thus, due to continuity, the two loci intersect at other points if

\[ -\frac{\partial D_1}{\partial n_1}(\bar{n}, \bar{n}) / \frac{\partial D_1}{\partial n_2}(\bar{n}, \bar{n}) > -1, \]
or

\[ f(\bar{y}) \left[ \frac{\partial y_1}{\partial n_1}(\bar{n}, \bar{n}) - \frac{\partial y_1}{\partial n_2}(\bar{n}, \bar{n}) \right] - 1 > 0 \] (4.4)

since \(\partial y_i/\partial n_2 < 0\). Note that, by symmetry, this condition implies that the same holds for \(i = 2\).

Let \(\eta_h\) be the elasticity of the externality function \(h\) evaluated at \(n = \bar{n}\):

\[ \eta_h = \frac{\bar{n} h'(\bar{n})}{h(\bar{n})}. \]
Note that an increasing value of $\eta_h$ means that the agglomeration externality, evaluated at the symmetric equilibrium, becomes more reactive to a small deviation in the mass of entrepreneurs.

Let $\eta_F$ be the elasticity of the distribution function $F$ evaluated at $\alpha = \bar{y}$:

$$\eta_F = \frac{\bar{y}f(\bar{y})}{F(\bar{y})}.$$  

Since $f(\bar{y})\bar{y} = \eta_F F(\bar{y})$ and $F(\bar{y}) = \bar{n}$ by definition of $\bar{n}$, it must be that $f(\bar{y})\bar{y}/\bar{n} = \eta_F$. Hence, the sufficient condition (4.4) implies the following result.

**Proposition 4.1.** Assume that the two countries are identical. Then, there always exists a symmetric entrepreneurship equilibrium, whereas asymmetric equilibria also exist if

$$\eta_h > \frac{(1 - \phi)^2 + (1 + \phi)^2/\eta_F}{4\phi}. \quad (4.5)$$

Since the right hand side of (4.5) decreases from $\infty$ to $1/\eta_F$ as $\phi$ increases from 0 to 1, for low degrees of trade openness the externality at the symmetric equilibrium must be strong for the condition (4.5) to hold. Consequently, once agglomeration economies are at work, trade liberalization is likely to foster the emergence of asymmetric industrial structures between trading partners that are otherwise identical.

It is readily verified that

$$\frac{\partial y_1}{\partial n_1}(\bar{n}, \bar{n}) = \frac{2\phi \bar{n}h'(\bar{n}) - (1 + \phi^2)h(\bar{n})}{(1 + \phi)^2\bar{n}h(\bar{n})} = \frac{2\phi}{(1 + \phi)^2} \bar{y} \left( \eta_h - \frac{1 + \phi^2}{2\phi} \right) \quad (4.6)$$

and

$$\frac{\partial y_1}{\partial n_2}(\bar{n}, \bar{n}) = -\bar{y} \frac{2\phi[\bar{n}h'(\bar{n}) + h(\bar{n})]}{(1 + \phi)^2\bar{n}h(\bar{n})} = -\frac{2\phi}{(1 + \phi)^2} \bar{y} (\eta_h + 1).$$
As expected, an increase in the number of entrepreneurs $n_1$ gives rise to two opposite effects in country 1. The former stems from the agglomeration externality that, everything else equal, fosters an increase in the entrepreneurs’ income by lowering the fixed cost they bear to launch a new variety. The latter is due to the more intense competition unleashed by the larger number of locally produced varieties; as usual, it tends to lower $y_1$. When there is autarky, the agglomeration externality has no impact since $y_1(n_1, n_2) = \mu/\sigma n_1$ is independent of $h(\cdot)$. However, once the two economies are open to trade, this externality may affect the industrial structure of both countries. Note first that, when there are no externalities ($h'(n) \equiv 0$), the only force at work is the competition effect so that

$$\frac{\partial y_1}{\partial n_1}(\bar{n}, \bar{n}) < 0.$$  

As seen in Section 3, the symmetric equilibrium is thus unique. The competition effect still dominates when the intensity of the agglomeration externality is weak. On the other hand, once the agglomeration externality effect is sufficiently large, there also exist asymmetric entrepreneurship equilibria, involving a larger number of entrepreneurs in one country than in the other. Specifically, (4.6) shows that the net effect of increasing $n_1$ is positive for country 1 entrepreneurs when the agglomeration externality at the symmetric equilibrium is sufficiently strong for

$$\eta_h > \frac{(1 + \phi^2)}{2\phi}. \quad (4.7)$$

to hold. This condition may be given a nice interpretation. For that, observe that $\partial y_1/\partial n_1$ can be decomposed as follows. Let $y_1^H$ be the first term in (2.4) and $y_1^F$ the second term. Then, we have

$$\frac{\partial y_1^H}{\partial n_1}(\bar{n}, \bar{n}) = \frac{\phi}{(1 + \phi)^2} \frac{\bar{y}}{\bar{n}} \left( \eta_h - \frac{1}{\phi} \right)$$

and

$$\frac{\partial y_1^F}{\partial n_1}(\bar{n}, \bar{n}) = \frac{\phi}{(1 + \phi)^2} \frac{\bar{y}}{\bar{n}} (\eta_h - \phi).$$
Both effects are positive when $\eta_h$ exceeds $1/\phi$ and $\phi$. Condition (4.7) thus holds if and only if $\eta_h$ is larger than the arithmetic mean of $1/\phi$ and $\phi$. In the limit when there is no trade cost, the condition boils down to $\eta_h > 1$.

Note, finally, that an increase in $n_2$ has only a competition effect in country 1 since the externality is localized. This implies that $\partial y_1/\partial n_2$ is always negative.

In what follows, we show that the heterogeneity of individuals has a stabilizing effect in that heterogeneity tends to work in the opposite direction from externality. To this end, we use the concepts of local and of global heterogeneity.

4.2 Local heterogeneity and stability

To study the local stability of equilibria, we consider the myopic best response dynamics given by

$$
\begin{align*}
\dot{n}_1(t) &= F[y_1(n_1(t), n_2(t))] - n_1(t) \\
\dot{n}_2(t) &= F[y_2(n_1(t), n_2(t))] - n_2(t).
\end{align*}
$$

(4.8)

Observe that the set of entrepreneurship equilibria is identical to the set of rest points of this dynamics. We identify a local condition on the distribution of types for the symmetric equilibrium $(\bar{n}, \bar{n})$ to be stable with respect to the dynamics (4.8).

We know from the foregoing that the symmetric equilibrium $(\bar{n}, \bar{n})$ is stable if

$$
-\frac{\partial D_1(\bar{n}, \bar{n})}{\partial n_1} / \frac{\partial D_1(\bar{n}, \bar{n})}{\partial n_2} < -1
$$

or, equivalently, if

$$
\eta_h < \frac{(1 - \phi)^2 + (1 + \phi)^2}{\eta F}. 
$$

(4.9)

We borrow the concept of spread from Rothschild and Stiglitz (1970) and restrict our attention to spread around the symmetric equilibrium $\bar{y}$. 

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**Definition 4.1.** A distribution $\tilde{F}$ is a spread of $F$ around $\bar{y}$ if $\tilde{F}(\bar{y}) = F(\bar{y})$, and $\tilde{F}(\alpha) > F(\alpha)$ for all $\alpha < \bar{y}$ and $\tilde{F}(\alpha) < F(\alpha)$ for all $\alpha > \bar{y}$.

When $\tilde{F}$ (with the density $\tilde{f}$) is a spread of $F$, it must be that

$$\tilde{f}(\bar{y}) < f(\bar{y}).$$

By continuity of the two densities, this inequality must hold in some neighborhood $(\bar{y}-\varepsilon, \bar{y}+\varepsilon)$ of $\bar{y}$. In other words, spreading $F$ around $\bar{y}$ implies that its density gets smaller in $(\bar{y}-\varepsilon, \bar{y}+\varepsilon)$. This in turn means that individuals whose types are in the vicinity of $\bar{y}$ are more dispersed under $\tilde{F}$ than under $F$. Hence, by spreading the distribution of types around $\bar{y}$, we can make the population more heterogeneous around $\bar{y}$. This is why we may refer to it as local heterogeneity.

We are now equipped to show that a sufficient amount of heterogeneity around $\bar{y}$ leads to the stability of $(\bar{n}, \bar{n})$. Indeed, when $\tilde{f}(\bar{y})$ tends to zero, $\eta_F$ also tends to zero, so that the right hand side of (4.9) goes to infinity. This implies that $(\bar{n}, \bar{n})$ is stable as long as $\tilde{f}(\bar{y})$ is sufficiently small. Thus, we have:

**Proposition 4.2.** If the distribution of types around $\bar{y}$ is sufficiently heterogeneous, then the symmetric equilibrium $(\bar{n}, \bar{n})$ is locally stable.

Intuitively, this proposition may be understood as follows. When the system (4.1) is perturbed around $(\bar{n}, \bar{n})$, individuals who are affected by the resulting change in income are those whose types are close to $\bar{y}$, whereas individuals whose types are away from $\bar{y}$ remain unaffected. At $(\bar{n}+\varepsilon, \bar{n}-\varepsilon)$ where $\varepsilon > 0$, some country 1 individuals have a higher income and some country 2 individuals have a lower income than what they earn at $(\bar{n}, \bar{n})$. The fraction of country 1 individuals affected by this income rise is given by

$$\tilde{f}(\bar{y}) \left( \frac{\partial y_1}{\partial n_1} \varepsilon - \frac{\partial y_1}{\partial n_2} \varepsilon \right) + o(\varepsilon).$$
If many individuals are concentrated around $\bar{y}$, then $\tilde{f}(\bar{y})$ is much greater than $\varepsilon$. Thus, because of the externality function, the income of country 1 potential entrepreneurs is further increased in an interval that includes $(\bar{n} + \varepsilon, \bar{n} - \varepsilon)$. This in turn sparks a further increase in $n_1$, which moves further away from $\bar{n}$. On the contrary, if individuals are widely dispersed around $\bar{y}$, then $\tilde{f}(\bar{y})$ is much lower than $\varepsilon$. In this case, the individuals affected by the income rise are too few for the externality to amplify the perturbation. As a result, $n_1$ goes back towards $\bar{n}$, which is stable. A similar argument applies to $n_2$.

4.3 Global heterogeneity and uniqueness

To capture the idea of global heterogeneity, we borrow the parameterization proposed by Herrendorf et al. (2000). Given $F$, let $\bar{n} \in (0, 1)$ and $\bar{y} \in (\alpha, \bar{\alpha})$ be as in (4.2) and (4.3), respectively. Then, define $F(\cdot | \gamma, \bar{y})$ as

$$F(\alpha | \gamma, \bar{y}) = F(\gamma \alpha + (1 - \gamma)\bar{y})$$

where $\gamma \in (0, \infty)$. Note that

$$f(\alpha | \gamma, \bar{y}) = \gamma f(\gamma \alpha + (1 - \gamma)\bar{y})$$

is the density of $F(\alpha | \gamma, \bar{y})$ on the support

$$\left[ \alpha - \frac{1 - \gamma}{\gamma} (\bar{y} - \alpha), \bar{\alpha} + \frac{1 - \gamma}{\gamma} (\bar{\alpha} - \bar{y}) \right].$$

(4.10)

Thus, a lower value of $\gamma$ implies a more heterogeneous population of individuals. Indeed, as $\gamma$ decreases, the whole density of types gets more spread over a broader domain. By taking such a transformation of $F$, the population of individuals exhibits a wider array of types while, for all types, less individuals share the same one. Observe, however, that $F(\bar{y} | \gamma, \bar{y}) = F(\bar{y})$ so that the value of $F(\alpha | \gamma, \bar{y})$ at $\bar{y}$ is the same as the value of the initial distribution there. Consequently, $(\bar{n}, \bar{n})$ remains the unique symmetric equilibrium of
the economy when types are distributed according to the $\gamma$-transformation of $F$.

Note also that $\eta_F(\cdot|\gamma, \bar{y}) = \gamma \eta_F(\cdot)$, whereas the system (4.1) must be replaced by

$$D_i(n_1, n_2|\gamma, \bar{y}) = F(y_i(n_1, n_2)|\gamma, \bar{y}) - n_i$$

so that $(\bar{n}, \bar{n})$ is the intersection point of the two loci $D_1(n_1, n_2|\gamma, \bar{y}) = 0$ and $D_2(n_1, n_2|\gamma, \bar{y}) = 0$.

We have the following.

**Proposition 4.3.** There exists $\bar{\gamma} > 0$ such that the symmetric equilibrium $(\bar{n}, \bar{n})$ is unique and globally stable when $\gamma < \bar{\gamma}$.

**Proof.** See the Appendix.

Thus, despite the presence of an agglomeration externality, when individuals display enough contrasted attitudes toward entrepreneurship, the symmetric outcome is the only equilibrium.

We provide a numerical example to illustrate Propositions 4.2 and 4.3. Consider a uniform distribution of types over (4.10) with $y_0 = y_2(0, 1) = y_2(1, 0)$, $\alpha = y_0$, and $\bar{\alpha} = 2\bar{y} - y_0$, where $\bar{y}$ is given by (4.3). The externality function is such that

$$h(n_i) = 1 + A n_i^\beta.$$

The parameter values are as follows: $m_1 = m_2 = 1$, $\beta = 3.3$, $A = 100$, $\sigma = 3$, $\phi = 0.17$, and $\mu = 10$. Under these parameters, the symmetric equilibrium is given by $(\bar{n}, \bar{n}) = (0.5, 0.5)$.

Figures 3(a) to 3(c) describe the changes in the correspondence of equilibria associated with $\gamma$ decreasing from 1 to 0.5. The equilibrium condition for country 1 is represented by the solid line, while the dashed line describes this condition for country 2. In Figure 3(a) where $\gamma = 1$ and $[\alpha, \bar{\alpha}] = [-1.95, 15.28]$, it is readily verified that the symmetric equilibrium is unstable. In Figure 3(b) where $\gamma = 0.75$ and $[\alpha, \bar{\alpha}] = [-1.95, 15.28]$,
the symmetric equilibrium becomes stable although there exist two asymmetric stable equilibria. In Figure 3(c), which is drawn for $\gamma = 0.5$ and $[\alpha, \bar{\alpha}] = [-6.26, 19.60]$, the symmetric equilibrium is unique and globally stable. Note that the lower bound of the support becomes negative when $\gamma$ is close to zero, thus violating our assumption that types are non-negative. We have chosen to keep the example as is because it is extremely simple and illustrative.

Let us take a pause in order to better understand why heterogeneity may
affect so deeply the nature of results. When individuals are identical, they all react in the same way, thus generating bang-bang aggregate behavior. The most famous example is undoubtedly given by the standard Bertrand model in which all consumers patronize the cheapest firm. In this case, it is easy to figure out why, when an agglomeration externality is at work, a minor deviation from the symmetric equilibrium gives rise to an unravelling process. By contrast, when individuals are locally heterogeneous, their aggregate behavior around the symmetric equilibrium is smoothed out. Furthermore, as individuals become globally more heterogeneous, i.e., as $\gamma$ decreases, the mass of individuals who choose to become an entrepreneur, regardless of the decisions made by the others, increases, and so does the mass of those who choose to be workers. In such a context, the sluggish behavior of individuals is less and less driven by the externality effect and, eventually, leads to the uniqueness, hence the global stability, of the symmetric equilibrium. Herrendorf et al. (2000) have made the same point in a model of occupational choice with infinitely lived agents and a closed economy, while we study the impact of trade within a general equilibrium model.

The above result is reminiscent of de Palma et al. (1985) and Anderson et al. (1990) who prove that a sufficient amount of heterogeneity guarantees the existence of an equilibrium in location and voting games. When individuals are heterogeneous, their sluggish aggregate behavior makes firms understand that they no longer gain a large share of customers by moving close to their ideal points. Furthermore, more heterogeneity makes the symmetric equilibrium unique because this leads firms to choose a location which yields the best average match between firms and dispersed consumers. Tabuchi and Thisse (2002) also show how global heterogeneity across potential migrants damper the agglomeration process in the core-periphery model, where the concentration of workers is driven by a pecuniary externality. Like us, they prove that a sufficient amount of heterogeneity across individuals sustains
the symmetric pattern as the only stable equilibrium. In the same spirit, McKelvey and Palfrey (1995) have shown that any finite game has a unique equilibrium when there is enough heterogeneity. It should be emphasized that all these results are obtained under the assumption that randomness is independent across agents. The global game literature, on the contrary, has shown that small but highly correlated heterogeneity leads to uniqueness (Morris and Shin, 2003).\footnote{For a unified analysis of these results, see Morris and Shin (2005) and Ui (2006).}

In many contemporary models of trade, industrial organization and development, externalities are used to show that strongly asymmetric equilibria may arise. In this perspective, Propositions 4.2 and 4.3 are potentially important because there is ample evidence that heterogeneity across individuals is pervasive, whether externality effects are or not. We may, therefore, expect heterogeneity across individuals to play an important role in the determination of the industry structure and the effects of trade liberalization.

5 Conclusion

Our results shed light on a topic that has been too much neglected, that is, the impact of trade on the formation of entrepreneurship. In particular, we have shown that the creation of new firms critically depends on the level of trade costs and the relative size of the trading partners. During the integration process, two very distinct phases emerge. In the first one, the share of the large country industry grows at the expense of that of the small country, whereas the opposite holds in the second phase. Note, in passing, that our analysis reveals once more the importance of the size effect since the share of each country is the same and invariant with respect to the level of trade costs when countries are identical. Furthermore, unlike many new trade models, our setting allows for a variable number of varieties, both at
the national and international levels. In this respect, it is worth noting that trade liberalization, first, leads the industry to shrink at the global level and, then, to expand. Thus, even though each variety is consumed both locally and abroad, consumers have access to a number of goods that varies with the level of trade costs.

Since our model bears some resemblance with economic geography, it is worth pursuing the comparison sketched in the foregoing. Through the creation of new firms, a country may develop a (much) bigger industrial sector than its trading partners. When agglomeration externalities are strong enough, one country might even accommodate a very large share of this sector. This is reminiscent of the core-periphery model proposed by Krugman (1991), in which agglomeration stems from the spatial mobility of firms and workers. However, contrary to us, low trade costs do not spark international convergence in Krugman’s model. The bell-shaped curve of spatial development mentioned above is obtained once crowding forces that make the large agglomeration less attractive are added to the model. Therefore, unlike what Mundell (1957) thought, commodity trade and factor mobility are not necessarily good substitutes.

It should be emphasized, however, that the possible emergence of trade-driven international disparities needs qualification. Indeed, when potential entrepreneurs show very different opportunity costs, heterogeneity across individuals tends to reduce such disparities. The comparison made with other models also suggests that agents’ heterogeneity could well be a general force that would destroy equilibria characterized by extreme features, thus pushing toward more balanced market outcomes. In a way, this is not a totally new idea. Ever since Hotelling (1929), we know that heterogeneity is often sufficient to get rid of knife-edge results obtained when economic agents are homogeneous. Clearly, more work is called for.

Finally, it is worth mentioning that our analysis confirms a well-established
result, namely income inequalities within countries vary with the degree of trade openness. However, we have seen that they also vary with the size of the trading partners, a fact that has been overlooked in the literature. Growing inequalities may induce national governments to implement redistributational policies. The decision of a country to do so should trigger international reactions because it affects the welfare level in the others through the access of their residents to the array of varieties. Some preliminary analysis suggests that, for some levels of trade costs, a simultaneous move toward less inequality could be harmful to both countries. We leave this important topic for future research.

Appendix

A.1 Proof of Proposition 4.3

Lemma A.1. Assume that the density $f$ is continuous. Then, $f(\cdot|\gamma, \bar{y})$ converges uniformly to zero as $\gamma \to 0$ on any compact interval in $\mathbb{R}$.

Proof. Fix any compact interval $I \subset \mathbb{R}$. Recall that the support of $F(\cdot|\gamma, \bar{y})$, given by

$$[\alpha - \frac{1 - \gamma}{\gamma}(\bar{y} - \alpha), \bar{\alpha} - \frac{1 - \gamma}{\gamma}(\bar{\alpha} - \bar{y})]$$

arbitrarily expands as $\gamma \to 0$. In what follows, we restrict ourselves to values of $\gamma$ that are small enough for the support of $F(\cdot|\gamma, \bar{y})$ to include the interval $I$. Since $f$ is continuous, so is $f(\cdot|\gamma, \bar{y})$. Hence, it is sufficient to show that $f(\cdot|\gamma, \bar{y})$ converges pointwise to zero on the compact interval $I$ as $\gamma \to 0$.

By continuity of $f$, there exists some $M > 0$ such that $f(\alpha) \leq M$ for all $\alpha \in [\alpha, \bar{\alpha}]$. It therefore follows that for any $\alpha \in I$,

$$f(\alpha|\gamma, \bar{y}) = \gamma f(\gamma \alpha + (1 - \gamma)\bar{y}) \leq \gamma M \to 0$$

as $\gamma \to 0$. ||
Proof of Proposition 4.3. Because of symmetry, it is sufficient to show that there exists $\gamma > 0$ such that for all $\gamma \in (0, \gamma)$, the slope of the $D_1$-locus at $(n_1, n_2)$ is smaller than $-1$ when $(n_1, n_2)$ is an equilibrium.

Recall (see Subsection 2.3) that under the assumption $h(0) > 0$, the equilibrium salary $y_1$ has a positive lower bound $y_{\min} > 0$ and an upper bound $y_{\max}$, which are independent of the distribution (and hence, of $\gamma$). When the interval $(\underline{\alpha}, \overline{\alpha})$ is wide enough to include $[y_{\min}, y_{\max}]$, we have $F(y_{\min}) > 0$. Any equilibrium thus belongs to the compact set $[F(y_{\min}), F(y_{\max})] \times [F(y_{\min}), F(y_{\max})]$. Since $y_{\min} < \bar{y} < y_{\max}$, for any $\gamma < 1$ we have $F(y_{\min}) < F(y_{\min} | \gamma, \bar{y})$ and $F(y_{\max} | \gamma, \bar{y}) < F(y_{\max})$.

Observe that $\partial y_1/\partial n_1$ and $\partial y_1/\partial n_2$ are continuous on $[F(y_{\min}), F(y_{\max})] \times [F(y_{\min}), F(y_{\max})]$, and thus there are constants $K_1 > 0$ and $K_2 > 0$ such that $|(\partial y_1/\partial n_1)(n_1, n_2)| \leq K_1$ and $|(\partial y_1/\partial n_2)(n_1, n_2)| \leq K_2$ for all $(n_1, n_2) \in [F(y_{\min}), F(y_{\max})] \times [F(y_{\min}), F(y_{\max})]$. It follows from Lemma A.1 that, for all $\epsilon > 0$, there exists $\bar{\gamma}(\epsilon) > 0$ such that $|f(y_1(n_1, n_2) | \gamma, \bar{y})| < \epsilon / \max\{K_1, K_2\}$ for all $\gamma < \bar{\gamma}(\epsilon)$ and any equilibrium $(n_1, n_2)$. Since $(\partial y_1/\partial n_1)(n_1, n_2) > 0$ and $(\partial y_1/\partial n_2)(n_1, n_2) < 0$, this implies that, for all $\epsilon > 0$, if $\gamma < \bar{\gamma}(\epsilon)$, then

$$
-1 < \frac{\partial D_1}{\partial n_1}(n_1, n_2 | \gamma, \bar{y}) = f(y_1(n_1, n_2) | \gamma, \bar{y}) \frac{\partial y_1}{\partial n_1}(n_1, n_2) - 1
\leq K_1 f(y_1(n_1, n_2) | \gamma, \bar{y}) - 1 < -1 + \epsilon
$$

and

$$
0 > \frac{\partial D_1}{\partial n_2}(n_1, n_2 | \gamma, \bar{y}) = f(y_1(n_1, n_2) | \gamma, \bar{y}) \frac{\partial y_1}{\partial n_2}(n_1, n_2)
\geq -K_2 f(y_1(n_1, n_2) | \gamma, \bar{y}) > -\epsilon
$$

for all equilibria $(n_1, n_2)$. Thus, setting $\bar{\gamma} = \bar{\gamma}(1/2)$, we have that if $\gamma < \bar{\gamma}$, then

$$
\frac{\partial D_1}{\partial n_1}(n_1, n_2 | \gamma, \bar{y}) \sqrt{\frac{\partial D_1}{\partial n_2}(n_1, n_2 | \gamma, \bar{y})} < -1
$$

for all equilibria $(n_1, n_2)$. ||
References


