Asymmetric Information, Auditing Commitment, and Economic Growth

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Abstract

This paper examines the commitment effects of auditing contract in a credit market in which the costly state verification (CSV) problem prevails. In particular, we compare two endogenous growth models of which one allows lenders to commit to a costly auditing strategy to identify the investment returns of borrowers and the other does not. To model the interaction between lenders and borrowers in the no-commitment case, we follow the framework pioneered by Khalil (1997). We find that the regime without commitment is associated with a higher loan rate, a higher auditing probability, lower economic growth rate and lower social welfare.

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1 Introduction

There has been a growing interest, and body of work, in the literature in applying principal-agent contract theories to analyze macroeconomic issues such as business cycles, capital accumulation, and economic growth. For example, Williamson (1986, 1987) study the impact of equilibrium credit rationing with the problem of costly state verification. Bernake and Gertler (1989) argue that when agency costs are inversely related to borrowers’ net worth, external random shocks that affect agency costs may initiate economic fluctuations. Bose and Cothren (1996, 1997) show adverse effects of ex-ante asymmetric information on growth in a model allowing for rationing and screening contracts. More recently, Ho and Wang (2005) examine the impact of asymmetric information in the credit market on taxation policy and growth in an endogenous growth model with tax-financed public capital. However, it is well recognized that many of these applications do not satisfy the time consistency property, as long as the self selection equilibrium is called for and there are costs associated with the principal in sustaining such an equilibrium. This problem of time (in)consistency can be simply stated as follows. In a typical contract environment with adverse selection, if agents will be self selected according to their own types in equilibrium, the principal has the incentive to forgo the costly activities, such as screening or auditing as the case may be, that are specified in equilibrium contracts and needed in order to induce the self selection of agents at the first place.

Specifically, we consider in the present paper a contract environment with the presence of adverse selection and costly state verification. Assume that investors in the economy seek for funds to implement their investment projects, which can yield two possible returns (high or low) depending on the realizations of external random shocks. As is typical in a principal-agent situation, the return of an investment project is known only to the investor (the agent) himself.\(^1\) In this context, the self-selection equilibrium contracts assume the form that the borrowers who report low return are audited with a positive probability while those who report high returns are never audited. In the literature, it is widely assumed that the lenders can commit to the auditing policy with some pre-announced probability of audit. But such an assumption on lenders’ commitment is indeed questionable since, once borrowers are induced to reveal their true output levels under the self-selecting contractual terms, it is no longer optimal (nor necessary) for lenders to audit, and hence incur the corresponding cost, anymore. Then, how can the contract with positive auditing probability be supported as an equilibrium? This issue may become more problematic as we take into account that in most of the literature mentioned above the interaction between borrowers and

\(^1\)Alternatively, one may consider the case where ex-ante information asymmetry is the primary concern, i.e., the risk types of investors are unknown to lenders.
lenders lasts for one period only, which precludes the possibility of using any reputation mechanisms to resolve the time inconsistency problem. This problem has strong empirical relevancy as one recognizes the prevalence of weak contractual enforcement in many underdeveloped countries.

This paper intends to resolve this time inconsistency problem and explore its implications on economic growth and welfare in an endogenous growth framework. Following the approach in Khalil (1997) and Khalil and Parigi (1998), where the lender cannot commit to a auditing strategy, the interaction between a lender and a borrower can be considered as a two-stage game. In the first stage of the game, lenders choose the optimal terms of the contracts. In the second stage, while taken the incentive contractual terms as given, lenders and borrowers choose their equilibrium auditing and cheating strategies respectively. In this stage, auditing takes place only if the lender can be provided with enough incentive to do so. This condition is satisfied only if the offered contract induces a certain level of cheating behavior in equilibrium from borrowers who experienced good output shocks. Therefore, both cheating and auditing will occur with a positive probability, implying a mixed-strategy equilibrium for both borrowers and lenders in the second stage of the game. In this mixed-strategy equilibrium, the probability of auditing makes the high risk borrowers to be indifferent between auditing and complying. Correspondingly, the probability of cheating also makes the lenders to have equal expected payoffs from the states of auditing and no-auditing.

We are going to develop and compare two endogenous growth models with credit market frictions similar to those in Bernanke and Gertler (1989) and Bhattacharya (1998). They are overlapping generations economies populated with heterogeneous agents who live for two periods. In each model, lenders/workers earn their wage income from supplying their endowed labor force in the labor market. Borrowers/enterpreneurs who are endowed with capital producing projects approach lenders for loans to implement their projects. However, while in the first model lenders can commit to a costly auditing strategy to induce self-selection from borrowers in equilibrium, they are not able to do so in the second model. As a consequence, interactions between lenders and borrowers can be modeled in a two-stage game. Through contrasting the two regimes, we want to study how the assumption regarding lenders’ ability to commit may affect the equilibrium contract terms, economic growth rate, and social welfare. It is shown that the regime without commitment to audit is associated with lower economic growth rate and lower social welfare, which seems to accord well with the view that weak contractual enforcement can adversely affect economic development, as argued by La Porta et al (1998) and Levine (1999). Empirically, the issue of commitment has a strong relevancy to economic development in view of the fact that contractual enforcement usually

\[2\] Bose & Cothren (1996) rely on a brand name that each lender purchases from a lender of the previous generation as a commitment device to solve the time inconsistency problem - only if a lender honors the brand name by implementing the terms of separating contracts, he can sell his own brand name in the future.
involves huge transaction costs due to some institutional factors.

More precisely, we study an overlapping generations economy populated with heterogeneous agents who live for two periods. Lenders who are endowed with one unit of time are responsible for credit provision. When lenders are young, they supply their endowed unit of labor force to earn wage rates. Subsequently, they supply their wages as the only source of investment funds in the credit market. When they become old, they consume the interest payment from the loan they made when young. Borrowers who are endowed with investment projects approach lenders for investment funds. The return of an investment project can take on two possible values, which are known only to their owners not to the lenders. It is this feature which gives rise to the problem of informational frictions. By adopting a costly auditing technology, lenders are able to detect borrowers’ true investment returns error free. If a borrower is caught cheating, his investment output will be fully appropriated by the lender. As a result, this borrower consumes nothing when he is old. The auditing probability is pre-announced in the contracts under the regime with commitment, whereas this probability is not explicitly specified as a part of the contract under the regime without commitment.

This paper has the following main findings. First, while both commitment and no-commitment contracts offer the same loan rate to borrowers who report low returns, the no-commitment contract is associated with a higher loan rate to those borrowers who report high returns. This is because when commitment to audit is not feasible, the optimal contract will induce some cheating activities from borrowers with high investment return, which in turn will lower lender’s expected payoff in the high-return state since a fraction of borrowers in this state will under report. As a result, lenders have to charge a higher loan rate from borrowers who truthfully report high returns in order to compensate for the loss of revenue arising from the cheating borrowers in this state. Second, comparing to its counterpart under the commitment regime, the equilibrium auditing probability when there is no commitment will be higher. This is because, since the higher loan rate under the no-commitment regime will also increase the incentives for these borrowers to cheat, the auditing probability then must increase in order to keep the borrowers indifferent between cheating and complying, as analogous to Khalil (1997). Since auditing activities expends capital goods, the regime with no-commitment is also associated with a lower economic growth rate. In addition, due to the higher loan rate that borrowers with high investment returns need to pay in no-commitment regime, social welfare is smaller as well.

The rest of the paper proceeds as follows. Section 2 lays out the basic environment. We devote section 3 to the regime when lenders can commit to audit. The regime under no commitment is characterized in section 4. Section 5 is the main section, in which we compare the economic growth
rates and the social welfare of these two regimes derived in previous sections. We conclude and discuss some possible extensions in section 6.

2 The Environment

The basic framework of our model is similar to Bernanke and Gertler (1989) and Bhattacharya (1998). In the economy, there is an infinite sequence of two-period lived overlapping generations. All generations are identical in size and composition, with each generation consisting of a continuum of agents whose measure is normalized to two. Young agents of each generation are equally divided into lenders/workers and borrowers/entrepreneurs. Lenders are workers who are endowed with one unit of labor when young, which is then supplied inelastically on the labor market at the competitive wage rate. Lenders’ wage income provides the only source of fund in the economy. On the other hand, each borrower is an entrepreneur who is endowed with a project that produces capital goods and needs be financial. For simplicity, we assume that $\pi_1 + \pi_2 = 1$. A borrower will become a firm owner in his old age. After earning the market wage, a young lender can lend his wage income to a borrower in exchange for consumption externally. Borrowers’ project output can take on one of two possible values. Specifically, an investment project can with probability $\pi_i$ convert one unit of time $t$ output (consumption goods) into $\kappa_i$ units of capital goods at time $t + 1$ with $i = 1, 2$. and goods in the next period. Alternatively, a lender has access to a default, risk-free technology that converts one unit of his time $t$ wage into $Q\varepsilon$ units of time $t + 1$ capital, where $\varepsilon$ is assumed to be small. All capital goods are supplied competitively at the market rental rate. It is also assumed that both borrowers and lenders are risk neutral and consume only when they are old.

The credit market operates as follows. In each period, a lender offers a loan contract. If the contract is not dominated by others, a borrower will approach him and sign the contract. Each lender will be approached by one borrower only and the competition in the credit market will drive the lender’s economic profit to the reservation level, which is normalized to zero.

To introduce asymmetric information, we assume that a lender can observe the output level of an individual borrower, by spending $\delta$ units of capital goods times the loan size. The output of the borrower will be appropriated by the lender if he is caught lying under auditing about his output.

Each firm at time $t$ produces the final output according to the Cobb-Douglas production function:

$$y_t = \Phi_t^\alpha k_t^\gamma l_t^{1-\gamma}$$

(1)

where $\Phi_t$ is the average capital stock per firm, $y_t$ is the output per firm, $k_t$ is the input of private capital per firm and $l_t$ is the labor input per firm. For expositional purpose, it is assumed that $\alpha = 1 - \gamma$. Firms hire private capital and labor competitively from the markets. Thus the rental
rate, $\rho_t$, and the wage rate, $w_t$, in period $t$ are equal to the marginal products of private capital and labor, respectively:

\[ \rho_t = \gamma l_t^{1-\gamma}, \quad (2) \]

\[ w_t = (1-\gamma)k_t l_t^{-\gamma}. \quad (3) \]

It is also assumed that physical capital depreciates completely after one period of use. Output as usual can be used for consumption or investment.

We maintain the following assumptions throughout the study. First, we assume that

\[ (A1 : ) Q \varepsilon > \kappa_1. \]

Second,

\[ (A2 : ) \pi_2(\kappa_2 - \kappa_1) - \delta > 0. \]

These two assumptions ensure that some auditing activities must occur in the low return state. Lastly, the net expected investment return is superior to that on the default technology. Specifically,

\[ (A3 : ) \pi_1 \kappa_1 + \pi_2 \kappa_2 - \delta > Q \varepsilon. \]

## 3 Auditing with Commitment

When commitment to audit is feasible, the loan contract at time $t$ offered to a borrower can be specified as $C_t = [\phi_t^1, \phi_t^2, R_t^1, R_t^2, q_t]^3$, where $\phi_t^1$ and $\phi_t^2$ are the auditing probabilities when reported output levels are low and high respectively, $R_t^1$ and $R_t^2$ are the gross loan rates (in real terms) when the reported output levels are low and high respectively, $q_t$ is the loan size.

The expected payoff to a borrower of generation $t$ is given by

\[ \pi_1(\kappa_1 \rho_{t+1} - R_t^1)q_t + \pi_2(\kappa_2 \rho_{t+1} - R_t^2)q_t, \quad (4) \]

In equilibrium, borrowers will exhibit self-selection by choosing the contracts that match with their own output. In other words, the following incentive compatibility constraints must be satisfied:

\[ (\kappa_1 \rho_{t+1} - R_t^1)q_t \geq (1 - \phi_t^2)(\kappa_1 \rho_{t+1} - R_t^2)q_t, \quad (5) \]

\[ (\kappa_2 \rho_{t+1} - R_t^2)q_t \geq (1 - \phi_t^1)(\kappa_2 \rho_{t+1} - R_t^1)q_t. \quad (6) \]

\[ ^3 \text{Following Khalil (1997) and Wang and Williamson (1998), we assume that the loan rate at the state of auditing is equal to that at the state of no-auditing to simplify the analysis.} \]
Because the credit market is assumed to be perfectly competitive, lenders always earn zero expected economic profit in equilibrium. This zero profit condition can be expressed as

$$\pi_1[\phi_1^1(R_1^1 - \delta \rho_{t+1}) - (1 - \phi_1^1)R_1^1]q_t + \pi_2[\phi_2^2(R_2^2 - \delta \rho_{t+1}) - (1 - \phi_2^2)R_2^2]q_t = Q\rho_{t+1}q_t. \quad (7)$$

The left hand side of this equation is the expected income from making loans and the right hand side is the forgone income of the loan. The equilibrium contracts must also satisfy the following feasibility condition:

$$q_t \leq w_t. \quad (8)$$

Now, we define an equilibrium in the credit market as follows:

**Definition 1.** An equilibrium in the credit market with commitment to audit is represented by a sequence of \( C_t \) where the contract \( C_t = [\phi_1^1, \phi_2^2, R_1^1, R_2^2, q_t] \), maximizes (4) subject to (5) - (8) taking the sequence of \( \{\rho_t\}, \{w_t\} \) as given.

According to (A1)-(A3), it can be shown that \( \kappa_i \rho_{t+1} - R_i^1 \) must be positive in equilibrium. It follows immediately that (8) must hold with equality sign in equilibrium, which determine the loan size for borrowers.

It is a standard result in models of this sort that, in equilibrium, only the incentive compatibility constraint for \( i = 2 \) is binding but not for \( i = 1 \). We will proceed by assuming that this is the case. A proof for this assumption is relegated to appendix after the complete equilibrium contracts are derived. As a result, the binding incentive compatibility constraint for \( i = 2 \) yields us

$$\phi_1^1 = \frac{R_2^2 - R_1^1}{\kappa_2 \rho_{t+1} - R_1^1}. \quad (9)$$

It can be shown that the expected payoff to a borrower is strictly decreasing with the auditing probabilities, \( \phi_1^1 \) and \( \phi_2^2 \). Since the incentive compatibility constraint for \( i = 1 \) borrowers is never binding, in equilibrium, it will be optimal to set \( \phi_2^2 = 0 \) implying that lenders never audit borrowers who report high output level. In addition, according to (9), \( \phi_1^1 \) is strictly decreasing with \( R_1^1 \). Hence, lenders will set \( R_1^1 = \kappa_1 \rho_{t+1} \) in order to maximize the borrower’s expected payoff. As a consequence, from (7), the equilibrium loan rate for borrowers with high output level is

$$R_2^2 = \frac{Q\varepsilon \rho_{t+1}}{\pi_2} - \frac{\pi_1 \rho_{t+1}}{\pi_2} (\kappa_1 - \delta \phi_1^1). \quad (10)$$

Substituting (10) into (9) in association with \( R_1^1 = \kappa_1 \rho_{t+1} \) gives the auditing probability for \( i = 2 \)

$$\phi_1^1 = \phi = \frac{Q\varepsilon - \kappa_1}{\pi_2 (\kappa_2 - \kappa_1) - \delta \pi_1}. \quad (11)$$
Now we summarize the above results in the following proposition.

**Proposition 1.** In each period $t$, the equilibrium loan contract is given by $C_t = [\phi^1_t, \phi^2_t, R^1_t, R^2_t, q_t]$ with

$$
\phi^1_t = \phi = \frac{Q\varepsilon - \kappa_1}{\pi_2(\kappa_2 - \kappa_1) - 3\pi_1}, \quad \phi^2_t = 0, \quad R^1_t = \kappa_1\rho_{t+1}, \quad R^2_t = \frac{Q\varepsilon\rho_{t+1}}{\pi_2} - \frac{\pi_1\rho_{t+1}}{\pi_2}(\kappa_1 - \delta\phi), \quad q_t = w_t.
$$

In equilibrium, all firms hire an equal amount of labor. Since there is an equal number of borrowers and lenders, the number of labor force per firm, $l_t = 1$ at each time period.

Note that next period’s capital is produced by the successful investment projects of current borrowers. Since those borrowers that report high output level are never audited while those with low output level are (with probability $\phi$), recalling the investment technology of the borrowers and full depreciation of capital, the economy-wide capital stock at $t+1$ is given by:

$$
K_{t+1} = (\pi_1\kappa_1 + \pi_2\kappa_2 - \delta\pi_1\phi)w_t.
$$

Since the total number of firms is equal to 1 and $w_t = (1-\gamma)k_t$, the growth rate of capital stock per firm at $t+1$ is given by

$$
\frac{k_{t+1}}{k_t} = g_t = (1-\gamma)(\pi_1\kappa_1 + \pi_2\kappa_2 - \delta\pi_1\phi) \quad (12)
$$

The social welfare is given by

$$
W = \pi_1(\kappa_1\rho_{t+1} - R^1_t)q_t + \pi_2(\kappa_2\rho_{t+1} - R^2_t)q_t + Q\varepsilon\rho_{t+1}q_t. \quad (13)
$$

Auditing with commitment enables lenders to induce self selection from borrowers. However, this assumption is indeed problematic because it will not be necessary for lenders to audit once borrowers reveal their true return by choosing an appropriate loan payment. In the following section, we are going to characterize the optimal contracts under which this assumption no longer holds and study its effects on economic growth and social welfare.

### 4 Auditing without Commitment

When lenders cannot commit to their auditing strategies, the loan contract at time $t$ offered to a borrower is characterized as $C_t = [R^1_t, R^2_t, q_t]$. Note that the auditing probability is not specified in the contracts, implying that lenders cannot commit to any pre-announced auditing strategies.

Let $\tau^i_t$ with $i = \{1, 2\}$ be the probability that a borrower $i$ will pretend as a borrower $j$, $i \neq j$. As standard, borrowers that experience adverse output shock always comply, hence we have $\tau^1_t = 0$. 

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7
We relegate the proof of this claim to the appendix. Without causing any confusion, we henceforth let $\tau_t^2 = \tau_t$. As a result, the expected payoff to a borrower of generation $t$ takes the following form:

$$\pi^1(\kappa_1 \rho_{t+1} - R^1_t)q_t + \pi^2\{\tau_t(1 - \phi^1_t)(\kappa_2 \rho_{t+1} - R^2_t) + (1 - \tau_t)[\phi^2_t(\kappa_2 \rho_{t+1} - R^2_t) + (1 - \phi^2_t)(\kappa_2 \rho_{t+1} - R^2_t)]\}q_t.$$

(14)

Since the loan market is perfectly competitive, the zero profit condition of lenders can be written as:

$$(\pi^1 + \pi^2 \tau_t)\{\phi^1_t\left[\frac{\pi^2 \tau_t}{\pi^1 + \pi^2 \tau_t}(\kappa_2 \rho_{t+1} - \delta \rho_{t+1}) + \frac{\pi^1}{\pi^1 + \pi^2 \tau_t}(R^1_t - \delta \rho_{t+1})\right] + (1 - \phi^1_t)\left[\frac{\pi^2 \tau_t}{\pi^1 + \pi^2 \tau_t}R^1_t + \frac{\pi^1}{\pi^1 + \pi^2 \tau_t}R^1_t\right]\}q_t + \pi^2(1 - \tau_t)[\phi^2_t(R^2_t - \delta \rho_{t+1}) + (1 - \phi^2_t)R^2_t]q_t$$

$$= Q \epsilon r_{t+1} q_t.$$

(15)

On the left of this equation, the first term represents a lender’s expected payoff that can be collected from a borrower who report low investment output. Specifically, if a lender audits such a borrower, he will discover that this borrower is indeed lying with probability $\frac{\pi^2 \tau_t}{\pi^1 + \pi^2 \tau_t}$. Under this situation, the lender can appropriate the borrower’s investment output. As a result, the lender’s net profit is equal to $\kappa_2 \rho_{t+1} - \delta \rho_{t+1}$ per unit of loan. With probability $\frac{\pi^1}{\pi^1 + \pi^2 \tau_t}$, the lender will find that the borrower is telling the truth and the lender’s net profit is equal to $R^1_t - \delta \rho_{t+1}$. If the lender does not audit, he is able to collect $R^1_t$ per unit of loan irrespective of whether the borrower cheats. The second term on the left is lender’s expected payoff that can be collected from a borrower who claims to have high output. The right hand side of this equation is simply the opportunity cost of making the loan available.

If borrowers with low output level never cheat, lenders never audit borrowers that claims to have low output. That is, $\phi^2_t = 0$. We will proceed by assuming that this is the case. A proof of this assumption is relegated to appendix.

To capture the idea of auditing without commitment, we model the interaction between lenders and borrowers as a two-stage game which can be solved by first characterizing the lender’s second-stage equilibrium auditing strategy and the borrower’s second-stage equilibrium cheating strategy given the terms of the incentive contracts determined in the first stage of the game. Following Khalil (1997), the second-stage equilibrium is shown to be a mixed-strategy equilibrium. Intuitively, if there is no cheating behavior in the equilibrium, lenders will never audit. Therefore, some cheating behavior must be observed for the auditing strategy to be ex-post optimal. However, if lenders always audit, borrowers will never cheat. Then lenders should not audit. The above arguments suggest that the optimal contract must induce random auditing from lenders and random cheating behaviors from borrowers.
Specifically, the following conditions must hold if there exists such a mixed-strategy equilibrium at the second stage of the game between lenders and borrowers:

\[(1 - \phi^1_t)(\kappa_2\rho_{t+1} - R^1_t)q_t = (\kappa_2\rho_{t+1} - R^2_t)q_t, \quad (16)\]

\[
\frac{\pi_2\tau_t}{\pi_1 + \pi_2\tau_t}(\kappa_2\rho_{t+1} - \delta\rho_{t+1})q_t + \frac{\pi_1}{\pi_1 + \pi_2\tau_t}(R^1_t - \delta\rho_{t+1})q_t = (\frac{\pi_2\tau_t}{\pi_1 + \pi_2\tau_t}R^1_t + \frac{\pi_1}{\pi_1 + \pi_2\tau_t}R^1_t)q_t. \quad (17)\]

The left hand side of equation (16) is a borrower’s expected payoff when he cheats. The right hand side indicates his expected payoff when he complies. This equation says that borrowers are indifferent between cheating and complying in equilibrium. The left hand side of equation (17) represents a lender’s expected payoff when he audits and the right hand side of it is his payoff when auditing does not take place. They tell that in equilibrium lenders are indifferent between auditing and not auditing when any borrowers approach them for loans.

The equilibrium contracts must also satisfy the following feasibility condition:

\[q_t \leq w_t. \quad (18)\]

Now, we define an equilibrium in the credit market as follows:

**Definition 2.** An equilibrium in the credit market without commitment to screen is represented by \(\tau_t\) and \(\phi_t\) and a sequence of \(\{C_t\}\) where the contract \(C_t = [R^1_t, R^2_t, q_t]\), maximizes (14) subject to (15) - (18) taking the sequence of \(\{\rho_t\}\), \(\{w_t\}\) as given.

Since it can be proved that \(\kappa_1\rho_{t+1} - R^1_t > 0\), equation (18) must hold with equality sign.

Equation (16) yields \(\phi^1_t\) as a function of the contract terms:

\[\phi^1_t = \frac{R^2_t - R^1_t}{\kappa_2\rho_{t+1} - R^1_t}. \quad (19)\]

Similarly, equation (17) gives the cheating probability of those borrowers with high output level as a function of contract terms:

\[\tau_t = \frac{\delta\pi_1\rho_{t+1}}{\pi_2(\kappa_2\rho_{t+1} - \delta\rho_{t+1} - R^1_t)}. \quad (20)\]

If we substitute (16) into borrowers’ expected payoff function, we get:

\[\pi^1(\kappa_1\rho_{t+1} - R^1_t)q_t + \pi^2(\kappa_2\rho_{t+1} - R^2_t)q_t. \quad (21)\]

Likewise, if we substitute (17) into lenders’ zero profit condition, we obtain:

\[(\pi_1 + \pi_2\tau_t)R^1_t + \pi_2(1 - \tau_t)R^2_t = Q\varepsilon\rho_{t+1}. \quad (22)\]
Since it can be proved that a borrower’s expected payoff is strictly increasing with $R^1_t$, it will be optimal to set $R^1_t = \kappa_1 \rho_{t+1}$. A proof of this claim can be found in appendix. Substituting this result into (20) yields us

$$\tau_t = \tau = \frac{\delta \pi_1}{\pi_2 (\kappa_2 - \kappa_1)}.$$  \hspace{1cm} (23)

Substituting (23) in association with $R^1_t = \kappa_1 \rho_{t+1}$ into (22), we get

$$R^2_t = \frac{(\kappa_2 - \kappa_1)(Q \varepsilon - \pi_1 \kappa_1) - \delta Q \varepsilon}{\pi_2 (\kappa_2 - \kappa_1)} \rho_{t+1}.$$  \hspace{1cm} (24)

Substituting (24) and $R^1_t = \kappa_1 \rho_{t+1}$ into (19) gives us the equilibrium auditing probability

$$\phi^1_t = \phi = \frac{(Q \varepsilon - \kappa_1)(\kappa_2 - \kappa_1)}{(\kappa_2 - \kappa_1)[\pi_2 (\kappa_2 - \kappa_1) - \delta]}.$$  \hspace{1cm} (25)

Now we summarize the above results in proposition 2.

**Proposition 2.** In each period $t$, the equilibrium loan contract is given by $C_t = [R^1_t, R^2_t, q_t]$ with $R^1_t = \kappa_1 \rho_{t+1}$, $R^2_t = \frac{(\kappa_2 - \kappa_1)(Q \varepsilon - \pi_1 \kappa_1) - \delta Q \varepsilon}{\pi_2 (\kappa_2 - \kappa_1) - \delta} \rho_{t+1}$ and $q_t = w_t$.

Be aware of that a fraction of borrowers with high output pretend to have low output with probability $\tau$ in equilibrium and the borrowers who report low output level are audited with probability $\phi^1_t$. In addition, when auditing takes place, lenders can appropriate non-complying borrowers’ project output. As a result, the aggregate capital stock at period $t + 1$ in this situation is equal to:

$$K_{t+1} = \{\pi_1 \kappa_1 + \pi_2 \tau (\phi^1_t + (1 - \phi^1_t)) \kappa_2 + (1 - \tau) \kappa_2 - \delta (\pi_1 + \pi_2 \tau) \phi^1_t\} q_t,$$
$$K_{t+1} = \{\pi_1 \kappa_1 + \pi_2 \kappa_2 - \delta (\pi_1 + \pi_2 \tau) \phi\} q_t.$$

Since the number of firms is equal to 1 and $w_t = (1 - \gamma)k_t$, the growth rate of capital stock per firm at period $t + 1$ is equal to

$$\frac{k_{t+1}}{k_t} = g_t = (1 - \gamma)[\pi_1 \kappa_1 + \pi_2 \kappa_2 - \delta (\pi_1 + \pi_2 \tau) \phi].$$  \hspace{1cm} (26)

In this regime, the social welfare is represented by:

$$W = \pi_1(\kappa_1 \rho_{t+1} - R^1_t)q_t + \pi_2(\kappa_2 \rho_{t+1} - R^2_t)q_t + Q \varepsilon \rho_{t+1} q_t.$$  \hspace{1cm} (27)

Now we are ready to discuss the growth and welfare implications of auditing contract under no commitment. This is the focus of the next section.
5 Comparison of economic growth and social welfare

Throughout this section, we denote variables and optimal contract terms from the regime without commitment with a “tilde”.

**Proposition 1.** Given \( \rho_{t+1} \), the loan rate to borrowers who report high output level is higher in the regime without commitment. That is, \( \tilde{R}_t^2 > R_t^2 \).

Proof: It is easy to show that

\[
\tilde{R}_t^2 - R_t^2 = \left[ \frac{(\kappa_2 - \kappa_1)(Q\varepsilon - \pi_1\kappa_1) - \delta Q\varepsilon}{\pi_2(\kappa_2 - \kappa_1) - \delta} - \frac{(\kappa_2 - \kappa_1)(Q\varepsilon - \pi_1\kappa_1) - \delta\pi_1\kappa_1}{\pi_2(\kappa_2 - \kappa_1) - \delta\pi_1} \right] \rho_{t+1} \\
= \frac{\delta^2 \pi_2(Q\varepsilon - \kappa_1) \rho_{t+1}}{\pi_2(\kappa_2 - \kappa_1) - \delta[\pi_2(\kappa_2 - \kappa_1) - \delta\pi_1]} > 0.
\]

Hence, this lemma is proved. QED

This result can be explained as follows. In order to make the auditing strategy to be ex post optimal under no-commitment, the optimal contract must be designed to induce some cheating behavior from those borrowers with high output level. In equilibrium, only a fraction of these borrowers’ cheating behavior is detected and the other fraction will only give the loan rate which should charge those borrowers with low output. This will reduce lenders’ expected payoff in the high-return state. To recover the loss, \( R_t^2 \) must increase.

**Proposition 2.** Lenders tend to audit borrowers who report low output level more frequently in the regime without commitment. That is, \( \tilde{\phi}^1 > \phi^1 \).

Proof: Since \( \tilde{\phi}^1 \) is increasing with \( \tilde{R}_t^2 \) and \( \phi^1 \) is increasing with \( R_t^2 \), the result of proposition 1 immediately implies that \( \tilde{\phi}^1 > \phi^1 \). QED

This result is comparable to that of Proposition 3 in Khalil (1997). Note that the gain from cheating for a borrower with high output level is represented by \( R_t^2 - R_t^1 \) per unit of loan. If \( R_t^2 \) increases, the incentive to under-report the output level increases. Thus, a higher auditing probability is required to satisfy (16).

A simple comparison between (12) and (26) indicates that the auditing contract with commitment and that without imply different economic growth rates. In particular, the regime with commitment to audit always gives higher economic growth rate. We show this result formally in the following proposition:
Proposition 3. The auditing contract with commitment always gives a higher economic growth rate than the auditing contract without commitment does. That is, $g_t > \tilde{g}_t$.

Proof: Since $\phi^1 > \phi^1_1$ and $1 > \tau > 0$, $g_t > \tilde{g}_t$ follows immediately by comparing (12) and (26). QED

This result has a very intuitive explanation. Under no-commitment to audit, the economic growth rate is lower because lenders tend to undertake more frequent auditing activities in this regime, which are resource wasting.

Whether the optimal contracts for the regime with commitment or those for the regime without prevails depends on the relative expected payoffs between these two regimes. Since lenders earn exactly the same amount of expected payoffs in these two regimes\(^4\), the difference between borrowers’ expected payoffs in these two regimes becomes crucial to the concern on regime switching condition. It can be shown that $W > \tilde{W}$. Therefore, regardless of the stage of economic development, lenders always prefer to offer the auditing contract with commitment. In view of the importance of this result, we state and proof it in a proposition:

Proposition 4. For given $w_t$ and $\rho_{t+1}$, lenders always prefer to offer the auditing contract with commitment $C_t$ to borrowers. That is, $W > \tilde{W}$.

Proof: Since $R^1_t = \tilde{R}^1_t$ and $R^2_t < \tilde{R}^2_t$, $W > \tilde{W}$ follows. Hence, we prove the result. QED

6 Conclusion

Two endogenous growth models with asymmetric information between lenders and borrowers are developed in this paper. In the first model, lenders cannot costlessly observe the investment returns of borrowers who approach them for investment funds. But by adopting a costly auditing technology, lenders are able to identify the true output levels of borrowers and thus self-selection is achieved. In the second model, as opposed to other contributions in this line of research, we assume that lenders cannot commit to their auditing strategy a la Khalil (1997). As a direct consequence of this assumption, the interaction between lenders and borrowers becomes a two-stage game. In the second stage of the game, a mixed strategy equilibrium is found in which lenders audit and borrowers cheat with positive probabilities. In the first stage, lenders decide the optimal contractual terms which are in consistent with the existence of the mixed strategy equilibrium in

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\(^4\)In particular, each lender’s expected payoff is equal to $Q_\epsilon \rho_{t+1}$ per unit of loan.
the second stage. We find that the loan rate and auditing probability are higher, while economic growth rate and social welfare lower, in the regime without commitment.

In typical studies of asymmetric information, the announced contractual enforcement (e.g., auditing and monitoring) by lenders is implicitly assumed to be taken at its face value and the separating contracts are then designed accordingly. Such an assumption can be quite problematic for at least two types of reasons. First, lenders may be unwilling to carry out the required costly enforcement activities once they know the carefully designed contracts would indeed work to induce self selection. Second, lenders may be unable to carry out the enforcement, to the fullest extent at least, due to the weak legal and contractual environment. Our current analysis shows that the inability to commit to contract enforcement, acting as an additional source of informational friction, has non-trivial consequences. The lack of commitment in contract enforcement leads to non-separating equilibrium and more stringent contractual terms in the credit market, which in turn result in lower growth and welfare in the economy.

**Appendix**

**Derivation of equilibrium loan contracts with commitment to audit**

**Claim 1:** Given the assumptions A2) and A3), in equilibrium, \( \kappa_2 \rho_{t+1} - R_t^2 \) must be positive.

Proof: In equilibrium, \( R_t^2 = \frac{(Q \epsilon - \pi_1 \kappa_1)(\kappa_2 - \kappa_1) - \delta \pi_1 \kappa_1}{\pi_2(\kappa_2 - \kappa_1) - \delta \pi_1} \rho_{t+1} \). It is easy to check that

\[
\kappa_2 \rho_{t+1} - \frac{(Q \epsilon - \pi_1 \kappa_1)(\kappa_2 - \kappa_1) - \delta \pi_1 \kappa_1}{\pi_2(\kappa_2 - \kappa_1) - \delta \pi_1} \rho_{t+1} = \frac{(\kappa_2 - \kappa_1)(\pi_1 \kappa_1 + \pi_2 \kappa_2 - Q \epsilon - \delta \pi_1) \rho_{t+1}}{\pi_2(\kappa_2 - \kappa_1) - \delta \pi_1},
\]

which must be positive based on assumptions A2), A3) and \( 0 < \pi_1 < 1 \). QED

**Claim 2:** The expected payoff to a borrower is strictly decreasing with the auditing probability \( \phi_t^2 \). Therefore, it is optimal to set \( \phi_t^2 = 0 \).

Proof: The expected payoff to a borrower is

\[
\pi_1 (\kappa_1 \rho_{t+1} - R_t^1) + \pi_2 (\kappa_2 \rho_{t+1} - R_t^2) = (\pi_1 \kappa_1 + \pi_2 \kappa_2) \rho_{t+1} q_t - (\pi_1 R_t^1 + \pi_2 R_t^2) q_t.
\]

With \( q_t = w_t \) and making use of the zero-profit condition (7), the above expression becomes

\[
(\pi_1 \kappa_1 + \pi_2 \kappa_2) \rho_{t+1} w_t - Q \epsilon \rho_{t+1} w_t - \pi_1 \delta \phi_t \rho_{t+1} w_t - \pi_2 \delta \phi_t^2 \rho_{t+1} w_t.
\]
which is decreasing with $\phi_1^2$. Free from the self-selection constraints, the optimal solution is to set $\phi_1^2$ as low as possible, i.e., at zero. QED

Claim 3: In equilibrium, the expected payoff to a borrower is strictly increasing with $R_1^t$. Therefore, it will be optimal to set $R_1^t = \kappa_1 \rho_{t+1}$.

Proof: From claim 2, the borrower’s expected payoff is given by

$$(\pi_1 \kappa_1 + \pi_2 \kappa_2) \rho_{t+1} w_t - Q \varepsilon \rho_{t+1} w_t - \pi_1 \delta \phi_1^1 \rho_{t+1} w_t$$

which is decreasing with $\phi_1^1$ as well. From (9) and $\kappa_2 \rho_{t+1} - R_1^2 > 0$, we obtain

$$\frac{\partial \phi_1^1}{\partial R_1^1} = -\frac{\kappa_2 \rho_{t+1} - R_1^2}{(\kappa_2 \rho_{t+1} - R_1^1)^2} < 0.$$ 

It then follows that the borrower’s expected payoff is strictly increasing with $R_1^1$. Thus we prove the claim. QED

Claim 4: In equilibrium, the incentive compatibility constraint (5) holds with strictly inequality while (6) holds with equality.

Proof: It is easy to show that (6) holds with equality after substituting in the terms of the equilibrium loan contracts. For (5), we have

$$(\kappa_1^1 \rho_{t+1} - R_1^2) q_t$$

$$= 0$$

$$> (1 - \phi_1^2)(\kappa_1 \rho_{t+1} - R_1^2) q_t$$

$$= (1 - \phi_1^2)[\kappa_1 - (Q \varepsilon - \pi_1 \kappa_1)(\kappa_2 - \kappa_1) - \delta \pi_1 \kappa_1 \rho_{t+1}]$$

$$= -(1 - \phi_1^2) \frac{(\kappa_2 - \kappa_1)(Q \varepsilon - \kappa_1) \rho_{t+1}}{\pi_2(\kappa_2 - \kappa_1) - \delta \pi_1}.$$ 

Thus the claim is proved. QED

**Derivation of equilibrium loan contracts in the absence of commitment to audit**

Claim 5: Under the assumptions $A1)$ and $A3)$, in equilibrium, $\kappa_2 \rho_{t+1} - R_1^2$ must be positive.

Proof: In equilibrium, $R_1^2 = \frac{(Q \varepsilon - \pi_1 \kappa_1)(\kappa_2 - \kappa_1) - \delta Q \varepsilon}{\pi_2(\kappa_2 - \kappa_1) - \delta} \rho_{t+1}$, it is easy to check that

$$\kappa_2 \rho_{t+1} - \frac{(Q \varepsilon - \pi_1 \kappa_1)(\kappa_2 - \kappa_1) - \delta Q \varepsilon}{\pi_2(\kappa_2 - \kappa_1) - \delta} \rho_{t+1}$$

$$= \frac{[(\kappa_2 - \kappa_1)(\pi_1 \kappa_1 + \pi_2 \kappa_2 - Q \varepsilon) - \delta(\kappa_2 - Q \varepsilon)] \rho_{t+1}}{\pi_2(\kappa_2 - \kappa_1) - \delta} > 0.$$
The last inequality follows from assumptions A1) and A3). QED

Claim 6: Borrowers with low output never have incentive to misrepresent to have high output. That is, \( \tau^1_t = 0 \).

Proof: To prove this claim, it would suffice to show that the expected payoff to those borrowers with low investment return from being compliance is strictly better than that from being non-compliance in equilibrium. Specifically, the expected payoff to a borrower with low investment return from being compliance is equal to:

\[
\pi_1(\kappa_1 \rho_{t+1} - R^1_t)q_t
\]

\[
= 0
\]

\[
> \pi_1(1 - \phi^2_t)(\kappa_1 \rho_{t+1} - R^2_t)q_t
\]

\[
= \pi_1(1 - \phi^2_t)(\kappa_1 - \frac{\kappa_1 - \kappa_2(Q_e - \pi_1 \kappa_1) - \delta Q_e}{\pi_2(\kappa_2 - \kappa_1) - \delta})q_t
\]

\[
= -\frac{\pi_1(1 - \phi^2_t)(Q_e - \kappa_1)(\kappa_2 - \kappa_1 - \delta)}{\pi_2(\kappa_2 - \kappa_1) - \delta} \rho_{t+1}
\]

Hence, the claim is proved. QED

Claim 7: The expected payoff to a borrower is strictly decreasing with the auditing probability \( \phi^2_t \). Therefore, it is optimal to set \( \phi^2_t = 0 \).

Proof: After substituting in (17), the lender’s zero profit condition becomes:

\[
(\pi_1 + \pi_2 \tau_t)R^1_t q_t + \pi_2(1 - \tau_t)[\phi^2_t(R^2_t - \delta \rho_{t+1}) + (1 - \phi^2_t)R^2_t]q_t = Q_e \rho_{t+1}q_t.
\]

which can be rewritten:

\[
(\pi_1 R^1_t + \pi_2 R^2_t)\rho_{t+1}q_t = Q_e \rho_{t+1}q_t - \pi_2 \tau_t(R^2_t - R^1_t)q_t - \pi_2(1 - \tau_t)\delta \rho_{t+1}q_t.
\]

If we substitute the above equation into the borrower’s expected payoff, we obtain:

\[
(\pi_1 \kappa_1 + \pi_2 \kappa_2)\rho_{t+1}q_t - Q_e \rho_{t+1}q_t - \pi_2 \tau_t(R^2_t - R^1_t)q_t - \pi_2(1 - \tau_t)\delta \rho_{t+1}q_t.
\]

which is strictly decreasing with \( \phi^2_t \), implying that it will be optimal to set \( \phi^2_t = 0 \). QED

Claim 8: In equilibrium, the expected payoff to a borrower is increasing with \( R^1_t \). Therefore, it will be optimal to set \( R^1_t = \kappa_1 \rho_{t+1} \).

Proof: Substituting (16) into borrowers’ expected payoff function gives:

\[
(\pi_1 \kappa_1 + \pi_2 \kappa_2)\rho_{t+1}q_t - (\pi_1 R^1_t + \pi_2 R^2_t)q_t.
\]
If we substitute (22) into the above expression, we obtain:

\[ U_t = \left( \pi_1 \kappa_1 + \pi_2 \kappa_2 \right) \rho_{t+1} q_t - \left( \pi_1 R^1_t + \frac{Q \rho_{t+1}}{1 - \tau_t} - \frac{\pi_1 + \pi_2 \tau_t R^1_t}{1 - \tau_t} \right) q_t \]

\[ = \left( \pi_1 \kappa_1 + \pi_2 \kappa_2 \right) \rho_{t+1} q_t - \frac{Q \rho_{t+1}}{1 - \tau_t} q_t - \frac{\tau_t}{1 - \tau_t} \left( Q \rho_{t+1} - R^1_t \right) q_t \]

with \( \tau_t = \frac{\delta \rho_{t+1}}{\pi_2 (\kappa_2 \rho_{t+1} - \delta \rho_{t+1} - R^1_t)} \). Then,

\[ \frac{\partial U_t}{\partial R^1_t} = \frac{\partial \left( \frac{\tau_t}{1 - \tau_t} \right) (Q \rho_{t+1} - R^1_t) q_t}{\partial R^1_t} + \frac{\tau_t q_t}{1 - \tau_t} \]

\[ = \frac{\delta \rho_{t+1}^2 q_t (\pi_2 \kappa_2 - \pi_2 Q \epsilon - \delta)}{(\pi_2 \kappa_2 \rho_{t+1} - \pi_2 R^1_t - \delta \rho_{t+1})^2}. \]

From assumption A3), \((\pi_2 \kappa_2 - \pi_2 Q \epsilon - \delta) - \pi_2 (Q \epsilon - \kappa_1) > 0\), implying that \(\pi_2 \kappa_2 - \pi_2 Q \epsilon - \delta > \pi_2 (Q \epsilon - \kappa_1) > 0\). The last inequality follows from assumption A1). Therefore, \(\frac{\partial U_t}{\partial R^1_t}\) must always be positive. Hence the claim is proved. QED
References


