QGAITL: A quadratic generalized version of the almost ideal and translog demand systems

Kang Ernest Liu*
National Chung Cheng University, Chia-Yi 621, Taiwan

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Abstract

A new demand system, the QGAITL model, nesting the quadratic almost ideal, translog and linear expenditure systems as its special cases, is introduced and estimated in this paper, and moreover, two of the nested models are also new. Employing urban household data from Jiangsu China in 2001, empirical evidence shows that the QGAITL is superior to its nested models, whether or not demographic effects are incorporated.

JEL: D11, D12
Key words: nested model, AIDS, Translog, urban China

* Corresponding Author: Kang Ernest Liu
Department of Economics, National Chung Cheng University
168 University Road, Ming-Hsiung, Chia-Yi 621, Taiwan
Phone: 886-5-2720411 ext. 34120
Fax: 886-5-2720816
E-Mail: ECDKL@CCU.EDU.TW
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1. Introduction

Selection of the appropriate functional forms for demand analysis is one of the most crucial issues in empirical studies. In the literature, the almost ideal (AI) demand system of Deaton and Muellbauer (1980) and the translog (TL) model of Christensen et al. (1975) are the two most commonly-used functional form specifications.

Several new models were developed during the past two and half decades, which were modified on the basis of the AI and the TL flexible functional forms. For example, Lewbel (1989) nested the AI and the TL models, which is called either the Lewbel demand system or the AITL model, while Banks et al. (1997) introduced a quadratic version of the AI model (QAI). In addition, the ‘translating’ procedure, which was interpreted as an introduction of the ‘committed quantities’ into the original models, was used to modify the original TL to the generalized TL (GTL) by Pollak and Wales (1980) and the original AI into the generalized AIDS (GAI) by Bollino (1987), respectively; Bollino and Violi (1990) provided a generalized version of the almost ideal and translog demand systems (GAITL) by incorporating the committed quantities into the Lewbel demand system. Recently, Moro (2003) introduced a quadratic generalization of the Lewbel demand system (QAITL), which nests the QAI and the quadratic TL (QTL) by Beach and Holt (2001) as special cases; unfortunately,
the committed quantities are not considered in his model, and moreover, no empirical
evidence is provided for supporting the superiority of his newly developed model.

This paper attempts to fill some gaps found in this domain and develops a new
model which extends Moro’s model (2003) by considering the committed quantities
as suggested in the literature. This newly developed model is called the quadratic
generalized version of the almost ideal and translog demand systems (QGAITL). On
the basis of Chinese urban household data in Jiangsu from the year 2001, empirical
evidence is provided and supports this newly developed QGAITL model is superior to
all its nested models, including Moro’s (2003) QAITL model.

The plan of this paper is as follows. Section 2 derives this quadratic
generalized version of the almost ideal and translog demand systems. The data is
described, and the results are presented and analysed in Section 3. We end on the
concluding remarks of Section 4.

2. The QGAITL model

Let $u$ be a given utility value and $p$ be an $n$ vector of prices. The expenditure
function $M = E(u, p)$ is of the form:

$$E(u, p) = c(p) + \exp\left\{\left[\frac{a(p) + b(p)}{(\ln u)^{-1}} - g(p)\right]\right\}d(p),$$  

where 

$$a(p) = \alpha_0 + \sum_{i=1}^n \alpha_i \ln p_i + (1/2) \cdot \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} \ln p_i \ln p_j,$$

$$b(p) = \exp\left(\sum_{i=1}^n \beta_i \ln p_i\right).$$
\[ c(p) = \sum_{i=1}^{n} p_i \xi_i, \]
\[ d(p) = \sum_{i=1}^{n} \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j, \]
\[ g(p) = \sum_{i=1}^{n} \delta_i \ln p_i. \]

Price indices, \( a(p), b(p), c(p), d(p) \) and \( g(p) \), are functions of parameters in Greek letters and prices in terms of either original or logged prices. In order to satisfy the homogeneity of the expenditure function, demand restrictions on parameters are given by:

\[ \sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \beta_i = 0, \quad \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} = 0, \quad \gamma_{ij} = \gamma_{ji}, \quad \text{and} \quad \sum_{i=1}^{n} \delta_i = 0. \] (2)

From equation (1), the supernumerary expenditure (Bollino, 1987) can be expressed as

\[ M^* = M - c(p) = M - \sum_{k=1}^{n} p_k \xi_k. \]

Applying Shephard’s lemma, the Marshallian demands of the QGAITL model in budget shares \( (w_i) \) are given by:

\[ w_i = p_i \xi_i / M + [M^*/M] \cdot \left\{ \alpha_i + \sum_{j=1}^{n} \gamma_{ij} (\ln p_j - \ln M^*) \right\} \]
\[ + \beta_i [d(p) \ln M^* - a(p)] + \delta_i / b(p) [d(p) \ln M^* - a(p)]^2 / d(p). \] (3)

This newly developed QGAITL nests twelve other models as its special cases, including the quadratic generalized version of the almost ideal (QGAI) and the quadratic generalized version of translog (QGTL) models both of which are also new. Figure 1 shows the relationships among these different demand systems including the testing procedures.
3. The empirical results

2001 Chinese urban household data from Jiangsu province are employed and illustrated in this study. The database is obtained from the National Bureau of Statistics (NBS) in China. The detailed description of the dataset can be found in Liu (2003). Four major food consumption categories, including grain, pork, fresh vegetables, and fresh fruits, are selected for a total of 800 observations.

Results of estimation are also presented in Figure 1 with the number of parameters to be estimated shown in parentheses. Using the full information maximum likelihood (FIML) estimator, the log-likelihood value (LnL) of the QGAITL model is 2455 (parameter estimates of the QGAITL model are in the appendix table). The likelihood ratio (LR) statistics of the nested models against the QGAITL are also revealed in Figure 1. Obviously, the LR tests show that all the nested models, including the QAIAL model (Mono, 2003), are rejected at the significant level of 1% or better except the GAITL model by Bollino and Violi (1990) with a p-value of 2.28%, which is, however, quite significant from a statistical viewpoint.

A possible misspecification can be caused by neglecting demographic effects. Following Bollino (1987), a simple modification of the QGAITL model is specified and estimated. Assume that each price $p_i$ depends upon a demographic modifying
function which is linear in household size $hs$:

$$p_i^* = (1 + \varphi_i \cdot hs) \cdot p_i,$$

(4)

where $p_i^*$ represents a modified price and $\varphi_i$ indicates parameters in terms of demographic variable, $hs$. Hence, the demographic effects can be tested again using the LR tests. The log-likelihood value of the QGAITL model with incorporation of demographic effects is 2478. The LR value of the QGAITL against the modified QGAITL model with the demographic effects is 46.56. This value is large enough to reject the original QGAITL model in which the demographic effects are ignored. Therefore, the empirical evidence shows that the quadratic generalized version of the almost ideal and translog demand systems proposed in this paper is superior to its nested models whether demographic effects are considered or not.

4. Concluding remarks

This paper has specified and estimated a quadratic generalized version of the almost ideal and translog demand systems. The QGAITL model nests twelve other models, including two new ones. Employing 2001 Chinese urban household data from Jiangsu province, empirical results show that the QGAITL model is superior to its nested models, whether or not demographic effects are incorporated.
References:


Pollak, R.A. and T.J. Wales, 1980, Comparison of the quadratic expenditure system
and the Translog demand system with alternative specifications of demographic effects, Econometrica 48, 595-612.
Figure 1. The QGAITL nested models
Appendix A: Parameter Estimates and Standard Errors for the QGAITL model

<table>
<thead>
<tr>
<th>Param</th>
<th>Coeff (S.E.)</th>
<th>Param</th>
<th>Coeff (S.E.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.6564 (0.3627)*</td>
<td>$\zeta_3$</td>
<td>0.2987 (5.2508)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.1861 (0.3554)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.3670 (0.2672)</td>
<td>$\zeta_4$</td>
<td>6.3191 (1.9517)**</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.4260 (0.0549)***</td>
<td>$\gamma_{11}$</td>
<td>-0.6971 (0.1411)***</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.0513 (0.0713)</td>
<td>$\gamma_{12}$</td>
<td>0.1256 (0.0997)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.3129 (0.0588)***</td>
<td>$\gamma_{13}$</td>
<td>0.2297 (0.0565)***</td>
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<tr>
<td>$\delta_1$</td>
<td>0.0182 (0.0097)*</td>
<td>$\gamma_{14}$</td>
<td>0.1220 (0.0782)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.0007 (0.0088)</td>
<td>$\gamma_{22}$</td>
<td>-0.0437 (0.0515)</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-0.0247 (0.0056)***</td>
<td>$\gamma_{23}$</td>
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</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.8949 (2.8039)</td>
<td>$\gamma_{24}$</td>
<td>-0.0065 (0.0197)</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>-0.1184 (1.1942)</td>
<td>$\gamma_{33}$</td>
<td>-0.0981 (0.0848)</td>
</tr>
</tbody>
</table>

Note: Param indicates “parameter,” Coeff means “coefficient,” and S.E. denotes approximate “standard error.” *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.