Abstract

This paper investigates the optimal compensation scheme for workers in a team who value not only absolute but also relative incomes. A worker is said to be more ambitious if his utility places more weight on relative income. In this case, the firm can exploit the worker’s preference for relative comparison to design a compensation scheme which induces the same effort level with lower cost. Under the optimal compensation scheme, the workers’ wages are shown to depend on relative performance, and exhibit wage compression. More importantly, even if the production technology calls for absolute performance evaluation in the traditional principal-agent model, the optimal wage structure still relies on relative performance when workers are ambitious. Finally, in contrast to past literature, worker-heterogeneity is shown to reduce the firm’s profit.
1 Introduction

Theoretical works have long postulated that the utility of an economic agent might depend not only on individual income, but also on relative income.¹ This theoretical postulation is also supported by empirical findings. For example, using Dutch cross-section and panel data, respectively, Kapteyn et. al. (1980) and van de Stadt et. al. (1985) show that individual utility functions depend on relative, rather than absolute, incomes. Easterlin (1974, 1995) shows that self-reported happiness has a positive correlation with income across individuals within a country, but this correlation does not increase when the country grows richer over time. This is interpreted as evidence that individual well-being depends on relative, rather than absolute, level of income. Clark and Oswald (1996) uses British House Panel Survey data to show that workers’ reported levels of well-being are at best weakly correlated with their absolute income alone, but are significantly negatively correlated with comparison incomes.² A recently paper by Luttmer (2004) uses National Survey of Families and Households (NSFH) panel data matched to Public Use Micro-data Areas (PUMA) to test whether individuals feel worse when others around them earn more. With careful specifications that filter out individual fixed effects and local factors, he convincingly argues that individual utility should depend on relative in addition to absolute earning. This tendency to compare one’s income with others is also confirmed by recent experimental data. For example, Zizzo and Oswald (2001) designs an experiment in which the subjects can reduce others’ money at their own costs. They find that about 65% of the subjects are on average willing to sacrifice 25 cents in order to reduce their peer’s incomes by one dollar.³

¹See, for example, Boskin and Sheshinski (1978) on redistributive taxation, Frank (1985) on status seeking and Abel (2005) on taxation to achieve optimal balanced growth path.
²Two measures of comparison income are used in that paper: A standard mincer earnings and peer’s wages.
³For extensive discussion on the literature, see Lutter (2004) and Fershtman et. al. (2003b).
In the context of a firm, this implies that the utility of a worker depends not only on his own wage, but also on the wages of his co-workers as well. Specifically, a worker derives an additional utility when he earns a higher wage than his co-workers, and suffers a disutility when earning a lower one. Since this represents the desire to do better than his co-workers, we will call it the worker’s “ambition.”

If a worker is ambitious, his marginal utility for income will be higher, regardless of whether he earns more or less than his co-workers. This means that, given a fixed wage schedule, the utility gap between high and low wages will be greater, which in turn implies that in order to implement the same effort level, the firm only needs to set a smaller wage gap. In particular, the firm can lower the wage for a worker who produces higher output (and is paid more than this co-worker) and at the same time retains the same incentives. There are three consequences for this. First, the firm only needs to expend lower wage cost to provide the same incentives to the workers. Consequently, the profit of the firm will increase. In other words, the firm can increase its profit by hiring ambitious workers. Second, there is wage compression in the sense that the wage gap between workers with high and low outputs will be narrowed. Third and most importantly, we also show that even when the information structure does not require relative compensation in the traditional principal-agent model, the optimal wage structure here is still strictly a relative performance evaluation. This is of great interest because it has been a puzzle in the personnel economics literature as to why, given the drawbacks of relative performance evaluation,\(^4\) is it still so widely used?\(^5\) There can be many explanations for it, and this paper offers an answer from a new perspective. The reason is not to infer a worker’s effort level more precisely by observing other workers’ output (as is the rationale in the standard principal-agent model), but to exploit the workers’ concern for relative income and to save wage cost.

\(^4\)See, for example, Lazear (1989) and Chen (2003).

\(^5\)DeVaro (2002) finds empirical evidence that promotion is strongly related to relative performance.
Recently there are many (mostly experimental) papers which show, or take as assumption, that people are strongly influenced by other-regarding preferences. In the context of the principal-agent relation, the basic ingredient of this literature is to view the relation between a worker (or an agent) and a firm (or a principal) as a gift-giving game. It then considers how the workers’ concern for fairness and the motive of reciprocity between the firm and the workers affect the firm’s wage policy and the workers’ effort decisions. (For a survey see Fehr and Schmidt, 2003.) Therefore, the main focus of this literature is the one-principal-one-agent relationship without formal contractual obligations, and it departs from the traditional principal-agent model in two aspects. First, the agent’s utility depends not only on how much he is paid, but also on how he is paid. Second, the level of wage directly influences the effort level of the agent, instead of through the incentive compatibility constraints.

In contrast to the gift-giving game model, our model focuses on the formal contractual relationship. We depart from the traditional principal-agent model only in that the worker’s utility is assumed to depend on other workers’ income as well. Moreover, unlike the gift-giving literature, the workers’ preferences depend on their co-workers’ wages, instead of on their concerns of fairness with the principal. Specifically, we consider a one-principal-many-agent model in which the psychological utility arises from the relation between the workers, instead of between the principal and the agent. The only papers we are aware of that also use the one-principal-many-agent framework in this regard are Charness and Kuhn (2004) and Fershtman et. al. (2003b). In the former, however, a worker’s effort level is assumed to be an increasing function of wage, so that the basic relation between the principal and the agent is still a gift-giving game. In particular, in their model the incentive compatibility constraint is assumed away, so that the only

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6 This model also differs from the literature that assumes altruistic utility function (see, e.g., Rotemberg, 1994, and the survey of Fehr and Schmidt, 2003, for details) in that this literature assumes a positive relation between utility and other agents’ wage, while in our model there is a negative relation.
way to increase effort is to raise wage instead of redesigning the output-contingent wage structure (as in our model). Moreover, in their model the workers differ in productivities, while in our model the workers differ in the degree they care about other workers’ incomes.

Fershtman et. al. (2003b) is perhaps the closest to our paper in the literature. If restricted to the context of a single firm, theirs is essentially a one-principal-many-agent model in which a worker is one of two types. He either cares about relative income, or not at all. Thus they are not concerned with how the degree in which the workers care about relative income affects the wage policy (as in our model). They derive the optimal effort decision of the workers and the equilibrium wage structure of the firm. There are two main differences between their model and ours. First, in their model the worker’s wage is assumed to be linear in his own output. This implicitly assumes away relative performance evaluation. In particular, a worker’s wage is independent of other workers’ wages or outputs. Our model, in contrast, investigates how a worker’s concern for relative income affects the firm’s decision to link his wage to other workers’ performances. Put differently, our model endogenizes the choice between absolute and relative performance evaluations. As we will show, even if production technologies are independent (so that relative performance evaluation is never needed on informational grounds), the optimal wage structure is necessarily a relative performance evaluation. Second, they assume the firm is able to offer different contracts to workers with different characteristics, and the first-best result can thus be obtained. That means there is essentially neither moral hazard nor adverse selection concern in their model, and the firm’s objective is virtually to maximize joint surplus. We assume that the firm is not allowed to offer different contracts to different workers, and the only way the workers are paid by different compensation schemes is through self-selection. That is, the firm must offer a menu of contracts for the workers to voluntarily choose the contracts they prefer. Our model thus combines the moral hazard and adverse selection considerations of contracts. On the other hand, their
model is more general in that they allow for continuous effort and output levels while we consider only the discrete case.

2 The Model

Consider a model that has the same informational structure as in Che and Yoo (2001). A principal (firm) hires two workers, each to perform a project. The worker has a binary choice of effort level \( k \in \{0, 1\} \), at a cost of \( ke \), for \( e > 0 \). This means that he supplies either high (\( k = 1 \)) or low (\( k = 0 \)) effort level. Effort levels are not observable to the firm. High effort level is more valuable to the firm in the sense to be specified shortly.

After effort is exerted, the principal learns of each worker \( i \)'s performance (output) \( y_i \), which can be either high (\( y_i = 1 \)) or low (\( y_i = 0 \)). The probability distribution of \( y_i \) depends on worker \( i \)'s effort level and a common shock. A favorable shock occurs with probability \( \sigma \in (0, 1) \), under which both workers will have high performance for sure, regardless of their effort levels. On the other hand, an unfavorable shock occurs with probability \( 1 - \sigma \), under which the probability for a worker to obtain a high output is \( q_1 \) when he supplies high effort level and \( q_0 \) if low, where \( 1 > q_1 > q_0 \geq 0 \). Since with probability \( \sigma \) both workers get high output, their performances are correlated, which provides reason for relative performance evaluation. It follows that each worker obtains high output with probability \( \sigma + (1 - \sigma)q_k \) and the low output with probability \( (1 - \sigma)(1 - q_k) \), given his effort decision \( k \). We assume that high output is so valuable to the firm that it intends to implement high effort for both workers.

Wages can only be contingent on each agent’s output. Let \( w_{yi,yj}^i \) denote the wage for worker \( i \) when his output is \( y_i \) and his co-worker’s output is \( y_j \). The wage structure of the firm is thus \( (w^1, w^2) \), where \( w^i = (w_{11}^i, w_{10}^i, w_{01}^i, w_{00}^i) \) is the contract for worker \( i \). Wages are subject to limit liability so that the principal cannot offer negative wages to
the workers, i.e., $w^i_{kj} \geq 0$ for all $i, k$ and $j$.

All parties are risk neutral. The main assumption of our model is that a worker will derive utility (or suffer disutility) from receiving a higher (lower) wage than his co-worker. There are many possible ways to model this. The simplest specification to capture this idea is to assume that worker $i$ has a utility function

$$u_i(x_i) = x_i + \alpha_i(x_i - x_j) = (1 + \alpha_i)x_i - \alpha_i x_j.$$  

(1)

Here, $\alpha_i$ is a measure of how worker $i$’s utility is dependent on his income relative to that of his co-worker. It can be seen from (1) that the marginal utility for income is greater if the worker cares about relative income. Moreover, since the value of $\alpha_i$ measures how strong is worker $i$’s desire to do better than his co-worker, we will use the value of $\alpha_i$ to measure how “ambitious” worker $i$ is. We also assume that $0 < \alpha_i < 1$, meaning that the absolute concern for wage accounts for more in a worker’s utility than the relative concern. Given contracts $w^1$ and $w^2$ and the effort decisions $k \in \{0, 1\}$ by worker $i$ and $l \in \{0, 1\}$ by worker $j$, worker $i$’s expected utility is:

$$u_i(k, l; w^1, w^2) \equiv$$

$$(\sigma + (1 - \sigma)q_kq_l)[(1 + \alpha_i)w^i_{11} - \alpha_i w^j_{11}] + (1 - \sigma)q_k(1 - q_l)[(1 + \alpha_i)w^i_{10} - \alpha_i w^j_{01}] +$$

$$(1 - \sigma)(1 - q_k)q_l[(1 + \alpha_i)w^i_{01} - \alpha_i w^j_{10}] + (1 - \sigma)(1 - q_k)(1 - q_l)[(1 + \alpha_i)w^i_{00} - \alpha_i w^j_{00}].$$

(2)

Contracts $w^1$ and $w^2$ induce both agents to supply high effort level in a Nash equilibrium if and only if

$$(IC_1) \quad u_1(1, 1; w^1, w^2) - e \geq u_1(0, 1; w^1, w^2),$$

and

$$(IC_2) \quad u_2(1, 1; w^1, w^2) - e \geq u_2(1, 0; w^1, w^2).$$

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Footnote:

7This is exactly the specification used in Fershtman, et. al. (2003a, b). Another possible specification is the ERC model (Bolton and Ockenfels, 2000) in which worker $i$’s utility can be written as $u_i(w_i, w_i/w_j)$. This utility is inconvenient to use in our context since, as can be seen in our later discussion, the wage of a worker is often zero.
The firm’s objective is to design a wage contract to minimize the expected cost of inducing both workers to supply high effort level, subject to limited liability constraints. That is,

\[
\min_{w^1, w^2} c(1, 1; w^1, w^2) \equiv (\sigma + (1 - \sigma)q_1^2)(w_{11}^1 + w_{11}^2) + (1 - \sigma)q_1(1 - q_1)(w_{10}^1 + w_{01}^2) + (1 - \sigma)q_1(1 - q_1)(w_{11}^1 + w_{11}^2) + (1 - \sigma)(1 - q_1^2)(w_{10}^1 + w_{01}^2) \quad (3)
\]

s.t. (IC1), (IC2), \( w^1 \geq 0 \) and \( w^2 \geq 0 \).

In the case when the workers do not care about relative incomes, i.e., when \( \alpha_1 = \alpha_2 = 0 \), Che and Yoo (2001) have shown that the optimal wage scheme is a relative performance evaluation (thereafter RPE), in which \( w^1 = w^2 = w^S = (0, w^S_{10}, 0, 0) \), where \( w^S_{10} = e^{(1-\sigma)(q_1-q_0)(1-q_1)} \). The worker’s expected utility is thus \( u^S(1, 1, w^S, w^S) = q_1 e/(q_1 - q_0) \). Under this scheme, a worker is paid a positive wage only if his performance is strictly better than his co-worker. This wage structure is actually equivalent to a promotion tournament. The essence of a promotion tournament is to provide incentives by setting up a prize (generally a promotion) that is awarded to a worker only if he out-performs all others. The workers are motivated to supply effort simply because of the prospect of becoming the winner.\(^8\) Under the RPE wage structure derived above, a worker is paid a positive wage only if his output is high and his co-worker’s output is low. This is exactly the way promotion tournament motivates workers, in which \( w^t_{10} \) is increasing in wage from promotion. Because of this, in the case when one worker has high output and the other low, we will call the former the “winner,” and the latter the “loser.”

### 3 The Case of Homogeneous Workers

This section considers the homogeneous case in which the workers are equally ambitious, that is, \( \alpha_1 = \alpha_2 = \alpha > 0 \). From (1) we can see that when the workers are ambitious, there is stronger incentive for workers to provide high effort level, since the marginal utility of

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\(^8\)See Lazear and Rosen (1981) and Green and Stokey (1983) for details on promotion tournament.
wage is higher. Because of this, in order to induce high effort level, the winner does not need to be paid as high as when he is not ambitious. We formally prove this in the following proposition.

**Proposition 1.** When workers are equally ambitious, the optimal contract for each worker is \( w^1 = w^2 = w^R = (0, w^R_{10}, 0, 0) \), where \( w^R_{10} = \frac{e}{(1-\sigma)(q_1-q_0)(1+\alpha-q_1)} \).

**Proof.** The incentive constraints for the workers, \((IC_1)\) and \((IC_2)\), can be re-written as:

\[
\begin{align*}
(IC_1) & \quad q_1[(1 + \alpha)w_{11}^1 - \alpha w_{11}^2] + (1 - q_1) [(1 + \alpha)w_{10}^1 - \alpha w_{01}^2] \\
& \quad - q_1[(1 + \alpha)w_{01}^1 - \alpha w_{01}^2] - (1 - q_1) [(1 + \alpha)w_{00}^1 - \alpha w_{00}^2] \geq \frac{e}{(1-\sigma)(q_1-q_0)},
\end{align*}
\]

and

\[
\begin{align*}
(IC_2) & \quad q_1[(1 + \alpha)w_{11}^2 - \alpha w_{11}^1] - q_1[(1 + \alpha)w_{01}^2 - \alpha w_{01}^1] \\
& \quad + (1 - q_1) [(1 + \alpha)w_{10}^2 - \alpha w_{01}^1] - (1 - q_1) [(1 + \alpha)w_{00}^2 - \alpha w_{00}^1] \geq \frac{e}{(1-\sigma)(q_1-q_0)}.
\end{align*}
\]

In a symmetric equilibrium, \( w_{kl}^1 = w_{kl}^2 \equiv w_{kl} \), and the firm’s minimization problem becomes (where superscripts denoting the identity of workers are omitted):

\[
\begin{align*}
\min_w & \quad 2\left[ (\sigma + (1 - \sigma)q_1^2)w_{11} + (1 - \sigma)q_1(1 - q_1)(w_{10} + w_{01}) + (1 - \sigma)(1 - q_1)^2w_{00} \right] \\
\text{s.t.} \quad & \quad (IC) \quad q_1w_{11} + (1 + \alpha - q_1)w_{10} - (\alpha + q_1)w_{01} - (1 - q_1)w_{00} \geq \frac{e}{(1-\sigma)(q_1-q_0)},
\end{align*}
\]

and \( w \geq 0 \).

Because the coefficients for \( w_{01} \) and \( w_{00} \) are positive in the objective function but negative in the constraint \((IC)\), and because of the limit liability constraints, it is optimal for both \( w_{01} \) and \( w_{00} \) to be zero. Moreover, since

\[
\frac{q_1}{1 + \alpha - q_1} < \frac{\sigma + (1 - \sigma)q_1^2}{(1 - \sigma)q_1(1 - q_1)},
\]

we can decrease the value of the objective function by increasing (decreasing) the value of \( w_{10} \) (\( w_{11} \)) while keeping \((IC)\) intact. That means that it is also optimal to set \( w_{11} = 0 \).
Therefore, the optimal solution is $w^R = (0, w_{10}^R, 0, 0)$, where $w_{10}^R$ is set to satisfy constraint $(IC)$. As a result,

$$w_{10}^R = \frac{e}{(1 - \sigma)(q_1 - q_0)(1 + \alpha - q_1)}.$$

It remains to show that the wage scheme $w^R$ indeed induces both workers to provide high effort level in a Nash equilibrium. It can be easily shown that

$$u_i(1, k; w^R) - e = u_i(0, k; w^R), \ k = \{0, 1\}.$$

This means that a worker prefers to provide high effort regardless of the other worker’s effort level. Consequently, the unique Nash equilibrium is for both workers to provide high effort level.

Proposition 1 shows that when workers are equally ambitious, then as in Che and Yoo (2001), the optimal contract is a tournament: the firm pays a worker positive wage if and only if he out-performs his co-worker. However, we now have $w_{10}^R < w_{10}^S$. That is, although the nature of the compensation scheme is still a tournament, the winner is now paid less. In the language of promotion tournament literature, this means that there is a wage compression. The intuition is clear: Since the winner derives additional utility by being paid more than his co-worker, the firm can provide enough incentive for the winner to exert high effort level with a lower wage. This result is consistent with several models that imply wage compression from different perspectives.\(^9\)

The following proposition shows how the presence of ambition affects the firm’s profit and the workers’ utilities.

**Proposition 2.** If the workers are equally ambitious, the profit for the firm is higher, and the expected utility for each worker is lower, than the case when the workers are unambitious.

Proof. Since $w_{10}^R < w_{10}^S$ as long as $\alpha > 0$, the firm’s wage cost is such that
\[
c(1, 1; w^R) = 2(1 - \sigma)q_1(1 - q_1)w_{10}^R = \frac{2q_1(1-q_1)e}{(q_1-q_0)(1+\alpha-q_1)} < \frac{2q_1e}{(q_1-q_0)} = c(1, 1; w^S).
\]
This means that the profit of the firm is higher when $\alpha > 0$. The expected utility for an ambitious worker is
\[
u_i(1, 1; w^R) = (1 - \sigma)q_1(1 - q_1)[(1 + \alpha)w_{10}^R - \alpha w_{10}^R] = (1 - \sigma)q_1(1 - q_1)w_{10}^R
\]
\[= \frac{q_1(1-q_1)e}{(q_1-q_0)(1+\alpha-q_1)} < \frac{q_1e}{(q_1-q_0)} = u^S.
\]
That is, a worker’s utility is lower when he is ambitious than when he is not. \(\Box\)

The intuition for Proposition 2 is as follows. Since the workers derive utility from making more money than their peer, the firm does not need to pay the winner as much in order to induce the same effort level. But if the workers are paid less, even with high output, what provides them enough incentive to exert high effort level? It comes from the fact that the workers suffer loss in utility when they are receiving lower wage than their peer. This creates enough utility gap to induce them to exert high effort level. Put differently, recall that the optimal contract is essentially a promotion tournament, and in that case what matters for the workers’ incentives is the utility gap between the winner and the loser, rather than the absolute values of utility. When the workers are ambitious, since they gain utility (beside that from wage itself) when they are paid more than their co-workers, and suffer utility when paid less, the utility gap between the winner and the loser is widened. The firm can therefore exploit this fact and lower the value of $w_{10}$, while still retaining enough incentive for the workers to exert high effort. As a result, the firm induces the same effort level with lower cost. The workers have lower utilities because although they are still paid a zero wage as a loser, the utility of being a loser is now negative.

The proof of Proposition 2 also shows that, since both $w_{10}^R$ and $u_i(1, 1; w^R)$ are decreasing in $\alpha$, the profit of the firm is increasing in $\alpha$ and the utility of the worker is
decreasing in $\alpha$. That means that the firm can make more profit by hiring workers who are more ambitious. This is in contrast to Charness and Kuhn (2004), who show that the firm’s profit stays the same when the workers care about relative incomes.\footnote{In their model, if the workers’ effort level responds to wage incentives only when his wage is lower than that of his co-worker (i.e. $\alpha > 0$ only when a worker’s wage is lower than his co-worker’s), then in even more contrast to our result, the firm is worse off.} The reason for the difference in result is that in their model the workers have different productivities, and that effort is assumed to be a linear function of own wage. Since (by their assumption) the workers concern equally about relative income, the firm needs to pay more (less) to worker with higher (lower) productivity in order to equalize the workers’ marginal productivities. This is only a reshuffle of wage bills, and has no effect on the total wage cost.\footnote{We have deliberately refrained from incorporating the differential ability case because if the workers differ in productivity, then difference in outputs might be due to ability. In that case a worker who is paid a lower wage might believe that the difference in wages reflects difference in productivities and is justified, and therefore might not suffer a disutility. See Section 5 for more discussion.}

Of special interest is the extreme case when $\sigma = 0$; that is, when the realization of the workers’ performances are independently distributed. In that case, it is well known in the principal-agent literature that the optimal wage for a worker is independent of the performance of others, since a worker’s performance offers no information in inferring the effort level of any other worker.\footnote{See, for example, Mookherjee (1984).} That is, the optimal wage for each worker is an absolute performance evaluation (thereafter $APE$). If this is also the case in our model, then since a worker’s wage depends only on his own performance, the contract for worker $i$ reduces to $w^i = (w^i_1, w^i_0)$.

Due to symmetry, the firm’s minimization problem becomes:

$$\begin{align*}
\min_{w_1, w_0} & \quad 2[q_1 w_1 + (1 - q_1) w_0] \\
\text{s.t.} & \quad (1 + \alpha)(q_1 - q_0)(w_1 - w_0) \geq e, \\
& \quad w_1 \geq 0, \quad w_0 \geq 0.
\end{align*}$$

(7)
The following result is immediate.

**Lemma 1.** If the realizations of the workers’ performances are independently distributed, then the optimal contract for each worker is an APE with \( w^A = (w^A_1, 0) \), where \( w^A_1 = \frac{e}{(q_1 - q_0)(1 + \alpha)} \).

The following proposition shows that APE is actually not optimal when the workers are ambitious, even if the information structure calls for an APE in the traditional principal-agent model.

**Proposition 3.** In the case when workers are homogeneous and there is no correlation in performance, the profit for the firm is higher under optimal RPE than under optimal APE.

**Proof.** According to Lemma 1, the firm’s expected wage cost under APE is

\[
c(1, 1; w^A) = 2q_1 w_1 = \frac{2q_1 e}{(q_1 - q_0)(1 + \alpha)}.
\]

From Proposition 2, the firm’s cost when \( \sigma = 0 \) is

\[
c(1, 1; w^R) = 2q_1 (1 - q_1) w^R_{10} = \frac{2q_1 (1 - q_1) e}{(q_1 - q_0)(1 + \alpha - q_1)} < \frac{2q_1 e}{(q_1 - q_0)(1 + \alpha)} = c(1, 1; w^A).
\]

That is, the firm can induce both workers to exert high effort at a lower cost under a RPE.

Proposition 3 is of great interest, since it offers an explanation for why the firm will still use relative performance evaluation, when a worker’s performance provides no information in inferring the effort level of any other worker. As is well known in the principal-agent literature, the main reason why a worker’s wage will depend on other workers’ performances is that if there is a common shock that affects the performance of all workers in the same way, then a RPE can filter out this common shock both to infer the workers’ effort more precisely, and to reduce the workers’ wage risks caused by the shock. The
optimal contract is thus a relative performance evaluation.\textsuperscript{13} Absent this consideration, a \textit{RPE} only brings in disturbance to a worker’s wage, and should not be used, as is shown in Lemma 1. However, relative-performance-based promotion is so commonly used that it seems to go beyond the consideration of screening out risks.\textsuperscript{14} Here we offer an explanation which does not depend on consideration of reducing risks. It comes from the workers’ desire to do better than their co-workers. If an ambitious worker’s wage is higher than his co-worker, he is willing to accept a lower wage in order to exert the same effort. The firm can thus save wage cost by artificially creating a wage structure that depends on \textit{RPE}. This is achieved by setting the worker’s wage to be zero if he is a loser (this comes from the limited liability constraint), and pays a winner only enough to induce high effort. Since the worker derives additional utility when he is a winner, and suffers additional utility as a loser, a lower wage for high performer suffices to provide enough utility gap in order to induce high effort.

4 The Case of Heterogeneous Workers

This section considers the case when the workers do not have the same degree of ambition. Without loss of generality, we assume that worker 1 is more ambitious than worker 2, that is, $\alpha_1 > \alpha_2$. Although the workers are heterogeneous in ambition, the firm might have difficulty in offering different wage contracts to different workers. There are two possible reasons for this. First, the firm might not know which worker is more ambitious, and therefore cannot offer wage contracts based on the worker’s characteristics. Second, and perhaps more importantly, the firm might be legally forbidden from discriminating between the workers. This is particularly so in our model because all workers have the same productivity, and are induced to exert high effort. If, say, worker 1 is paid less

\textsuperscript{13} See, for example, Mookherjee (1984).
\textsuperscript{14} See Devaro (2002).
than 2 when both have exerted high efforts and produce high outputs, then the firm risks being sued by the workers. As a result, there are two possible types of contracts that the firm can offer. The first is to offer a uniform contract to both types of worker. That is, although the workers have different degrees of ambition, the firm does not try to distinguish between them. We will call this a pooling contract. In the second kind of contract, the firm offers a menu of two contracts to discriminate between two workers. However, following the adverse selection literature, the workers choose the contracts they prefer, rather than being forced by the firm. Since the workers will reveal their own types by choosing different contracts, we call this a separating contract. The advantage of a menu of contracts is that it can exploit the difference in the worker’s degree of ambition. However, the disadvantage is that additional self-selection constraints have to be satisfied. The following proposition derives the optimal contracts for both cases.

**Proposition 4.**

(i) Under the optimal pooling contract, $w^1 = w^2 \equiv w^H = (0, w, 0, 0)$, where $w = e^{\frac{(1+\alpha_2)}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)}}$.

(ii) Under the optimal separating contract, $w^1 = (0, w^1_{10}, 0, w^1_{00})$, where $w^1_{10} = e^{\frac{(1+\alpha_2)}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)(1+\alpha_1)}}$, $w^1_{00} = \frac{q_1(\alpha_1-\alpha_2)e^{\frac{(1+\alpha_1)}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)(1+\alpha_1)}}}{(1-\sigma)(q_1-q_0)(1-\sigma)(1+\alpha_2-q_1)(1+\alpha_1)}$; and $w^2 = (0, w^2_{10}, 0, 0)$, where $w^2_{10} = e^{\frac{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)}}$.

**Proof.** See Appendix.

The optimal pooling contract, as in the case of homogeneous workers, is a RPE. Note that although the workers have different degrees of ambition, they have exactly the same utilities. This can be seen by plugging $w^H$ into the utility functions in (2). What needs to be explained is then why $w^H$ depends only on $\alpha_2$, and not on $\alpha_1$. The reason for this is as follows. Since worker 1 is more ambitious, he derives more utility for being a winner, and thus only requires a lower wage when he is a winner than worker 2 to derive the same
utility. So the value of wage paid to the winner, $w_{10}^i$, is binding only for worker 2 (and is thus a function of $\alpha_2$), who requires a higher wage as a winner to attain the same utility. By the same argument, the lower bound for the loser’s wage, $w_{01}^i$, should be binding for worker 1, who suffers more as a loser. However, limited liability requirement limits its bound as 0, which is independent of $\alpha_1$. Consequently, the value of $w^H$ is a function of $\alpha_2$ only, and wage structure is thus independent of $\alpha_1$.

A direct comparison between $w^H$ and $w^R$ shows that as long as $\alpha \geq \alpha_2$, the total wage cost for the firm will be higher, implying that the workers receive higher utilities in the heterogeneous case. The reason for this is that under a pooling contract, the firm is paying the workers as if both are the less ambitious worker, which requires higher wage as a winner. This means that although the firm still profits from hiring ambitious workers, its profit is lower than in the homogeneous case. In other words, the firm is worse off when it has a mixture of workforce with different degrees of ambition.

In the separating contract, the firm offers a menu of two wage contracts, each voluntarily selected by one type of worker. The benefit of a separating contract is that the firm can discriminate against the more ambitious worker 1 by paying him a lower wage as a winner than worker 2. This can be seen from Proposition 4 that $\bar{w}_{10}^2 > \bar{w}_{10}^1$. In order to prevent worker 1 from choosing $w^2$, he must be compensated in some other contingency when he is not the winner. It turns out that he is paid a positive wage when both workers have low outputs, i.e., $w_{00}^1 > 0$. The reason why it is $w_{00}^1$, rather than $w_{01}^1$ or $w_{11}^1$, that is positive is as follows. If $w_{01}^1 > 0$, then worker 1 is given positive wage as a loser. This weakens his incentive in effort, and is thus less desirable than setting $w_{11}^1 > 0$ or $w_{00}^1 > 0$.

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15 This is a reasonable restriction because worker 2 is the least ambitious worker. In general, an adequate comparison between homogeneous and heterogeneous cases might be to assume that $\alpha$ is the mean value of $\alpha_1$ and $\alpha_2$, where the mean is taken over the relative composition of type 1 and type 2 workers. In this case $\alpha_1 > \alpha > \alpha_2$.

16 Fershtman et al. (2003b) also show that wage cost will be higher under the heterogeneous case. However, since the workers exert more effort, the firm actually has higher output and, in contrast to our result, higher profit. We will discuss their results in more detail shortly.
Although raising the value of either $w_{11}$ or $w_{00}$ will increase the utility of worker 1, it will at the same time reduce worker 2’s utility, but by a different degree. A unit raise in $w_{11}$ (resp. $w_{00}$) will increase the utility of worker 1 by $(\sigma + (1 - \sigma)q_1^2)(1 + \alpha_1)$ (resp. $(1 - \sigma)(1 - q_1^2)(1 + \alpha_1)$) but reduce the utility of worker 2 by $(\sigma + (1 - \sigma)q_1^2)\alpha_2$ (resp. $(1 - \sigma)(1 - q_1^2)\alpha_2$). Simple comparative advantage comparison implies that it is less costly for the firm to raise the value of $w_{00}$.

Although worker 1 is paid a positive wage when both have low outputs, the wage structure still resembles a relative performance evaluation. In particular, worker 2 is paid a positive wage only when he is a winner. Similar to the pooling contract case, unless $\alpha < \alpha_2$, otherwise $w_{10}^1 > w_{10}^R$ and $w_{10}^2 > w_{10}^R$, which in turn implies that the firm has lower profit. That means unless the worker in the homogeneous case is even less ambitious than the least ambitious in the heterogeneous case, otherwise both workers are paid higher wage in the heterogeneous case.

Although the optimal wage structures are different for pooling and separating contracts, a common feature emerges: The firm is worse off with a mixed workforce. That is, worker heterogeneity encroaches on the ability of the firm to manipulate wage policy, and thus reduces its profit. This is in contrast to the result in either Fershtman et al. (2003b) or Charness and Kuhn (2004); both accentuate the merit of a mixed workforce in improving the profit of the firm. There are two reasons for this difference in result. First, in our model the firm cannot discriminate against the workers by giving different contracts to workers with different degrees of ambition. The only case in which the workers can be paid differently is through self-selection. As a result the wage contracts have to satisfy additional incentive compatibility constraints, which reduces the firm’s profit. Alternatively, the firm can offer a uniform contract to both types of workers, but in that case all workers have to be treated as low-ambition type. This, similarly, reduces the firm’s profit. On the other hand, Fershtman et. al. (2003b) and Charness and Kuhn
(2004) assume that the firm is allowed to offer different contracts to different types of workers, so there is no cost in treating different workers differently.\(^{17}\) Second, in their models the workers have a continuum of choices of effort. In that case workers are not always motivated to exert the highest effort possible. Different contracts tailored to different types of workers (without needing to satisfy the incentive compatibility constraints by assumption) enables the firm to motivate the workers to exert the optimal level of effort. Consequently, although workers are paid more in the heterogeneous case (which is the same as our result), their effort level also increases, which also increases the total profit of the firm.

It turns out that pooling and separating contracts yield exactly the same profits for the firm and the same utilities for the workers. That is, in turn of result, they are exactly the same for both the firm and the workers. This can be seen by comparing the expected wage for both types of workers under pooling and separating contracts (See Appendix). However, although the two contracts result in the same expected utility for both the principal and the agent, they have different nature. The pooling contract is strictly a relative performance evaluation in that only the winner receives positive wage. The separating contract, while setting a \(RPE\) for worker 2, also pays a positive wage to worker 1 when both have low outputs. Note that if we measure wage risk of a worker by the variance of his wage distributions, then both workers have lower wage risks under separating contract.\(^{18}\) In that case the separating contract will Pareto dominate the pooling contract, and is a better prediction of the wage structure that the firm will offer.

\(^{17}\)In terms of our discussion in the beginning of this section, in their models the firm has the advantage of directly discriminating between the workers without the disadvantage of having to satisfy the self-selection constraints.

\(^{18}\)Note that this is true even for worker 2.
5 Conclusion

This paper investigates the property of a firm’s compensation scheme when the workers’ utilities depend not only on their own incomes, but also on the incomes of their co-workers. The optimal contract is shown to exhibit wage compression, and relies on relative performance evaluation. Moreover, even if the information structure calls for absolute performance evaluation in the traditional principal-agent model, the optimal contract will still be a relative performance evaluation. Thus our model offers a strong rationale for why, despite many of its drawbacks, relative performance evaluation is so commonly used.

The firm’s profit is greater than when the workers only care about own-incomes. This is because given a fixed wage schedule, the utility gap for a worker between different wages levels is widened. The firm can thus lower the wage paid to the high-performer while still maintains the same incentives. We also show that, in contrast to past literature, worker-heterogeneity encroaches on the ability of the firm to profit from the workers’ concern for relative income, despite the fact that the optimal compensation scheme is still in relative performance evaluation.

A natural extension of the present model is the case when the workers differ not only in ambition, but also in productivity. For example, different workers might have different probabilities of producing high output even when they all exert high efforts. This, however, requires very careful specification of the worker’s utility function. For example, in our setup both workers are motivated to exert high effort. As a result, there is stronger reason that a worker will suffer psychological loss if he is paid less. If, however, the worker who is being paid lower wage is aware that his co-worker has higher productivity, then he might feel this difference in wage justified. Or, on the contrary, a low-ability worker might believe that he should be paid more if he has the same output as a high-ability worker, since this signals he has exerted more effort. And if not, he suffers disutility. Obviously, more fundamental investigation on how and in what way the
workers’ utilities depend on others’ incomes, when they are heterogeneous in abilities, is much needed in order to have a thorough understanding of team compensation.
References


Appendix

In this Appendix we consider the case where $\alpha_1 > \alpha_2$. The firm does not know which worker is more ambitious, so the firm offers a menu of wage contracts for them to choose from. We discuss the pooling and separating cases separately:

A. Pooling Contract

Under pooling contract, both types of workers are offered the same contract. The firm’s objective function is thus:

$$\min_w 2\left[(\sigma + (1 - \sigma)q_1^2)w_{11} + (1 - \sigma)q_1(1 - q_1)w_{10} + (1 - \sigma)q_1(1 - q_1)w_{01} + (1 - \sigma)(1 - q_1)^2w_{00}\right]$$

s.t. $w^1 \geq 0, \ w^2 \geq 0$, and

$$(IC_1) \quad q_1w_{11} + (1 + \alpha_1 - q_1)w_{10} - (\alpha_1 + q_1)w_{01} - (1 - q_1)w_{00} \geq \frac{e}{(1 - \sigma)(q_1 - q_0)};$$

$$(IC_2) \quad q_1w_{11} + (1 + \alpha_2 - q_1)w_{10} - (\alpha_2 + q_1)w_{01} - (1 - q_1)w_{00} \geq \frac{e}{(1 - \sigma)(q_1 - q_0)}.$$ 

Since the coefficients for $w_{01}$ and $w_{00}$ are positive in the objective function but negative in the $IC$ constraints, optimality requires $w_{01} = w_{00} = 0$. Moreover, note that

$$\frac{q_1}{1 + \alpha_1 - q_1} < \frac{q_1}{1 + \alpha_2 - q_1} < \frac{\sigma + (1 - \sigma)q_1^2}{(1 - \sigma)q_1(1 - q_1)}.$$ 

This means as long as $w_{11} > 0$, the firm can always decrease $w_{11}$ while increasing $w_{10}$ simultaneously by a rate of $1 : q_1/(1 + \alpha_2 - q_1)$. This will not violate the $IC$s, but will decrease the value of the objective function. Therefore, it is optimal to choose $w_{11} = 0$ and, to keep $w_{10}$ at a minimum value so that $(IC_2)$ is satisfied with equality. That is, the optimal contract is $w^H = (0, w_{10}, 0, 0)$, where

$$w_{10} = \frac{e}{(1 - \sigma)(q_1 - q_0)(1 + \alpha_2 - q_1)}.$$ 

Type $i$ worker’s utility under this contract is

$$(1 - \sigma)q_1(1 - q_1)(1 + \alpha_i)w_{10} - \alpha_iw_{10} = \frac{q_1(1 - q_1)e}{(q_1 - q_0)(1 + \alpha_2 - q_1)}.$$
As a result, both have the same utilities.

B. Separating Contract

The firm’s objective function is:

$$\min_{w^1, w^2} c = 2[(\sigma + (1 - \sigma)q_1^2)w_{11}^1 + (1 - \sigma)q_1(1 - q_1)w_{10}^1 + (1 - \sigma)q_1(1 - q_1)w_{01}^1
$$

$$+ (1 - \sigma)(1 - q_1)^2w_{00}^1]$$

$$= 2[(\sigma + (1 - \sigma)q_1^2)w_{11}^2 + (1 - \sigma)q_1(1 - q_1)w_{01}^2 + (1 - \sigma)q_1(1 - q_1)w_{10}^2
$$

$$+ (1 - \sigma)(1 - q_1)^2w_{00}^2].$$

s.t. \( w^1 \geq 0, \ w^2 \geq 0, \) and

\[(IC_1) \quad q_1[(1 + \alpha_1)w_{11}^1 - \alpha_1w_{11}^2] + (1 - q_1)[(1 + \alpha_1)w_{10}^1 - \alpha_1w_{01}^1]
$$

$$- q_1[(1 + \alpha_1)w_{01}^1 - \alpha_1w_{01}^2] - (1 - q_1)[(1 + \alpha_1)w_{00}^1 - \alpha_1w_{00}^2] \geq \frac{e}{(1 - \sigma)(q_1 - q_0)};$$

\[(IC_2) \quad q_1[(1 + \alpha_2)w_{11}^2 - \alpha_2w_{11}^1] - q_1[(1 + \alpha_2)w_{01}^2 - \alpha_2w_{01}^1]
$$

$$+ (1 - q_1)[(1 + \alpha_2)w_{10}^2 - \alpha_2w_{10}^1] - (1 - q_1)[(1 + \alpha_2)w_{00}^2 - \alpha_2w_{00}^1] \geq \frac{e}{(1 - \sigma)(q_1 - q_0)};$$

\[(IC_3) \quad (\sigma + (1 - \sigma)q_1^2)[(1 + \alpha_1)w_{11}^1 - \alpha_1w_{11}^2] + (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_1)w_{10}^1 - \alpha_1w_{01}^1]
$$

$$+ (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_1)w_{01}^1 - \alpha_1w_{01}^2] + (1 - \sigma)(1 - q_1)w_{00}^1 - \alpha_1w_{00}^2] - e
$$

$$\geq \sigma + (1 - \sigma)q_1^2[(1 + \alpha_1)w_{11}^1 - \alpha_1w_{11}^2] + (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_1)w_{01}^1 - \alpha_1w_{01}^2]
$$

$$+ (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_1)w_{10}^1 - \alpha_1w_{10}^2] + (1 - \sigma)(1 - q_1)w_{00}^1 - \alpha_1w_{00}^2] - e;$$

\[(IC_4) \quad (\sigma + (1 - \sigma)q_1^2)[(1 + \alpha_2)w_{11}^2 - \alpha_2w_{11}^1] + (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_2)w_{10}^2 - \alpha_2w_{10}^1]
$$

$$+ (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_2)w_{01}^2 - \alpha_2w_{01}^1] + (1 - \sigma)(1 - q_1)w_{00}^2 - \alpha_2w_{00}^1] - e
$$

$$\geq (\sigma + (1 - \sigma)q_1^2)[(1 + \alpha_2)w_{11}^1 - \alpha_2w_{11}^2] + (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_2)w_{01}^1 - \alpha_2w_{01}^2]
$$

$$+ (1 - \sigma)q_1(1 - q_1)[(1 + \alpha_2)w_{10}^1 - \alpha_2w_{10}^2] + (1 - \sigma)(1 - q_1)w_{00}^1 - \alpha_2w_{00}^1] - e.$$

\((IC_3)\) is the condition for worker 1 to choose \( w^1 \) over \( w^2 \), and \((IC_4)\) is for worker 2 to choose \( w^2 \) over \( w^1 \). As it turns out, \((IC_3)\) and \((IC_4)\) can hold simultaneously only when
both are satisfied with equality:

\[(IC_A) \quad (\sigma + (1 - \sigma)q_1^2)(w_{11}^1 - w_{11}^2) + (1 - \sigma)q_1(1 - q_1)(w_{10}^1 - w_{01}^2) + (1 - \sigma)q_1(1 - q_1)(w_{01}^1 - w_{10}^2) + (1 - \sigma)(1 - q_1)^2(w_{00}^1 - w_{00}^2) = 0.\]  

(A.1)

To solve the problem, first we note that since the coefficients of \(w_{01}^1\) and \(w_{01}^2\) are both negative in \((IC_1)\) and \((IC_2)\) and positive in the objective function, it is optimal to set \(w_{01}^1 = w_{01}^2 = 0\). We impose this result hereafter.

For the rest of the variables, we use the \textit{simplex algorithm method} to select the right ones that should be strictly positive in the solution. The method is composed of several steps.

\textit{Step 1.} Add slack variables into \((IC_1)\) and \((IC_2)\) so that they satisfy with equality:

\[(IC'_1) \quad q_1(1 + \alpha_1)w_{11}^1 - q_1 \alpha_1 w_{11}^2 + (1 - q_1)(1 + \alpha_1)w_{10}^1 + q_1 \alpha_1 w_{10}^2 - (1 - q_1)(1 + \alpha_1)w_{00}^1 - z_1 = \frac{e}{(1 - \sigma)(q_1 - q_0)};\]

\[(IC'_2) \quad q_1(1 + \alpha_2)w_{11}^2 - q_1 \alpha_2 w_{11}^1 + (1 - q_1)(1 + \alpha_2)w_{10}^2 + q_1 \alpha_2 w_{10}^1 - (1 - q_1)(1 + \alpha_2)w_{00}^2 - z_2 = \frac{e}{(1 - \sigma)(q_1 - q_0)}.\]  

\(z_1 \geq 0, \quad z_2 \geq 0.\)

\textit{Step 2.} Find the basic variables, then solve for these variables in terms of the remaining nonbasic ones.

By a standard but tedious process, it can be shown that the solution calls for choosing \(w_{10}^1, \ w_{10}^2\) and \(w_{00}^1\) as the basic variables. We then solve for them in terms of the rest of the variables.

\[
w_{10}^1 = \frac{(1 + \alpha_2)e}{(1 - \sigma)(q_1 - q_0)(1 + \alpha_2 - q_1)(1 + \alpha_1)} - \frac{q_1(1 + \alpha_1 + \alpha_2) - (1 + \alpha_1)(1 + \alpha_2)}{\beta(1 + \alpha_2 - q_1)(1 + \alpha_1)}w_{11}^1 + \frac{[1 - q_1(1 + \beta)](1 + \alpha_1 + \alpha_2 + \alpha_1(\alpha_2 + \beta + \alpha_2)\beta}{\beta(1 + \alpha_2 - q_1)(1 + \alpha_1)}w_{11}^2 + \frac{(1 - q_1)(1 + \alpha_2)}{(1 + \alpha_2 - q_1)(1 + \alpha_1)}w_{00}^2 + \frac{1}{(1 + \alpha_1)}z_1 + \frac{q_1}{(1 + \alpha_2 - q_1)(1 + \alpha_1)}z_2;\]
\[ w_{10}^2 = \frac{e}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)} + \frac{q_1(1+\beta)\alpha_2}{\beta(1+\alpha_2-q_1)}w_{11}^1 + 0 \cdot w_{11}^2 + \frac{1}{(1+\alpha_2-q_1)}w_{00}^0 + 0 \cdot z_1 + \frac{1}{(1+\alpha_2-q_1)}z_2; \]  
(A.3)

\[
w_{10}^1 = \frac{q_1(\alpha_1-\alpha_2)e}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)(1+\alpha_1)} - \frac{q_1[q_1(\alpha_1-\alpha_2)\beta + \Delta]}{\beta(1-q_1)(1+\alpha_2-q_1)(1+\alpha_1)}w_{11}^1 \\
+ \frac{q_1\Delta}{\beta(1-q_1)(1+\alpha_2-q_1)(1+\alpha_1)}w_{11}^2 + \frac{(1+\alpha_1-q_1)(1+\alpha_2)}{(1+\alpha_2-q_1)(1+\alpha_1)}w_0^0 \\
- \frac{q_1}{(1-q_1)(1+\alpha_1)}z_1 + \frac{q_1(1+\alpha_1-q_1)}{(1-q_1)(1+\alpha_2-q_1)(1+\alpha_1)}z_2.
\]

where \( \Delta = (1 + \alpha_1 - q_1)[(1 + \alpha_2) - q_1(1 + \beta)(1 + \alpha_1 + \alpha_2)] + \alpha_1 q_1(1 + \beta)(\alpha_1 - \alpha_2) > 0, \)
and \( \beta = \frac{(1-\sigma)q_1(1-q_1)}{\sigma + (1-\sigma)q_1^2}. \)

**Step 3.** Criterion for optimality: if all the coefficients in the objective function \( f \) in terms of the nonbasic variables are positive, then the solution is optimal.

Substitute (A.3) into the objective function, and also impose \((IC_A)\) constraint, then we have:

\[
c = 2(1-\sigma)q_1(1-q_1)[\frac{1}{\beta}w_{11}^1 + w_{10}^1 + \frac{(1-q_1)}{q_1}w_{00}^0] \\
= 2\left\{ \frac{q_1(1-q_1)e}{(q_1-q_0)(1+\alpha_2-q_1)} + \frac{(\sigma + (1-\sigma)q_1^2)q_1(1+\beta)\alpha_2}{(1+\alpha_2-q_1)}w_{11}^1 + \frac{(\sigma + (1-\sigma)q_1^2)(1+\alpha_2)(1-q_1(1+\beta))}{(1+\alpha_2-q_1)}w_{11}^2 \right\} (A.4)
+ \frac{(1-\sigma)(1-q_1)^2(1+\alpha_2)}{(1+\alpha_2-q_1)}w_{00}^2 + \frac{(1-\sigma)q_1(1-q_1)}{(1+\alpha_2-q_1)}z_2).
\]

We can see that all the coefficients in (A.4) are positive (the coefficient of \( w_{11}^2 \) is positive since \( 1 > q_1(1 + \beta) \)). \((IC_2)\) is always binding because the coefficient of \( z_2 \) is positive. Therefore, by setting all the nonbasic variables equal to zero in (A.3), the following solution is optimal:

\[
\begin{align*}
   w_{10}^1 &= \frac{(1+\alpha_2)e}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)(1+\alpha_1)}; \\
   w_{10}^2 &= \frac{e}{(1-\sigma)(q_1-q_0)(1+\alpha_2-q_1)}; \\
   w_{00}^1 &= \frac{q_1(\alpha_1-\alpha_2)e}{(1-\sigma)(q_1-q_0)(1-q_1)(1+\alpha_2-q_1)(1+\alpha_1)}; \\
   0 & \text{ otherwise.}
\end{align*}
\]  
(A.5)
Worker 1’s utility under this contract is

\[(1 - \sigma)q_1(1 - q_1)[(1 + \alpha_1)w_{10}^1 - \alpha_1 w_{10}^2] + (1 - \sigma)(1 - q_1)^2(1 + \alpha_1)w_{00}^1\]

\[= \frac{q_1(1-q_1)e}{(q_1-q_0)(1+\alpha_2-q_1)}.
\]

Likewise, worker 2’s utility is

\[(1 - \sigma)q_1(1 - q_1)[(1 + \alpha_2)w_{10}^2 - \alpha_2 w_{10}^1] - (1 - \sigma)(1 - q_1)^2\alpha_2 w_{00}^1\]

\[= \frac{q_1(1-q_1)e}{(q_1-q_0)(1+\alpha_2-q_1)}.
\]

Again, both workers have the same utilities. Moreover, their utilities are the same as in the pooling contract case.