Spatial Discrimination: Bertrand vs. Cournot with Asymmetric Demands∗

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Abstract

This paper develops a spatial model with homogeneous product and asymmetric demand structure to compare the delivered prices, aggregate profits and social welfare between Cournot and Bertrand competition. It focuses on the impacts caused by the asymmetric demand curves and differential transport costs. It shows that the market-size effect stemming from asymmetric demand structure is crucial in determining firms’ locations under Cournot competition, but insignificant under Bertrand competition. Moreover, when either market is excessively large, the equilibrium price of the large market is likely to be higher under Bertrand than under Cournot competition if the maximum transport cost (the cost of shipping a unit of output between the two markets) is high. Aggregate profits may be higher under Bertrand than under Cournot if the maximum transport cost is high. Also, social welfare under Cournot competition may exceed that under Bertrand competition if one of the markets is excessively large and the maximum transport cost is high. These results are in sharp contrast with the ones obtained in the existing literature.
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I. Introduction

Since Bertrand’s (1883) criticism of Cournot (1838), economists often use the game theory to represent oligopolistic competition and debate the relative merits of models using quantities or prices as firms’ strategic variables. The following questions are often raised and discussed in the literature. First, are the delivered prices lower under Bertrand than under Cournot equilibrium? Second, are the aggregate profits always lower under Bertrand than under Cournot competition? Third, from the standpoint of the government, which type of competition is more efficient? In a duopoly game in which two firms produce a homogeneous good and have an equal and constant marginal cost, it is well recognized that the equilibrium price is equal to (higher than) the marginal cost under Bertrand (Cournot) competition. These results are no longer true if the assumption of homogeneous product is relaxed. In the case of differentiated products, Bertrand prices are necessarily higher than the marginal cost. Even in this case, Cournot competition is still viewed as more “monopolistic” than Bertrand competition, as the price under Cournot competition is higher than that under Bertrand competition. Singh and Vives (1984) further

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1 Aggregate profits are defined as the sum of profits from markets 1 and 2.
2 This is not the case if supergame equilibria are considered. Price-setting supergame equilibria may generate higher prices than quantity-setting equilibria for either homogeneous or differentiated products. See Brock and Scheinkman (1981) and Deneckere (1983).
showed that in a differentiated duopoly profits are larger (smaller) in Cournot than in
Bertrand competition if the products are substitutes (complements), and that Bertrand
equilibrium is more efficient than Cournot equilibrium because the consumer surplus
and total surplus are higher at the former than at the latter equilibrium.³ Cheng
(1985) provided some insights into the results obtained by Singh and Vives (1984)
with a geometric analysis. In particular, he concluded that profits are higher under
Cournot than under Bertrand competition only if the demand structure is not
excessively asymmetric. Otherwise, one of the firms may earn a lower profit under
Cournot than under Bertrand competition. Unfortunately, Cheng (1985) did not go
further to explore the role of asymmetric demand structure.

The above analyses are examined in the context of a traditional non-spatial
framework in which transport costs are insignificant and negligible. In a spatial
model, Hamilton et al. (1989) set up a model of spatial discrimination with duopolists
competing in location and sales of a homogeneous product. In the model, consumers
with an identical downward sloping demand are uniformly distributed along a line
segment. They showed that: (1) prices are necessarily higher under Cournot than
under Bertrand competition; (2) profits are higher under Cournot competition only for
a low transport cost, but the reverse holds for a high transport cost; (3) the social

³ Vives (1985) also derived that given symmetric demands and unique equilibrium, the prices and
profits at equilibrium are larger and quantity smaller in Cournot than in Bertrand competition.
welfare is higher under Bertrand competition; (4) the two firms agglomerate at the center of the line segment in the Cournot case, but spread apart in the Bertrand case.

Note that if demands are symmetric, comparisons derived in a spatial economy by Hamilton et al. are not significantly different from those in a non-spatial economy. However, when demands are asymmetric, the price and profits in the large market are higher, creating a market-size effect, thereby attracting firms to locate closer to the large market. If the demand asymmetry is significant so as to make the market size effect striking, it is conceivable that the rankings of prices, profits and welfare under the two competition schemes are dependent on the locations of the firms. Therefore, it is of interest to take into account asymmetric demand structure in a spatial model to discuss the afore-mentioned questions.

In this paper, we assume that consumers reside in two distinct markets, which locate at two endpoints of a line segment. We will also assume the demand curves of the two markets to be asymmetric. The focus of this paper is therefore on the impacts arising from asymmetric demand curves and differential transport costs. A two-stage game is employed in which firms choose locations to maximize their profits in the first stage. In the second stage, firms can choose either outputs under Cournot

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4 The only difference arises from profits.
5 Haufler and Wooton (1999) pointed out that firms prefer to locate in a large market where they will be able to charge higher producer prices. Rowthorn (1992) emphasized the role of market size and concerned with an intra-industry trade and investment between countries at approximately the same level of development.
competition or prices under Bertrand competition. We shall solve these two cases for the afore-mentioned questions, respectively, and then compare the results with those in Hamilton et al. (1989) and in non-spatial literature.

The remainder of the paper is organized as follows. Section II explores the firms’ location choices under Cournot competition while Section III analyzes the firms’ location choices under Bertrand competition. Section IV compares the equilibrium prices, aggregate profits, and social welfare derived under the two types of competition. The final section concludes the paper.

II. Cournot Competition

Consider a simple framework: There are two firms, denoted as firm A and firm B, who can locate at any point along a line segment, such as a highway, with length $s$. There are two distinct markets, denoted as markets 1 and 2, locating at the two endpoints of the line segment as shown in Figure 1. The firms sell a homogeneous product with zero production cost to consumers who reside only in either of the two markets. The locations of firms A and B are assumed to be $x_A$ and $x_B$ apart from the left endpoint of the line segment, respectively. Moreover, firm A is assumed to locate to the left of firm B, i.e., $x_A \leq x_B$.

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6 The two end-point markets setting was first proposed by Hwang and Mai (1990) and subsequently employed by Gross and Holahan (2003) etc.
In order to exhibit the feature of asymmetric demand structure, we assume that the demand curves at markets 1 and 2 take the following forms:7

\[ q_i = \gamma a(1 - p_i), \]
\[ q_2 = a(1 - p_2). \]  \hspace{1cm} (1)

where \( q_i \) and \( p_i \) are the quantity demanded and delivered price in market \( i \) \((i = 1, 2)\); \( \gamma \) and \( a \) are positive constants and \( \gamma \) is a measure of relative market size. Without loss of generality, we assume throughout the paper that market 1 is larger than market 2 or equivalently \( \gamma \) is greater than unity. Note that if \( \gamma = 1 \), the two demands become symmetric.

The game in this paper consists of two stages. In the first stage, the two firms simultaneously select their locations. In the second stage, given the location decisions, the firms simultaneously choose their quantities (prices) if they play Cournot (Bertrand) competition. The sub-game perfect equilibrium of the model is solved by backward induction, beginning with the final stage.

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7 Motta and Norman (1996) and Haufler and Wooton (1999) also use these demand curves to exhibit asymmetric market size.
Assume in this section that firms play Cournot competition in both markets—each firm chooses a quantity schedule to maximize its profits given locations $x_A$ and $x_B$, and also the rival’s quantity schedule. Profits of each firm are defined as the sum of its profits from markets 1 and 2. They can be expressed as:

$$\pi^A = \{q_1^A - (1/\mu)(q_1^A)^2 + q_1^A q_1^B\} - t x_A q_1^A,$$

$$+ \{q_2^A - (1/\mu)(q_2^A)^2 + q_2^A q_2^B\} - t(s - x_A) q_2^A\},$$

(2.1)

$$\pi^B = \{q_1^B - (1/\mu)(q_1^B)^2 + q_1^B q_1^A\} - t x_B q_1^B,$$

$$+ \{q_2^B - (1/\mu)(q_2^B)^2 + q_2^B q_2^A\} - t(s - x_B) q_2^B\},$$

(2.2)

where $q_i^j (i = 1, 2, j = A, B)$ are firm $j$’s sales in market $i$; and $t$ is constant transport rate.

Maximizing each firm’s profits with respect to its sales for the two markets and solving these first-order conditions, we obtain the following Cournot quantity schedules:

$$q_1^{AC} = (\mu/3)(1 - 2tx_A + tx_B),$$

$$q_2^{AC} = (\mu/3)[1 - 2t(s - x_A) + t(s - x_B)],$$

$$q_1^{BC} = (\mu/3)(1 + tx_A - 2tx_B),$$

$$q_2^{BC} = (\mu/3)[1 + t(s - x_A) - 2t(s - x_B)],$$

(3)

where variables with superscript “C” denote that their values are associated with the Cournot equilibrium. Equation (3) shows, as expected, that firm $j$’s sales in market $i$ increase as its location becomes closer to the market.

The resulting delivered prices under Cournot competition can be obtained by substituting (3) into (1):
\[
\begin{align*}
p_1^C &= 1 - (1/3)(2 - tx_A - tx_B), \\
p_2^C &= 1 - (1/3)[2 - t(s - x_A) - t(s - x_B)].
\end{align*}
\] (4)

We now turn to the first stage problem. Anticipating the Cournot quantity schedules and delivered prices in the second stage, each firm chooses a location to maximize its profits given the rival’s location. Substituting (3) and (4) into (2.1) and (2.2), we obtain the first- and second-order conditions for profit-maximization with respect to each firm’s location as:

\[
\begin{align*}
\partial \pi^A / \partial x_A &= t(-q_1^{AC} + q_2^{AC}), \\
\partial^2 \pi^A / \partial x_A^2 &= (2a/3)(\gamma + 1)t^2 > 0, \\
\partial \pi^B / \partial x_B &= t(-q_1^{BC} + q_2^{BC}), \\
\partial^2 \pi^B / \partial x_B^2 &= (2a/3)(\gamma + 1)t^2 > 0.
\end{align*}
\] (5)

Equation (5) shows that the profit functions for firms A and B are all strictly convex with respect to location \(x_j\) (\(j = A, B\)), respectively, which implies a corner solution. The Nash equilibrium location can be derived by comparing the profits at the two corners, i.e., markets 1 and 2.

Note that given the assumptions of \(x_B \geq x_A\), there exist three possible solutions: \((x_A, x_B) = (0, 0), (0, s)\) and \((s, s)\). However, the last can be ruled out as firm A’s profits are necessarily higher to locate at market 1 than at market 2, should firm B choose to locate at market 2. This is derivable by the following condition:

\[
\pi^{AC}(s, s) - \pi^{AC}(0, s) = (4\gamma ats / 9)\{[(1/\gamma) - 1] - (ts / \gamma)\} < 0,
\] (6.1)
where $ts$ is called the maximum transport cost of shipping one unit of output from one endpoint market to the other. The solution can therefore be derived by comparing the profits of firm B at the two markets given firm A to stay at market 1:

$$
\pi_{BC}^{(0,s)} - \pi_{BC}^{(0,0)} = (4\gamma ts / 9) [ts - (1 - (1/\gamma))] < (>), \text{if } 1 - (1/\gamma) > (<) ts. (6.2)
$$

Equation (6.2) shows that the solution can be either $(0, 0)$ or $(0, s)$ as it depends on parameter combinations. The two firms would agglomerate at market 1 (i.e., $(x_A, x_B) = (0, 0)$) if $(1 - (1/\gamma) > ts)$; they would locate separately at the two opposite endpoints (i.e., $(x_A, x_B) = (0, s)$) if $(1 - (1/\gamma) < ts)$.

Intuitively, the optimal locations of the firms are jointly determined by the following two effects: the market-size effect and the competition effect. The market-size effect indicates that, other things being equal, firms tend to locate toward the larger market. The competition effect is a bit complicated and need some explanations. First, let us assume the two markets are identical in size and the two firms locate separately at the two ends of the line market. In this case, any increase in maximum transport cost would: (i) raise each firm’s competitiveness against its rival in its home market and therefore the profits from this market; (ii) lower the competitiveness in and hence profits from the foreign market. Nevertheless, the total profits go up as the gain from the home market is higher than the loss from the

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8 For expository convenience, we shall call the market where a firm locates home market and the other market foreign market.
foreign market. The high maximum transport cost can mitigate competition in both markets and make both firms better off. More specifically, as the maximum transport cost becomes higher, cross hauling becomes less significant and each firm’s profits from the two markets become higher. In the extreme case with a sufficiently high maximum transport cost, cross hauling does not occur at all. Each firm becomes a monopolist in its home market, enjoying a monopoly profit. This is the best outcome a firm can expect in the two-market model. In the other extreme with zero transport cost or the two firms locate at the same market, the competition in the two markets is the highest and each firm can earn only a duopoly profit from each market. This turns out to be the worst scenario for the two firms. The competition effect intends to capture the above-mentioned effect. As the maximum transport cost rises, each firm has an incentive to stay apart from each other to alleviate competition in the two markets and earn a higher aggregate profit.

According to (6.2), the optimal locations of the two firms are determined by the magnitudes of \(1-(1/\gamma)\) and \(ts\). The former reflects the market-size effect, which is positively related to the value of \(\gamma\). On the other hand, the latter captures the competition effect, which becomes higher as the maximum transport cost \(ts\) goes up.

Consequently, when market 1 is excessively large (i.e., \(1-(1/\gamma) > ts\), the market-size

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9 We can also find this kind of production efficiency from Bergstrom and Varian (1985), Long and Soubeyran (1997), and Salant and Shaffer (1999) etc.
effect outweighs the competition effect and the two firms will agglomerate at the left endpoint. On the contrary, if market 1 is not excessively larger than market 2 (i.e., \(1-(1/\gamma) < ts\)) such that the competition effect outweighs the market-size effect, then the two firms will take apart and locate at the opposite endpoints of the line segment.

We can thus establish:

**Proposition 1.** If market 1 is excessively larger than market 2, the two firms will agglomerate at market 1 under Cournot competition. Otherwise, firms will take apart and locate separately at the opposite endpoints of the line market.

This result is significantly different from the one derived by Hamilton et al. (1989). They assumed a symmetric demand structure with consumers uniformly distributed along the line segment and found that the two firms will agglomerate at the middle of a line segment. In contrast, we assume an asymmetric demand structure and conclude that the two firms will agglomerate at the larger market if one of the markets is excessively larger than the other, and locate separately at the two endpoints, if neither market is excessively large.

III. Bertrand Competition
In this section, we use the framework specified in our Cournot scenario to examine the case of Bertrand competition with a homogeneous product. As before, the game in question consists of two stages. The two firms choose locations in the first stage and then select their price schedules in the second stage.

In the second stage, the two firms simultaneously determine their prices $p_i^A$ and $p_i^B$ for market $i$. Sales for firm $j$ in markets 1 and 2 are then given by:

\[
q_1^i = \begin{cases} 
0 & \text{if } p_i^j > p_i^k, \\
\frac{(\gamma a/2)(1 - p_i^j)}{\gamma a(1 - p_i^k)} & \text{if } p_i^j = p_i^k, j, k = A, B, j \neq k, \\
\frac{\gamma a(1 - p_i^j)}{a(1 - p_i^k)} & \text{if } p_i^j < p_i^k,
\end{cases}
\]

(7.1)

\[
q_2^i = \begin{cases} 
0 & \text{if } p_i^j > p_i^k, \\
\frac{(a/2)(1 - p_i^j)}{a(1 - p_i^k)} & \text{if } p_i^j = p_i^k, j, k = A, B, j \neq k, \\
\frac{a(1 - p_i^j)}{a(1 - p_i^k)} & \text{if } p_i^j < p_i^k,
\end{cases}
\]

(7.2)

Note that the Bertrand game with a homogeneous product is a game with the property of winner-takes-all. That is the firm with a lower cost (marginal production cost plus maximum transport cost) will undercut the rival’s price and takes the whole market.

Given the assumptions of $x_B \geq x_A$ and zero marginal production cost, firm A is closer to market 1, and can use its location advantage to force out its rival from the market.

Similarly, firm B has an edge over firm A in market 2 and will use this advantage to price firm A out of the market. Furthermore, the winner’s delivered price in its advantageous market must be slightly lower than the maximum transport cost.\(^\text{10}\)

\(^{10}\) Without loss of generality, we assume that the winners’ equilibrium prices are set equal to the maximum transport cost.
Therefore, the delivered prices of markets 1 and 2 charged by firms A and B, respectively, under Bertrand competition are:

\[
p_1^A = m_1^A + tx_A = tx_B,
\]
\[
p_2^B = m_2^B + t(s - x_B) = t(s - x_A),
\]

where \(m_i^j\) denotes firm \(j\)'s mill price in market \(i\).

We now turn to the first stage problem with respect to the firms’ optimal locations. Making use of the equilibrium in the second stage, the profit functions of firms A and B can be specified as follows:

\[
\pi^A = (p_1^A - tx_A)q_1^A = \gamma at(x_B - x_A)(1 - tx_B),
\]
\[
\pi^B = [p_2^B - t(s - x_B)]q_2^B = at(x_B - x_A)[1 - t(s - x_A)].
\]

Totally differentiating (9) with respect to \(x_j\), respectively, we can derive the profit-maximizing conditions for firms’ locations as follows:

\[
\frac{\partial \pi^A}{\partial x_A} = -q_1^A = -\gamma at(1 - tx_B) < 0,
\]
\[
\frac{\partial \pi^B}{\partial x_B} = q_2^B = at[1 - t(s - x_A)] > 0.
\]

Equation (10) shows that firm A will locate at market 1 in which it enjoys a competitive advantage over firm B, while firm B at market 2 for a similar reason. This outcome is irrespective of market sizes and is in sharp contrast to the one we derived under the case of Cournot competition. The intuition of the result is straightforward. As discussed before, under Bertrand competition, firm A always captures the entire market 1 and firm B market 2 for any location configuration given
the assumption of $x_A \leq x_B$. Under such circumstances, an increase in $x_A$ will lower firm A’s mill price, resulting in a profit loss. The best location is a location at market 1. Similarly, increasing $x_B$ will move firm B closer to market 2, thereby raising its profits from market 2. Firm B will thus locate in market 2. Therefore, we can establish:

Proposition 2. The two firms always locate at the opposite endpoints of the line segment under Bertrand competition, irrespective of market size.

It warrants mention that under Bertrand competition, the two firms will never choose to agglomerate at market 1 even if this market is excessively larger than the other. The reason is simple. Suppose the two firms agglomerate at market 1. The two firms are identical and can earn only zero profit under Bertrand competition, which is the worst scenario. They become better off should any of them move away from market 1. Hence, unlike Cournot competition, demand asymmetry does not have any significant impact on plant locations under Bertrand competition. It also implies that the principle of maximum differentiation applies only to the case of Bertrand competition.
Ⅳ. Comparison of the Equilibria

Since the main purpose of this paper is to explore the role of asymmetric demand structure, we will focus upon the case where market 1 is excessively larger than market 2 \((1-(1/\gamma) > ts)\) throughout the rest of the paper.\(^{11}\) If so, the equilibrium location pair is \((x_A^C, x_B^C) = (0, 0)\) under Cournot competition and \((x_A^T, x_B^T) = (0, s)\) under Bertrand competition, where superscripts “C” and “T” denote Cournot and Bertrand equilibrium, respectively. Given these location equilibria, we can further compare the differences in price, profit, and social welfare under Cournot and Bertrand competition.

A. Delivered Prices

Substituting the equilibrium location pair \((x_A^C, x_B^C) = (0, 0)\) into (4), we can derive the delivered prices of the two markets under Cournot equilibrium as follows:

\[
\begin{align*}
  p_1^C(0,0) &= 1/3, \\
  p_2^C(0,0) &= (1/3)(1 + 2ts).
\end{align*}
\]

Alternatively, substituting \((x_A^T, x_B^T) = (0, s)\) into (8), we find that the optimal delivered prices at the two markets under Bertrand equilibrium are:\(^{12}\)

\[
\begin{align*}
  p_1^T(0,s) &= ts \quad \text{if } ts \leq 1/2, \\
  p_2^T(0,s) &= ts \quad \text{if } ts \leq 1/2.
\end{align*}
\]

\(^{11}\)We can prove that the rankings of equilibrium delivered prices, aggregate profits, and social welfare obtained in the case where neither market is excessively large (i.e., \(1-(1/\gamma) < ts\)), are the same as those derived in Hamilton et al. (1989). In order to save space, we will not go further here.

\(^{12}\)As production cost is assumed to be nil in this paper, \(1/2\) is also the highest (monopoly) price a firm would charge.
It is noteworthy that any price exceeding the monopoly price, 1/2, would lead to a lower profit and would never be charged by firms. Hence, price undercutting and then Bertrand competition could not operate if the maximum transport cost is higher than 1/2. For purpose of simplifying the analysis, we will discuss only the case where the maximum transport cost is lower than 1/2 throughout the rest of the paper.

Manipulating (11.1), (11.2), (12.1) and (12.2), we obtain:

\[ p_i^t(0,0) - p_i^t(0,s) = (1/3)(1 - 3ts) > (\,<)0 \text{ if } ts < (>)1/3, \]
\[ p_2^t(0,0) - p_2^t(0,s) = (1/3)(1 - ts) > 0 \text{ if } ts \leq 1/2. \] (13.1) (13.2)

Since the two firms agglomerate in the large market (i.e., market 1) under Cournot competition, the transport costs of shipping their products to market 1 are zero. The Cournot equilibrium price at market 1 is thus equal to 1/3. On the other hand, under Bertrand competition, the two firms locate at the two ends of the line segment. To avoid price undercutting from its rival, firm A would set its price equal to the maximum transport cost, \( ts \), minus \( \varepsilon \) where \( \varepsilon \) is an infinitely small number, and serve the entire market 1. Therefore, the larger the maximum transport cost, the higher is the equilibrium price of market 1. If the maximum transport cost is so high such that \( ts > 1/3 \), the equilibrium price of market 1 will become higher under Bertrand than under Cournot competition.

The intuition for (13.2) is straightforward. Under Bertrand competition, firm B,
who locates at market 2, would set its price equal to $t_s - \varepsilon$ and blockade the entry of its rival who locates at market 1. On the other hand, in the case of Cournot competition, the two firms locate at market 1 and compete in a Cournot fashion in market 2. Since competition is less severe under Cournot than under Bertrand competition, the Cournot equilibrium price at market 2 must be higher than that under Bertrand competition. Based on the above discussions, we can establish the following proposition.

Proposition 3. The equilibrium price of the large market is likely to be higher but that of the small market is definitely lower under Bertrand competition than under Cournot competition.

This outcome is in sharp contrast with the one obtained in the existing literature. In a non-spatial economy with differentiated products, it has been shown that equilibrium prices under Cournot competition are necessarily higher than those under Bertrand competition (see Hathaway and Rickard (1979), Singh and Vives (1984) and Cheng (1985)). In a spatial model with homogeneous goods as well as symmetric demand structure, Hamilton et al. (1989) also obtain the similar result. However, the present paper has shown that Cournot price could be lower than Bertrand price at the
large market if demands are sufficiently asymmetric.

B. Aggregate profits

Substituting (3) into (2.1) and (2.2) and noting that \((x_A^C, x_B^C) = (0, 0)\), we can derive the Cournot profits in markets 1 and 2, respectively, as follows:

\[
\begin{align*}
\pi_1^C(0,0) &= 2\gamma a/9, \\
\pi_2^C(0,0) &= (2a/9)(1-ts)^2.
\end{align*}
\] (14.1)

From (12.1), (12.2) and noting that \((x_A^T, x_B^T) = (0, s)\), the Bertrand profits of markets 1 and 2 are derivable, respectively, as follows:

\[
\begin{align*}
\pi_1^T(0,s) &= \gamma ats(1-ts) \quad \text{if } ts \leq 1/2, \\
\pi_2^T(0,s) &= ats(1-ts) \quad \text{if } ts \leq 1/2.
\end{align*}
\] (14.2)

The profit differences between Cournot and Bertrand equilibrium for markets 1 and 2 are then obtainable by comparing (14.1) and (14.2):

\[
\begin{align*}
\pi_1^C(0,0) - \pi_1^T(0,s) &= (\gamma a/9)(2-3ts)(1-3ts) > (\gamma a/9)(2-3ts)(1-3ts) > (0) \quad \text{if } ts < 1/3 (1/3 < ts \leq 1/2), \\
\pi_2^C(0,0) - \pi_2^T(0,s) &= (a/9)(2-11ts)(1-ts) > (a/9)(2-11ts)(1-ts) > (0) \quad \text{if } ts < 2/11 (2/11 < ts \leq 1/2).
\end{align*}
\] (15.1) (15.2)

(Insert Figures 2 and 3 here)

We see from (15.1) that the profits of market 1 is higher under Cournot than under Bertrand competition, if the maximum transport cost between the two markets is lower than 1/3 (this is the Cournot price if both firms pay no transport costs). The opposite holds if the condition is reversed. The total Cournot and Bertrand profits of
the two firms from market 1 are depicted in Figure 2. Under Cournot competition, firms agglomerate at market 1 without incurring transport cost and the equilibrium price of market 1 is 1/3. Thus, the total profits of market 1 under Cournot are equal to $2\gamma a / 9$ as indicated by equation (14.1). On the other hand, under Bertrand competition, firm A captures the entire market 1 by setting its equilibrium price equal to the maximum transport cost. Its profits grow when the maximum transport cost rises and reach the highest level when the maximum transport cost is equal to 1/2. Consequently, we can conclude that profits from market 1 are higher (lower) under Bertrand than Cournot equilibrium, as long as the Bertrand equilibrium price is higher (lower) than the Cournot equilibrium price, which is equal to 1/3.

Moreover, we find from (15.2) that Cournot profits are higher than Bertrand profits in market 2 if the maximum transport cost is lower than 2/11, and the ranking is reversed otherwise. This result can be described by Figure 3 in terms of profit curves. As shown in the figure, the Bertrand profits of market 2 increase when the maximum transport cost rises, reaching its maximum level when $t_s = 1/2$. As for the Cournot profits from market 2, they fall when the maximum transport cost rises. This is because the two firms agglomerate at market 1 and the profits are negatively related to transport costs. This explains why the Cournot profits from market 2 are lower than Bertrand profits, when the maximum transport cost becomes high. We
can thus establish the following proposition:

Proposition 4. Assume that market 1 is excessively large and that $ts \leq 1/2$. The aggregate profits are higher under Cournot than under Bertrand competition if the maximum transport cost is low. The opposite holds if the maximum transport cost is high.

This result weakens the conclusions made by Singh and Vives (1984) and Cheng (1985). In a non-spatial framework, they found that aggregate profits are higher under Cournot than under Bertrand competition if the products are substitutes and the demand structure is symmetric. Our finding also confirms Cheng’s conjecture that aggregate profits may be higher under Bertrand than under Cournot competition if the demands are sufficiently asymmetric.

C. Social Welfare

The social welfare is defined as the sum of the consumer and producer surplus from markets 1 and 2. The welfare difference between the Cournot and Bertrand equilibria is derivable as follows:

$$SW^C(0,0) - SW^F(0,s) = (a/18)[\gamma [(-1 + 3ts)(1 + 3ts)] - [(1 - ts)(1 + 17ts)]) \text{ if } ts \leq 1/2.$$  \hspace{1cm} (16)
We see from (16) that given $ts \leq 1/2$, the welfare difference is negative if the maximum transport cost is less than 1/3, but is positive if the maximum transport cost is greater than 1/3 and the relative market size ratio $\gamma$ is sufficiently large.\textsuperscript{13} This result can be explained by noting that the equilibrium price of market 1 under Cournot competition is 1/3, while the equilibrium price under Bertrand competition is equal to the maximum transport cost. As discussed before, the equilibrium price is always higher under Cournot than under Bertrand competition in market 2; hence the welfare in market 2 is higher under Bertrand competition. On the other hand, the equilibrium price in market 1 is higher (lower) and hence the welfare is lower (higher) under Cournot than under Bertrand competition, if the maximum transport cost is less (greater) than 1/3. Consequently, the welfare difference is negative if the maximum transport cost is less than 1/3. However, when the maximum transport cost is greater than 1/3 and the market size of market 1 is sufficiently large, the welfare difference in market 1 becomes positive and outweighs the negative welfare difference in market 2.\textsuperscript{14} Accordingly, we have:

Proposition 5. The social welfare is higher under Cournot than under Bertrand

\textsuperscript{13} Given $ts \leq 1/2$, the term $(1-ts)(1+17ts) > 0$, while the term $\gamma \left[(-1+3ts)(1+3ts)\right] \leq (>) 0$ if $ts < (>) 1/3$. It is clear that the welfare difference is positive if $ts > 1/3$ and $\gamma$ is sufficiently large.

\textsuperscript{14} For purpose of illustration, two numerical cases are provided. Let us assume $\gamma = 5$ for one case and $\gamma = 10$ for the other and $a = 1$ for both cases. We then figure out that for the case of $\gamma = 5$, the welfare difference is positive if $0.472 \leq ts \leq 0.75$. For the other case of $\gamma = 10$, it is positive if $ts \geq 0.4074$. 


competition, if market 1 is excessively large and \( \frac{1}{3} < \theta s \leq \frac{1}{2} \).

This result is significantly different from the one obtained by Singh and Vives (1984) and Hamilton et al. (1989) in which the social welfare is higher under Bertrand than under Cournot competition.

\section{Concluding Remarks}

This paper has developed a spatial model with homogeneous products and asymmetric demands to compare the delivered prices, aggregate profits and social welfare between Bertrand and Cournot equilibrium. It focuses on the impacts caused from the asymmetric demand curves and the differential transport costs. We obtain several striking results. First of all, we find that market asymmetry is crucial in determining the equilibrium locations under Cournot competition, but insignificant under Bertrand competition. In the case of Cournot competition, we have shown that the two firms separate and locate at the opposite endpoints of a line market if the demand is asymmetry with neither market excessively large, but agglomerate at the large market otherwise. This result is in contrast to the one derived by Hamilton et al. (1989). They assume symmetric demand and find that the two firms agglomerate at the center of a line market under Cournot competition. Secondly, Hamilton et al.
(1989) conclude that the equilibrium price is always lower under Bertrand than under Cournot competition, while we have shown that if one market is excessively larger than the other and the maximum transport cost is high, Bertrand equilibrium price in the large market may be higher than Cournot equilibrium price. Lastly, Hamilton et al. (1989) conclude that Bertrand equilibrium is more efficient than Cournot equilibrium. However, we find in this paper that the reverse holds if the demand structure is excessively asymmetric and the maximum transport cost is high.
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Fig. 2

Fig. 3