Inventories and Optimal Monetary Policy in a Small Open Economy

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Abstract: We study how inventory investment affects the design of optimal monetary policy in a New Keynesian small open economy model. We find that under producer currency pricing, when the intratemporal elasticity of substitution is smaller than 1, optimal monetary policy in our model with inventories is similar to a standard model without inventories. However, when the intratemporal elasticity of substitution is larger than 1, inventory investment increases the importance of nominal exchange rate stabilization relative to a standard model without inventories. We further show that a fixed exchange rate regime can be welfare superior compared to a strict domestic goods price inflation targeting and a strict consumer price inflation targeting in our model with inventories for plausible parameterization.

Keywords: Inventories; small open economy; New Keynesian model; optimal monetary policy

JEL classification: E2, E52, F41

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1 Introduction

While inventory investment is generally a very small fraction of real GDP of an economy, it has long been recognized that fluctuations in inventory investment play an important role in fluctuations of real GDP. For example, Blinder and Maccini (1991) document that even though inventory investment is only roughly one-half of 1 percent of real GDP in the US, in a typical postwar recession in the US, the fall in inventory investment accounts for 87% of the fall of output. Chung (1997) documents similar pattern for Canada, Japan and the UK, where the fall in inventory investment accounts for more than 70% of the fall of output in these countries during postwar recessions. This puzzling empirical fact sparks a lot of research on why inventory investment has such a disproportionate role in business cycle fluctuations. Nevertheless, despite the large literature on the positive aspects of inventory investment, few studies have investigated the normative aspects of inventory investment. An exception is Lubik and Teo (2009), who study how inventory investment affects the design of optimal monetary policy in a closed economy model. Lubik and Teo (2009) find that optimal monetary policy in a New Keynesian model with inventories deviates from a standard New Keynesian model without inventories as a strict inflation targeting is no longer the optimal policy in a model with inventories. The purpose of this paper is to extend Lubik and Teo (2009) and investigate how inventory investment affects the design of optimal monetary policy in an open economy model.

There are several reasons why inventories can affect the design of optimal monetary policy in a small open economy. First, inventories can affect the dynamics of production as well as trade, and the latter in particular could be of great importance for a small open economy. Alessandria et al. (2009), for instance, find that inventories play an important role in trade dynamics. Second, in the presence of inventories for exported goods and imported goods, exchange rate changes will have effects on the valuation of inventory stocks, which will in turn affect the welfare. Central bank will then have to take into consideration of the effects of monetary policy on the valuation of inventory stocks if it aims to maximize the welfare of the households. This will affect the tradeoffs that the central bank faces.

We introduce inventories into the standard small open economy New Keynesian Dynamic Stochastic General Equilibrium of Kollmann (2002). The model features a utility maximizing representative household, monopolistically firms and a central bank. Inventories are introduced into the model by assuming that inventory stock facilitates sales, as suggested in Bils and Khan (2000). This approach is also used by Jung and Yun (2005) and Lubik and Teo (2009, 2010) in closed economy models. It is consistent with a stock-out avoidance motive. Wen (2005) shows that the stockout avoidance theory explains the fluctuations of inventories at different cyclical frequencies better than alternative theories.

Our results can be summarized as follow. Under producer currency pricing and an

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intratemporal elasticity of substitution that is smaller than 1, optimal monetary policy for a small open economy model with inventories focuses mainly on stabilizing the domestic goods price inflation, similar to the case of a standard model without inventories. However, when the intratemporal elasticity of substitution is larger than 1, optimal monetary policy faces a tradeoff between stabilizing the domestic goods price inflation and stabilizing the nominal exchange rate. This is because exchange rate volatility increases the mean export price, which would lead to a lower export revenue when the elasticity of substitution between domestic goods and foreign goods is larger than 1. The lower export revenue is undesirable since it reduces real income and consumption. We find that the presence of inventories amplifies this effect since the higher export price also affects the valuation of the goods in the inventory stock, which is now expected to bring in less export revenue, and hence reduces welfare further. This increases the importance of stabilizing nominal exchange rate. We show that under plausible parameterization, a fixed exchange regime can be welfare superior to a strict domestic goods price inflation targeting and a strict CPI inflation targeting in a model with inventories, for a value of elasticity of substitution between domestic goods and foreign goods larger than 7, which might be empirically relevant. In contrast, in standard models without inventories, the elasticity of substitution between domestic goods and foreign goods has to be unrealistically large for a fixed exchange rate regime to be welfare superior to a strict domestic goods price inflation targeting and a strict CPI inflation targeting.

Our work is related to two literatures. First, it is related to the large and growing literature of optimal monetary policy in New Open Economy Macroeconomics. Corsetti et al. (2009) provide a survey of that literature. The studies that are closest to us are Kollmann (2002) and Sutherland (2006). Kollmann (2002) finds that the optimal Taylor style interest rate rule in a small open economy with a low intratemporal elasticity of substitution focuses on stabilizing inflation and does not react much to the nominal exchange rate. Sutherland (2006) on the other hand, finds that a high intratemporal elasticity of substitution increases the importance of nominal exchange rate stabilization. However, unlike our paper, Sutherland (2006) finds that the critical value for a fixed exchange rate to be welfare superior to a strict domestic goods price inflation targeting and a strict CPI inflation targeting is too high to be empirically realistic in his model, which does not have inventories.

The second literature that is related to our work is the large literature on inventory. Blinder and Maccini (1991), Ramey and West (1999) and Khan (2003) provide excellent surveys of this literature. Most of the work in this literature focuses on explaining the empirical regularities of inventories and their relations to GDP and its components. While much of the work in this literature use partial equilibrium models, inventories have also been studied in general equilibrium models, such as Kydland and Prescott (1982) and Christiano (1988). More recent studies such as Fisher and Hornstein (2000), Khan and Thomas (2007) study inventories in S-s environments. Nonetheless, as mentioned above, the normative aspect of inventories has not been explored in the literature, except in the
closed economy model of Lubik and Teo (2009). Our work thus extends the normative analysis of inventories to the case of a small open economy.

The rest of the paper is organized as follow. Section 2 develops the small open economy New Keynesian model with inventories. Section 3 discusses the solution method and calibration issues. Section 4 presents the results while Section 5 presents the sensitivity analysis. Section 6 concludes.

2 The Model

We introduce inventories into the standard New Keynesian small open economy DSGE model of Kollmann (2002). We follow the approach in Bils and Khan (2000) and assume that inventory stock facilitates sales. There is a representative household, a continuum of monopolistically competitive firms and a central bank in the model.

2.1 Final good production

The final goods, \( Z_t \), is a CES aggregate of domestic intermediate goods and imported intermediate goods:

\[
Z_t = \left\{ \alpha \frac{1}{\vartheta} \left( Q_t^d \right)^{\frac{\vartheta-1}{\vartheta}} + (1 - \alpha) \frac{1}{\vartheta} \left( Q_t^m \right)^{\frac{\vartheta-1}{\vartheta}} \right\}^{\frac{\vartheta}{\vartheta-1}},
\]

where \( \alpha > 0 \) affects the ratio of domestic intermediate goods to final goods in the steady state, \( \vartheta > 0 \) is the intratemporal elasticity of substitution between domestic intermediate goods and imported intermediate goods. \( Q_t^d \) and \( Q_t^m \) are Dixit-Stiglitz aggregators of domestic and imported intermediate goods, given by:

\[
Q_t^d = \left\{ \int_0^1 \left( \frac{N_{t}^{dx}(s)}{N_{t}^{dx}} \right)^{\frac{1}{\vartheta}} Q_t^d(s)^{\frac{\vartheta-1}{\vartheta}} ds \right\}^{\frac{\vartheta}{\vartheta-1}},
\]

\[
Q_t^m = \left\{ \int_0^1 \left( \frac{N_{t}^{mx}(s)}{N_{t}^{mx}} \right)^{\frac{1}{v}} Q_t^m(s)^{\frac{v-1}{v}} ds \right\}^{\frac{v}{v-1}},
\]

where \( Q_t^d(s) \) and \( Q_t^m(s) \) are domestic intermediate goods and imported intermediate goods of variety \( s \), respectively. \( v > 1 \) is the elasticity of substitution among intermediate goods of different varieties. \( N_{t}^{dx} \) is the stock of goods available for sale for domestic intermediate goods, while \( N_{t}^{mx} \) is the counterpart for imported intermediate goods. \( N_{t}^{dx} \) and \( N_{t}^{mx} \) are the aggregate stock of goods available for sale respectively. We will explain why stocks of goods available for sale are introduced into the Dixit-Stiglitz aggregators below.
Solving a cost minimization problem for final good production, we can obtain the following demand functions:

\[ Q^d_t(s) = \left( \frac{N^d_{tx}(s)}{N^d_t} \right)^\gamma \left( \frac{P^d_t(s)}{P^d_t} \right)^{-\nu} Q^d_t, \quad (4) \]

\[ Q^m_t(s) = \left( \frac{N^m_{tx}(s)}{N^m_t} \right)^\gamma \left( \frac{P^m_t(s)}{P^m_t} \right)^{-\nu} Q^m_t, \quad (5) \]

\[ Q^d_t = \alpha \left( \frac{P^d_t}{P_t} \right)^{-\vartheta} Z_t, \quad (6) \]

\[ Q^m_t = (1 - \alpha) \left( \frac{P^m_t}{P_t} \right)^{-\vartheta} Z_t, \quad (7) \]

where \( P^d_t(s) \) and \( P^m_t(s) \) are the prices of intermediate goods of variety \( s \) in the units of the currency of the small open economy. \( P^d_t, P^m_t, \) and \( P_t \) are price indices given by:

\[ P^d_t \equiv \left\{ \int_0^1 \left( \frac{N^d_{tx}(s)}{N^d_t} \right)^\gamma P^d_t(s)^{1-\nu} ds \right\}^{\frac{1}{1-\nu}}, \quad (8) \]

\[ P^m_t \equiv \left\{ \int_0^1 \left( \frac{N^m_{tx}(s)}{N^m_t} \right)^\gamma P^m_t(s)^{1-\nu} ds \right\}^{\frac{1}{1-\nu}}, \quad (9) \]

\[ P_t \equiv \left\{ \alpha \left( \frac{P^d_t}{P_t} \right)^{1-\vartheta} + (1 - \alpha) \left( \frac{P^m_t}{P_t} \right)^{1-\vartheta} \right\}^{\frac{1}{1-\vartheta}}. \quad (10) \]

Introducing stocks of goods available for sale into the Dixit-Stiglitz aggregators in equations (2) and (3) lead to demand functions in which higher stocks of goods available facilitates sales as can be seen from equations (4) and (5). The idea that higher stocks of goods available for sale facilitate sales is first introduced by Bils and Kahn (2000) in a partial equilibrium model. Jung and Yun (2005) and Lubik and Teo (2009, 2010) extend that idea to closed-economy DSGE models. As can be seen from equations (4) and (5), the parameter \( \gamma > 0 \) captures the elasticity of demand with respect to the stock of goods available for sale. Note that like Jung and Yun (2005) and Lubik and Teo (2009, 2010), we assume that the sales of firm \( i \) are increasing in the ratio of its stock of goods available to the aggregate stock of goods available instead of in its stock alone. This assumption greatly simplifies the algebras and can be motivated by assuming that a firm’s stock of goods can facilitate sales only if it is larger relative to its competitors.\(^2\) The stock of

\(^2\)In addition, this assumption also leads to a tractable log-linearized Phillips curve (Lubik and Teo, 2010).
goods available terms in (2) and (3) can alternatively be thought of as taste shifters which affect the representative household’s preference over the intermediate goods (Kryvtsov and Midrigan, 2009). 3

2.2 Intermediate goods firms

The production function for domestic intermediate good firm of variety \( s \) is:

\[
Y_t(s) = \theta_t K_t(s) \psi L_t(s)^{1-\psi},
\]

where \( Y_t(s) \) is the output of firm \( s \). \( \theta_t \) is an economy-wide productivity parameter. \( K_t(s) \) and \( L_t(s) \) are the capital and labor hour used by firm \( s \). The parameter \( \psi \in (0, 1) \) affects the ratios of factor payments to total revenue. Firm \( s \) chooses \( K_t(s) \) and \( L_t(s) \) to minimize the total cost of production given by \( R^k_t K_t(s) + W_t L_t(s) \), where \( R^k_t \) and \( W_t \) are respectively, the nominal rental rate of capital and nominal wage rate, subject to the constraint of the production function, (11). The first order conditions are:

\[
R^k_t = \psi M C_t \frac{Y_t}{K_t},
\]

\[
W_t = (1 - \psi) M C_t \frac{Y_t}{L_t},
\]

where \( M C_t \) is the Lagrange multiplier associated with the constraint, which can also be interpreted as the nominal marginal cost. 4

Good \( s \) is sold domestically and abroad. The demand function for the sales abroad, \( Q_t^x(s) \), is assumed to resemble the counterpart for domestic demand:

\[
Q_t^x(s) = \left( \frac{N_t^{dx}(s)}{N_t} \right)^\gamma \left( \frac{P_t^x(s)}{P_t} \right)^{-\nu} Q_t^x,
\]

\[
Q_t^x = \kappa \left( \frac{P_t^x}{P_t} \right)^{-\theta},
\]

where \( P_t^x \) is the foreign CPI price index. The parameter \( \kappa \) is a scale parameter. Note that we assume that foreign intratemporal elasticity of substitution takes on the same value as the domestic intratemporal elasticity of substitution following Kollmann (2002) and Sutherland (2006). \( Q_t^x \) and \( P_t^x \) are indices given by:

\[
Q_t^x = \left\{ \int_0^1 \left( \frac{N_t^{dx}(s)}{N_t} \right) \frac{\gamma}{\nu} Q_t^x(s)^{\nu-1} ds \right\}^{\nu-1},
\]

\( ^3 \)With this interpretation, the representative household’s preference for a particular intermediate goods is increasing in its stock of goods available, perhaps because a higher stock of goods available reduces the representative household’s shopping time.

\( ^4 \)Given the structure of the model, nominal marginal cost will be equalized across firms, so there is no index \( s \) for the nominal marginal cost.
\[ P_t^x \equiv \left\{ \int_0^1 \left( \frac{N_{tx}^d(s)}{N_{tx}^x(s)} \right) P_t^x(s)^{1-v} ds \right\}^{\frac{1}{1-v}}. \] (17)

To simplify the algebra, we will assume that firms face menu cost instead of following Kollmann (2002) in assuming that firms set prices in Calvo (1983) fashion. The profits of a domestic intermediate good producer, \( \pi_t^{dx}(s) \), and an intermediate good importer, \( \pi_t^{mx}(s) \), are:

\[ \pi_t^{dx}(s) = P_t^d(s) Q_t^d(s) + e_t P_t^x(s) Q_t^x(s) - MC_t Y_t(s) - AC_t^{dx}(s), \] (18)

\[ \pi_t^{mx}(s) = P_t^m(s) Q_t^m(s) - e_t P_t^x Y_t^m(s) - AC_t^{mx}(s), \] (19)

where \( AC_t^{dx}(s) \), \( AC_t^{x}(s) \) and \( AC_t^{mx}(s) \) are menu cost functions given in the appendix. \( e_t \) is the nominal exchange rate, expressed as units of domestic currency per 1 unit of foreign currency. Intermediate good importer imports \( Y_t^m(s) \) of homogenous foreign goods at a price \( P_t^x \) and transform them into differentiated goods to be sold in the domestic market. Firms maximize profits by choosing prices and the stock of goods available for sale subject to the demand functions and the laws of motion for stock of goods available for sale given by:

\[ N_t^{dx}(s) = Y_t(s) + (1 - \delta_N) \left[ N_{t-1}^{dx}(s) - Q_{t-1}(s) - Q_{t-1}^x(s) \right], \] (20)

\[ N_t^{mx}(s) = Y_t^{mx}(s) + (1 - \delta_N) \left[ N_{t-1}^{mx}(s) - Q_{t-1}(s) \right], \] (21)

where \( \delta_N \in [0, 1] \) is the depreciation rate of stock of goods available for sale.

In the benchmark model, we will depart from Kollmann (2002) and assume that exporters set their prices in their own currency while importers set their prices in foreign currency, that is, we assume that price setting is in the form of producer currency pricing (PCP). We will investigate the robustness of the results for the case of local currency pricing (LCP). In the appendix, we show that the optimal price-setting conditions for the case of PCP are:

\[ 0 = (1 - v) \frac{Q_t^d(s)}{P_t} + v(1 - \delta_N) E_t \rho_{t,t+1} \frac{MC_{t+1} Q_t^d(s)}{P_{t+1}} - \phi \left( \frac{P_t^d(s)}{\Pi P_{t-1}^d(s)} - 1 \right) \frac{Q_t}{\Pi P_t^d(s)} \frac{P_t^d}{P_t} \]

\[ + E_t \rho_{t,t+1} \phi \left( \frac{P_t^d(s)}{\Pi P_t^d(s)} - 1 \right) \frac{P_{t+1}^d(s)}{\Pi (P_t^d(s))^2} \frac{P_{t+1}^d Q_{t+1}^d}{P_{t+1}}. \] (22)
\[
0 = (1 - v) \frac{Q_t^x(s)}{P_t} + v(1 - \delta_N)E_t \rho_{t,t+1} \frac{MC_{t+1} Q_t^x(s)}{P_{t+1}} - \phi \left( \frac{e_t P_t^x(s)}{\Pi_{t-1}^* P_{t-1}^x(s)} - 1 \right) \frac{Q_t^x}{\Pi_{t-1}^* P_{t-1}^x(s) - e_t P_t^x(s)} + E_t \rho_{t,t+1} \phi \left( \frac{e_{t+1} P_{t+1}^x(s)}{\Pi_{t+1}^* P_{t+1}^x(s)} - 1 \right) \frac{e_{t+1} P_{t+1}^x(s)}{\Pi_{t+1}^* (P_{t+1}^x(s))^2} Q_{t+1} \frac{e_{t+1} P_{t+1}^x(s)}{e_{t+1}}, \tag{23}
\]

\[
0 = (1 - v) Q_t^m(s) + v(1 - \delta_N)E_t \frac{1}{R_t^*} P_{t+1}^* \frac{Q_t^m(s)}{P_t^m(s) / e_t} - \phi \left( \frac{P_t^m(s) / e_t}{\Pi^* P_{t-1}^m(s) / e_t - 1} \right) \frac{Q_t^m}{\Pi^* P_{t-1}^m(s) / e_t} + E_t \frac{1}{R_t^*} \phi \left( \frac{P_{t+1}^m(s) / e_{t+1}}{\Pi^* P_{t+1}^m(s) / e_{t+1}} - 1 \right) \frac{P_{t+1}^m(s) / e_{t+1}}{\Pi^* (P_{t+1}^m(s) / e_{t+1})^2} Q_{t+1} \frac{e_{t+1} P_{t+1}^m(s)}{e_{t+1}}, \tag{24}
\]

where \( \phi > 0 \) is a menu cost parameter, \( \rho_{t,t+1} \) is a pricing kernel for evaluating profit\(^5\), \( R_t^* \) is the gross world interest rate. \( \Pi_t = \frac{P_{t-1}^*}{P_{t-1}} \) and \( \Pi_{t}^* = \frac{P_{t}^*}{P_{t-1}^*} \) are the gross CPI inflation rates of the domestic economy and the world economy, respectively. Variables without time subscripts denote the deterministic steady state of the corresponding variables.

For the case of LCP, equations (23) and (24) are replaced by:

\[
0 = (1 - v) e_t Q_t^x(s) / P_t + v(1 - \delta_N)E_t \rho_{t,t+1} \frac{MC_{t+1} Q_t^x(s)}{P_{t+1}^* P_t^x(s)} - \phi \left( \frac{P_t^x(s)}{\Pi^* P_{t-1}^x(s)} - 1 \right) \frac{Q_t^x}{\Pi^* P_{t-1}^x(s) P_t^x(s)} + E_t \rho_{t,t+1} \phi \left( \frac{P_{t+1}^x(s)}{\Pi^* P_{t+1}^x(s)} - 1 \right) \frac{P_{t+1}^x(s)}{\Pi^* (P_{t+1}^x(s))^2} Q_{t+1} \frac{e_{t+1} P_{t+1}^x(s)}{e_{t+1}}, \tag{25}
\]

\[
0 = (1 - v) Q_t^m(s) / e_t + v(1 - \delta_N)E_t \frac{1}{R_t^*} P_{t+1}^* \frac{Q_t^m(s)}{P_t^m(s)} - \phi \left( \frac{P_t^m(s)}{\Pi^* P_{t-1}^m(s) e_t - 1} \right) \frac{Q_t^m}{\Pi^* P_{t-1}^m(s) e_t} + E_t \frac{1}{R_t^*} \phi \left( \frac{P_{t+1}^m(s)}{\Pi^* P_{t+1}^m(s) e_{t+1}} - 1 \right) \frac{P_{t+1}^m(s)}{\Pi^* (P_{t+1}^m(s))^2} Q_{t+1} \frac{e_{t+1} P_{t+1}^m(s)}{e_{t+1}}, \tag{26}
\]

\(^5\)Kim (2000) shows that pricing kernel equals the discounted ratio of marginal utility of consumption across periods, i.e., \( \rho_{t,t+1} = \beta \frac{c_t}{c_{t+1}} \).
The first order conditions for the stock of goods available for sale are:

\[
\gamma \frac{P^d_t(s) Q^d_t(s) + e_t P^x_t(s) Q^x_t(s)}{N^dx_t(s) P_t} - \frac{MC_t}{P_t} + (1-\delta_N)E_t\rho_{t,t+1} \frac{MC_{t+1}}{P_{t+1}} \left( 1 - \gamma \frac{Q^d_t(s) + Q^x_t(s)}{N^dx_t(s)} \right) = 0,
\]

(27)

\[
\gamma \frac{P^m_t(s) Q^m_t(s)}{N^m_t(s) e_t} - P^*_t + (1-\delta_N)E_t \frac{1}{R^*_t} P^*_t \left( 1 - \gamma \frac{Q^m_t(s)}{N^m_t(s)} \right) = 0.
\]

(28)

Note that in the optimal pricing equations in (22) to (26), marginal costs enter the equations in expectations. Present and future marginal costs are in turn related through the ratio of sales \((Q^d_t(s), Q^x_t(s))\) to stocks of goods available for sale in the optimality conditions of stock of goods available for sale. In contrast, in a model without inventories, contemporaneous marginal costs will enter into the pricing equations directly.

### 2.3 The representative household

The representative household’s intertemporal utility is defined by consumption \(C_t\) and labor hour \(L_t\):

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),
\]

(29)

where \(\beta \in [0,1]\) is the subjective discount factor. We assume that the period utility function is separable in its arguments following Kollmann (2002) but we allow for more general functional forms:

\[
U(C_t, L_t) = \frac{C_t^{1-\xi} - 1}{1-\xi} - \frac{L_t^{1+\omega}}{1+\omega},
\]

(30)

where \(\xi\) is the coefficient of risk aversion and \(\omega\) is the inverse of Frisch labor supply elasticity.

The representative household owns the capital stock \(K_t\), which evolves according to the law of motion:

\[
K_{t+1} = K_t (1-\delta) + I_t - \frac{1}{2} \Phi \frac{(K_{t+1} - K_t)^2}{K_t},
\]

(31)

where \(\frac{1}{2} \Phi \frac{(K_{t+1} - K_t)^2}{K_t}\) with \(\Phi > 0\) is a quadratic capital adjustment cost. The representative household also owns all the domestic firms in the economy. At period \(t\), the representative household can purchase a one-period domestic bond \(A_{t+1}\) and a one-period foreign currency bond, \(B_{t+1}\), which mature in period \(t + 1\). The gross nominal interest rates that \(A_{t+1}\) and \(B_{t+1}\) pay are denoted by \(R_t\) and \(R^f_t\) respectively. The budget constraint of the representative household is:

\[
A_{t+1} + e_t B_{t+1} + P_t (C_t + I_t) = A_t R_{t-1} + e_t B_t R^f_{t-1} + R^f_t K_t + W_t L_t + \int_0^1 \pi^{dx}_t(s) ds,
\]

(32)
where \( \int_{0}^{1} \pi^M_t(s) ds \) reflects the dividends that the representative household receives from owning domestic firms.

The first order conditions for the utility maximization problem of the representative household are:

\[
1 = \beta R_t E_t \left\{ \frac{P_tC_t^f}{P_{t+1}C_{t+1}^f} \right\},
\]

(33)

\[
1 = \beta R^f_t E_t \left\{ \frac{P_tC_t^f e_{t+1}}{P_{t+1}C_{t+1}^f e_t} \right\},
\]

(34)

\[
1 = \beta E_t \left\{ \frac{C_t^f}{C_{t+1}^f} \frac{R^{t+1}}{R_{t+1}} + 1 - \delta + \Phi \frac{(K_{t+2} - K_{t+1})}{K_{t+1}} + \frac{1}{2} \Phi \frac{(K_{t+2} - K_{t+1})^2}{K_{t+1}^2} \right\},
\]

(35)

\[
\frac{W_t}{P_t} = C_t^f L_t^w.
\]

(36)

Equations (33) and (34) are Euler equations for bonds. Equation (35) is Euler equation for capital. Equation (36) equates the marginal benefit and marginal cost of labor hour. Following Kollmann (2002), we will introduce an uncovered interest parity shock, \( \phi_t \), into the Euler equation for foreign bond, so that Equation (34) becomes:

\[
1 = \phi_t \beta R^f_t E_t \left\{ \frac{P_tC_t^f e_{t+1}}{P_{t+1}C_{t+1}^f e_t} \right\}.
\]

(37)

The uncovered interest parity shock is introduced so that equations (33) and (37) imply up to a log-linear approximation:

\[
E_t \Delta e_{t+1} \simeq \hat{R}_t - \hat{R}^f_t - \hat{\phi}_t,
\]

(38)

where \( \Delta e_{t+1} \equiv e_{t+1}/e_t \) and a caret over a variable denotes the log deviation of that variable from its steady state. The uncovered interest parity shocks capture departures from the UIP condition, which are well-documented in the empirical literature.

2.4 Market clearing conditions

Firms are symmetric in equilibrium, so we can drop the \( s \) indices in the equations above. Market clearing for the final goods requires:

\[
Z_t = C_t + I_t + \frac{AC_d^f}{P_t} + \frac{AC_r^f}{P_t}.
\]

(39)

Note that for a given quantity of final goods, \( Z_t \), the menu cost reduces the goods available for consumption and investment purpose. Hence, the menu cost is a source of inefficiency.
in this economy. For the purpose of explaining the results below, we can define a variable $RC_t$, as the ratio of menu cost to final goods, to capture the resource cost stemming from price adjustments:

$$RC_t \equiv \frac{AC^{cd}_t}{P_t} + \frac{AC^{cz}_t}{P_t}.$$  

Market clearing for domestic bond requires:

$$A_t = 0.$$  

Finally, we follow Kollmann (2002) and assume that the interest rate that the representative household receives from holding the international bond, $R^d_t$, deviates from the world interest rate, $R^*_t$, by a factor of $\frac{\lambda B_{t+1}}{P^*_t Q^*_t}$, where $\lambda > 0$ is a parameter which captures the degree of integration of international financial market:

$$\frac{R^d_t}{\Pi^*} = \frac{R^*_t}{\Pi^*} - \frac{\lambda B_{t+1}}{P^*_t Q^*_t}.$$  

The term $\frac{\lambda B_{t+1}}{P^*_t Q^*_t}$ also serves the purpose of closing the small open economy model (Schmitt-Grohé and Uribe, 2003).

### 2.5 Monetary policy

For monetary policy, we will first consider the Ramsey optimal policy.\(^6\) The Ramsey optimal policy can be computed by setting up a Lagrangian problem, in which the central bank maximizes the conditional lifetime utility of the representative household subject to the first-order conditions of the private agents and the market clearing conditions of the economy. The optimality conditions for the Ramsey problem are obtained by differentiating the Lagrangian with respect to each of the endogenous variables and setting the derivatives to zero. We compute this numerically by using the Matlab procedures developed by Levin and Lopez-Salido (2004).

In addition to the Ramsey optimal policy, following Devereux et al. (2006) and Sutherland (2006),\(^7\) we will consider the following simple rules:

- $\Pi^d_t = \Pi^d_t$, \hspace{1cm} (43)
- $\Pi_t = \Pi_t$, \hspace{1cm} (44)
- $\Delta e_t = 1$, \hspace{1cm} (45)

where $\Pi^d_t \equiv \frac{P^d_t}{P^*_t}$ is the gross domestic goods price inflation rate. We will call the rules in equations (43)-(45) as strict domestic goods price inflation targeting (DPIT), strict CPI inflation targeting (CPIT) and fixed exchange rate (FE), respectively.

\(^{6}\)For more detail discussions of Ramsey optimal policy in New Keynesian model, see Khan et al. (2003), Levin et al. (2006) and Schmitt-Grohé and Uribe (2007).

\(^{7}\)However, Devereux et al. (2006) and Sutherland (2006) do not consider Ramsey optimal policy.
2.6 Exogenous variables

The exogenous processes in the model are all assumed to follow first-order autoregressive processes:

\[
\theta_t = (1 - \rho^\theta) + \rho^\theta \theta_{t-1} + \varepsilon^\theta_t, \tag{46}
\]

\[
\Pi_t^* = (1 - \rho^*) \Pi^* + \rho^\Pi^*_{t-1} + \varepsilon^\pi_t, \tag{47}
\]

\[
R_t^* = (1 - \rho^R) R^* + \rho^R R^*_{t-1} + \varepsilon^R_t, \tag{48}
\]

\[
\varphi_t = (1 - \rho^\varphi) + \rho^\varphi \varphi_{t-1} + \varepsilon^\varphi_t, \tag{49}
\]

where \(\rho^\theta, \rho^*, \rho^R, \rho^\varphi \in [0, 1]\) and \(\varepsilon^\theta_t, \varepsilon^\pi_t, \varepsilon^R_t, \varepsilon^\varphi_t\) are mean zero i.i.d. innovations with standard deviations of \(\sigma^\theta, \sigma^*, \sigma^R, \sigma^\varphi\).

2.7 Welfare criterion

We use the conditional expected lifetime utility of the representative household at time zero as the welfare measure, \(CV_0\):

\[
CV_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\xi} - 1}{1 - \xi} - \frac{L_t^{1+\omega}}{1 + \omega} \right). \tag{50}
\]

We follow Schmitt-Grohé and Uribe (2007) and compute the expected lifetime utility conditional on the initial state being the deterministic steady state. This has the benefit that the economy always starts from the same initial point, since for a given set of parameter values, the steady states of this model are the same for all monetary policies considered in this paper.

We follow Lucas (1987) and report the welfare as a fraction, \(\zeta\), of steady state consumption that the household is willing to give up to be as well off under the steady state, as under a given monetary policy regime \(a\). Formally, \(\zeta\) is given implicitly by:

\[
\sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[ (1 - \frac{\zeta}{100}) C \right]^{1-\xi} - 1}{1 - \xi} - \frac{L^{1+\omega}}{1 + \omega} \right\} = CV_0^a. \tag{51}
\]

Higher values of \(\zeta\) correspond to lower welfare.

3 Solution method and calibration

Since the model does not have an analytical solution, we solve the model numerically by taking second-order Taylor approximation methods (Schmitt-Grohé and Uribe, 2004). The second-order approximations are computed using the software package, Dynare (Juillard, 1996).
To use the second-order approximation methods, we need to calibrate the model parameters. Since we would like to compare the results to a standard New Keynesian small open economy model, all parameters, except for those related to inventories ($\gamma$ and $\delta_N$), are calibrated following Kollmann (2002), who calibrate his model to quarterly data for Japan, Germany and the UK. The time period is a quarter. The subjective discount factor, $\beta$, is set to $1/1.01$. The steady state gross inflation rates of the domestic economy and the world economy, $\Pi$ and $\Pi^*$ are set to 1. $\alpha$ is set so that the steady state import to GDP ratio is 30%. The elasticity of substitution among goods, $v$ is set to 6, so that the steady state markup is 20%. The technology parameter, $\psi$, is set to 0.24 while the depreciation rate of capital stock, $\delta$, is set to 0.025. The elasticity of substitution between domestic goods and foreign goods, $\vartheta$ is set to 0.6 in the benchmark model. The inverse of Frisch labor elasticity, $\omega$ is set to zero while the coefficient of risk aversion $\xi$ is set to 1. The capital adjustment cost parameter, $\Phi$, is set to 15. $\lambda$, which captures the degree of integration with international financial market is set to 0.0019. The menu cost parameter, $\phi$, is calibrated so that log-linearized Phillips curve of the model in this paper is identical to the log-linearized Phillips curve in Kollmann (2002) with Calvo-style price setting, which has an average price change duration of 4 quarters. The parameter $\kappa$ is calibrated so that the ratio of import price to domestic goods price is 1 in the steady state. For the exogenous shock processes, we set $\rho^\delta = 0.9, \rho^\vartheta = 0.8, \rho^R = 0.75, \rho^\varphi = 0.5, \sigma^\theta = 0.01, \sigma^* = 0.005, \sigma^R = 0.004, \sigma^\varphi = 0.033$.

For parameters related to inventories, we calibrate them as follow. We calibrate the depreciation rate of inventory stock, $\delta_N$, to 0.05. A depreciation rate of 0.05 is close to the value estimated by Iacoviello et al. (2010) in an estimated DSGE model for the United States, which could be a good approximation for Japan, Germany and the UK. For the elasticity of sales with respect to the stock of goods available, $\gamma$, we calibrate it to 0.491 so that in the steady state, the ratio of inventory stock to output is approximately $2/3$.

4 Results

4.1 Benchmark parameterization

Table 1 shows the results for the benchmark parameterization. For comparison, we also show the results for a standard New Keynesian small open economy DSGE model without inventories. As can be seen from the table, the Ramsey optimal policy for our model with inventories shares many similar characteristics with its counterpart for the standard model without inventories. Like the case of the standard model without inventories, the Ramsey optimal policy for the model with inventories delivers a relatively low standard deviation.

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8Ramey and West (1999, p869) report that the ratio of inventory stock to GDP in annual data is approximately 1/6 for G7 countries, which translate to a ratio of inventory stock to GDP of 2/3 in quarterly data. A value of $\gamma$ of 0.491 is also within the range of 0.37 to 0.89 considered by Jung and Yun (2005).
of domestic goods price inflation while it allows the standard deviations of nominal depreciation and CPI inflation to be high. This is consistent with the findings of Sutherland (2006) that when the elasticity of substitution between domestic goods and foreign goods is smaller than 1, optimal monetary policy puts more weights on stabilizing the domestic goods price inflation.

Although the Ramsey optimal policy puts more weights on stabilizing the domestic goods price inflation, the standard deviation of domestic goods price inflation at about 1.6% in annual rate, while it is relatively small, is still substantial. This is also consistent with the findings of Sutherland (2006) who finds that for small open economy models, a complete stabilization of domestic goods price inflation is not optimal, except for the special case that the elasticity of substitution among domestic goods and foreign goods is exactly 1. The intuition for this result is that when the elasticity of substitution among domestic goods and foreign goods is different from 1, some variations in the domestic goods price inflation is desirable so as to generate variations in the nominal exchange rate, in order to affect the terms of trade (Sutherland, 2006). In fact, as can be seen from the results for DPIT, a complete stabilization of domestic goods price inflation leads to a welfare cost of 0.17 to 0.2% of steady state consumption, compared to the Ramsey optimal policy, which is substantial in the realm of business cycle analysis. It is also interesting to note that the rankings among DPIT, CPIT and FE in both the models with and without inventories, are consistent with the findings of Sutherland (2006), with DPIT being the best simple rule among the three that we consider while FE being the worst. It is also worthwhile to note that CPIT and FE lead to welfare differences of 0.5 to 1.1% of steady state consumption compared to DPIT, which is large in the realm of business cycle analysis. CPIT and FE lead to substantial welfare cost in the benchmark parameterization for two reasons. First, they lead to a more variable domestic goods price inflation, which leads to inefficiency through a higher menu cost (as shown in the $RC_t$ row in Table 1). Second, they induces a more stable nominal exchange rate, which is undesirable when the elasticity of substitution among domestic goods and foreign goods is less than 1 as shown by Sutherland (2006). Specifically, a more stable nominal exchange rate leads to a lower mean export price ($p_x^t$), which translates into a lower mean export revenue ($p_x^t Q_x^t$) when the elasticity of substitution between domestic goods and foreign goods is less than 1. Hence, FE which leads to the most stable nominal exchange rate also leads to the lowest mean export revenue among the three simple rules. This subsection thus shows that introducing inventories does not alter the ranking of simple monetary policy rules under the benchmark parameterization.

4.2 Higher intratemporal elasticity of substitution

The results in the subsection above are obtained under the assumption that the intratemporal elasticity of substitution, $\vartheta$, is 0.6. However, there are some uncertainties over the value of $\vartheta$. For instance, Lai and Treffer (2002) report that the estimates for the elasticity of substitution for aggregate manufacturing are between 5 and 8 from a panel of devel-
oped and developing countries. Anderson and van Wincoop (2003) conclude from a survey of empirical evidences on trade elasticities that a value between 5 and 10 is reasonable. Hummels (2001) finds that the elasticities of substitution range from 3 to 8 for most goods but can be as high as 79 for some goods using disaggregated data from US and 5 other countries. On the other hand, studies that make use of high frequency data to estimate the elasticity of substitution between domestic and foreign goods typically find much smaller estimates, ranging from 0.2 to 3.5 (Ruhl, 2008). While the empirically relevant value of the intratemporal elasticity of substitution remains an unresolved issue, some theoretical studies have shown that it can affect the welfare ranking of different monetary policies. For instance, Sutherland (2006) shows that the intratemporal elasticity of substitution is a crucial parameter that influences whether monetary policy should stabilize the nominal exchange rate. Considering the findings above, we will first consider the robustness of the results in detail for a high intratemporal elasticity of substitution, $\vartheta$, of 10, followed by more sensitivity analysis for other values of $\vartheta$.

Table 2 shows the results for the case of $\vartheta = 10$. As can be seen from the table, in the model with inventories, Ramsey optimal policy now stabilizes nominal exchange rate depreciation more than it stabilizes domestic goods price inflation and CPI inflation. For the model without inventories, while Ramsey policy still stabilizes domestic goods price inflation more than it stabilizes nominal depreciation, the standard deviation of nominal depreciation is several times smaller compared to the benchmark case of $\vartheta = 0.6$. This is consistent with the findings in Sutherland (2006) that optimal monetary policy would put more weight on stabilizing the nominal exchange rate as the elasticity of substitution between domestic goods and foreign goods increases. The intuition for this result is that nominal exchange rate volatility increases the risk premium for export price and hence increases the mean export price. When the elasticity of substitution between domestic goods and foreign goods is larger than 1, the higher mean export price leads to a lower export revenue, $\pi^x_t Q^x_t$, which lowers the real income and hence consumption of the small open economy model. Hence, optimal monetary policy faces a tradeoff between stabilizing domestic goods price inflation, which minimizes the menu cost of price adjustment and stabilizing the nominal exchange rate when the elasticity of substitution between domestic goods and foreign goods is larger than 1. This makes CPIT a better policy than DPIT and FE for $\vartheta = 10$, which stands in contrast to the benchmark case of $\vartheta = 0.6$, for both the models with and without inventories. CPIT becomes a better policy than DPIT and FE because the CPI price level is a weighted average of domestic goods price and imported

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9 Teo (2009a,b) extends the findings in Sutherland (2006) and shows that the elasticity of substitution for export is more important compared to the elasticity of substitution between domestic goods and imported goods in affecting the welfare ranking of different monetary policies. In contrast, in Sutherland (2006), the elasticity of substitution for export and the elasticity of substitution between domestic goods and imported goods are assumed to take the same value. We will however, focus on the case for which elasticity of substitution for export takes on the same value as the elasticity of substitution between domestic goods and imported goods in this paper.
goods price as can be seen from equation (10). With producer currency pricing, imported goods price is mainly affected by the nominal exchange rate. Hence stabilizing the CPI inflation is equivalent to stabilizing a weighted average of domestic goods price inflation and nominal exchange rate for the case of $\vartheta = 10$.

It is interesting to note that for $\vartheta = 10$, FE is better than DPIT for the model with inventories while the opposite is true for the model without inventories. This suggests that inventories increase the importance of stabilizing nominal exchange rate as the elasticity of substitution between domestic goods and foreign goods increases, compared to the model without inventories. To investigate this issue further, Figures 1 and 2 show the welfare costs for the three simple rules as $\vartheta$ varies, for the model with inventories and the model without inventories, respectively. As can be seen from Figure 1, for the model with inventories, FE starts to dominate DPIT for $\vartheta > 6$ and for $\vartheta > 29$, FE even becomes the best rule among the three simple rules that we consider. In contrast, for the standard model without inventories, Figure 2 shows that FE only dominates DPIT for $\vartheta > 20$ and it only beats CPIT for $\vartheta > 88$, which is too large to be empirically realistic. While a value of $\vartheta$ of 29 for FE to be the best rule for the model with inventories is still likely to be too large to be empirically realistic, Figures 1 and 2 do highlight the message that the presence of inventories makes stabilizing nominal exchange rate more important as $\vartheta$ increases. The intuition for this result is as follow: nominal exchange rate volatility increases the risk premium for export price, which reduces the export revenue for $\vartheta > 1$. This is undesirable since it reduces real income and hence consumption. In the presence of inventories, a higher mean export price has a larger effect on the welfare since it also affects the valuation of the goods in the inventory stock. This is because the higher mean export price implies that the goods in the inventory stock is expected to bring in less export revenue and hence less income, which reduces welfare. Hence, for $\vartheta > 1$, the presence of inventories amplifies the negative effect of nominal exchange rate volatility on the welfare, making stabilizing the nominal exchange rate a more important consideration.

5 Sensitivity analysis

The subsection above has shown that inventories increase the importance of stabilizing nominal exchange rate. The crucial mechanisms behind this result are the effect of inventories on the risk premium of the export price and the associated expenditure switching effects. In this subsection, we explore the robustness of our results to several modifications to the benchmark model which might affect the two mechanisms highlighted above. 10

10In addition to the sensitivity analysis reported below, we have also considered increasing $\omega$ so that the Frisch labor supply elasticity is more consistent with microeconomic evidence but we find that it does not have quantitatively important effects on our results.
5.1 Local currency pricing

In the results reported above, expenditure switching effect features prominently in the welfare ranking among the simple policy rules that we consider. This effect would become weaker if price-setting is in the form of local currency pricing (LCP). For LCP, export price is set in the currency of the export market while import price is set in the currency of the domestic currency. This means that export price and import price will be less affected by nominal exchange rate volatility. Hence, optimal monetary policy for the case of LCP will likely focus more on stabilizing the domestic goods price inflation. Figure 3 shows the welfare ranking among the 3 simple rules for the model with inventories under LCP. As can be seen from Figure 3, FE is the worst policy among the 3 simple rules for all the values that we consider. CPIT only beats DPIT for $\vartheta > 23$, which is a rather high critical value. The results thus confirm the conjecture that the presence of inventories does not increase the importance of stabilizing the nominal exchange rate when price setting is LCP.

5.2 Degree of price rigidity

In the benchmark model, we set the menu cost parameter to be consistent with an average price change duration of 4 quarters. An average price change duration of 4 quarters is a commonly used degree of price rigidity in the DSGE literature. It is also consistent with the evidence summarized by Taylor (1999) for the case of US. However, recent study by Bils and Klenow (2004) and Klenow and Kryvtsov (2008) find a much lower degree of price rigidity for the US, with an average price change duration of around 2 quarters. While countries in the Euro Area tend to have higher degree of price rigidity compared to the US (Dhyne et al. 2006), countries such as Canada and the UK tend to have lower degrees of price rigidity like the US (Amirault et al., 2006, Hall et al., 2000).

Recall from the Section 4 that for intratemporal elasticity of substitution larger than 1, optimal monetary policy faces a tradeoff between stabilizing domestic price inflation so as to minimize the menu cost of price adjustment and stabilizing the nominal exchange rate so as to generate higher export revenue. This tradeoff clearly can be affected by the degree of price rigidity. For a lower degree of price rigidity, the resource loss from menu cost will become smaller, which is likely to increase the importance of stabilizing the nominal exchange rate. Figure 4 considers the case for which the menu cost parameter is set to be consistent with an average price change duration of 2 quarters for the model with inventories under PCP. As can be seen from Figure 4, the critical value of $\vartheta$ for FE to beat DPIT is now reduced to 5 while the critical value of $\vartheta$ for FE to be the best policy among the 3 simple rules is now reduced dramatically to 9. This is an important finding since a critical value of $\vartheta$ of 9 is within the range of values that might be reasonable (Anderson and van Wincoop, 2003).

---

5.3 Risk aversion

Since a crucial mechanism that drives the results above is the effect of inventories on the risk premium of the export price, the degree of risk aversion of the representative household, and hence the firms that are owned by the representative household, is likely to matter for the welfare ranking of different monetary policies. In the benchmark model, we set the coefficient of risk aversion, $\xi$, to 1. In Figure 5, we consider a higher risk aversion $\xi$ of 5, which is within the range of empirically plausible values (Lucas, 1987), for the model with inventories under PCP. For this experiment, we keep the average price change duration at 2 quarters as in Figure 3. As can be seen from the figure, the higher risk aversion reduces the critical value of $\theta$ for FE to be the best policy among the three simple rules to only 7.\(^{12}\) Hence, a higher risk aversion strengthens the importance of stabilizing the nominal exchange rate in our model with inventories.

5.4 Elasticity of demand with respect to goods available for sale

In the benchmark model, we set the elasticity of demand with respect to goods available for sale, $\gamma$, to 0.491 so that the ratio of inventory stock to output is 2/3 in quarterly data. In this subsection, we consider the sensitivity of our results to this parameter. We consider reducing $\gamma$ to 0.403, so that the ratio of inventory stock to output is 1/3 in quarterly data. We choose a ratio of inventory stock to output of 1/3 in quarterly data because Ramey and West (1999) report that about 50% of total inventory stock in the US is held by retailers and wholesalers, while the rest is in the form of manufacturing finished goods, work in process, raw materials and others. Hence, calibrating the ratio of inventory stock to output to 1/3 has the effect of treating the inventories in our model as corresponding to retail and wholesale inventories in the data. For this experiment, we keep $\xi$ at 5 and average price change duration at 2 quarters like Figure 5. As can be seen from Figure 6, the qualitative results are similar to the case of Figure 5. Quantitatively, the critical value of $\theta$ for FE to be the best policy among the 3 simple rules decreases slightly to 8. Figure 6 thus suggests that our results are not sensitive to the value of $\gamma$, for empirically reasonable value of inventory stock to output ratio.

6 Conclusion

We study optimal monetary policy in a small open economy New Keynesian DSGE model with inventories. We find that under PCP, when the elasticity of substitution among domestic goods and foreign is larger than 1, the presence of inventories increases the importance

\(^{12}\)In contrast, in a model without inventories, even with $\xi = 5$ and an average price change duration of 2 quarters, the critical value for FE to be the best policy among the 3 simple rules is 13, which is larger than the range of values that Anderson and van Wincoop (2003) consider as reasonable.
of nominal exchange rate stabilization relative to a standard model without inventories. We further show that under plausible parameterization, a fixed exchange rate regime can be welfare superior to a strict domestic goods price inflation targeting and a strict consumer price inflation targeting for empirically plausible value of elasticity of substitution among domestic goods and foreign goods in our model with inventories. The results suggest that nominal exchange rate stabilization could be an important consideration for small open economies.

We conclude the paper by discussing some future research directions. First, it would be interesting to distinguish between input and output inventories in the model since these two types of inventories might have very different behaviors. Humphreys et al. (2001), Iacoviello et al. (2009) are some work along this line. Second, it would also be interesting to explore how inventories affect the rankings of monetary policies in a setting where trade is lumpy and inventory adjustment is of S,s type. Alessandria et al. (2009) show that such a setting is able to capture the behavior of import and import prices after large devaluations.

References


Table 1: Results for benchmark parameterization ($\theta = 0.6$)

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<th>DPIT</th>
<th>CPIT</th>
<th>FE</th>
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<tr>
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Note: The means and standard deviations are in percentage deviations from the steady state values.
Table 2: Results for high elasticity of substitution for export ($\varphi = 10$)

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<td>0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.30</td>
</tr>
<tr>
<td>$p_t^x$</td>
<td>-0.23</td>
<td>0.08</td>
<td>-0.25</td>
<td>-0.30</td>
</tr>
<tr>
<td>$p_t^x Q_t^x$</td>
<td>2.11</td>
<td>-0.73</td>
<td>2.21</td>
<td>2.70</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi_t^d$</td>
<td>1.44</td>
<td>0.00</td>
<td>1.52</td>
<td>4.05</td>
</tr>
<tr>
<td>$\Pi_t$</td>
<td>1.20</td>
<td>3.66</td>
<td>0.00</td>
<td>3.51</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>5.66</td>
<td>11.61</td>
<td>5.79</td>
<td>4.79</td>
</tr>
<tr>
<td>$\Delta e_t$</td>
<td>1.58</td>
<td>3.15</td>
<td>1.17</td>
<td>0.00</td>
</tr>
<tr>
<td>$RER_t$</td>
<td>1.29</td>
<td>2.07</td>
<td>1.24</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note: The means and standard deviations are in percentage deviations from the steady state values.
Figure 1: Welfare cost for the model with inventories

Figure 2: Welfare cost for the model without inventories
Figure 3: Welfare cost for the model with inventories under LCP

Figure 4: Welfare cost for the model with inventories under lower degree of price rigidity
Figure 5: Welfare cost for the model with inventories under higher risk aversion

Figure 6: Welfare cost for the model with inventories under lower elasticity of demand with respect to stock of goods available
7 Appendix

7.1 Price-setting for the case of LCP

Since $P^x_t(s)$ is denoted in foreign currency while $P^m_t(s)$ is denoted in domestic currency, it will be more convenient to first explain the price-setting for the case of LCP. The menu cost functions for price adjustment for the case of LCP are assumed to be:

$$AC^d_t(s) = \frac{\phi}{2} \left( \frac{P^d_t(s)}{P^d_{t-1}(s)} - 1 \right)^2 P^d_t Q^d_t,$$ (52)

$$AC^x_t(s) = \frac{\phi}{2} \left( \frac{P^x_t(s)}{P^x_{t-1}(s)} - 1 \right)^2 \epsilon_t P^x_t Q^x_t,$$ (53)

$$AC^m_t(s) = \frac{\phi}{2} \left( \frac{P^m_t(s)}{P^m_{t-1}(s)} - 1 \right)^2 P^m_t Q^m_t,$$ (54)

where $\phi$ is a menu cost parameter. The menu costs penalize deviation of goods price inflation from the steady state inflation rate. Domestic intermediate good firm $s$ maximizes discounted intertemporal profits given by:

$$E_t \sum_{\tau=0}^{\infty} \rho_{t,t+\tau} \frac{\pi^d_{t+\tau}}{P_t^{d_{t+\tau}}},$$ (55)

through choosing $P^d_t(s)$, $P^x_t(s)$ and $N^d_{t+\tau}(s)$ subject to the constraints of equations (4), (14), (18), (20), (52) and (53). The first-order conditions are given by (22), (25) and (27).

Imported goods firm $s$ wants to maximize discounted intertemporal profits given by:

$$E_t \sum_{\tau=0}^{\infty} R_{t,t+\tau} \frac{\pi^m_{t+\tau}}{\epsilon_t^{t+\tau}},$$ (56)

where $R_{t,t+\tau} = \prod_{k=1}^{k=\tau-1} (R^s_{t+k})^{-1}$ for $\tau > 1$ is the discount factor that imported goods firms use to evaluate profits.\(^{13}\) Imported goods firm $s$ maximizes equation (56) by choosing $P^m_t(s)$ and $N^m_{t+\tau}(s)$ subject to the constraints of equations (5), (19), (21) and (54). The first-order conditions are given by equations (26) and (28).

\(^{13}\)We follow Kollmann (2002) and assume that imported goods firms are owned by foreigners.
7.2 Price-setting for the case of PCP

For the case of PCP, we need to define two auxiliary variables: Let $P_{t}^{x,PCP}(s)$ denotes export price in domestic currency term and $P_{t}^{m,PCP}(s)$ denotes import price in foreign currency term. Their relations with $P_{t}^{x}(s)$ and $P_{t}^{m}(s)$ are given by:

$$P_{t}^{x}(s) = P_{t}^{x,PCP}(s) / e_{t},$$

(57)

$$P_{t}^{m}(s) = e_{t} P_{t}^{m,PCP}(s).$$

(58)

For the case of PCP, firms choose $P_{t}^{x,PCP}(s)$ and $P_{t}^{m,PCP}(s)$ to maximize profits. Menu cost functions for adjusting $P_{t}^{x,PCP}(s)$ and $P_{t}^{m,PCP}(s)$ are given by:

$$AC_{t}^{x}(s) = \frac{\phi}{2} \left( \frac{P_{t}^{x,PCP}(s)}{\Pi P_{t-1}^{x,PCP}(s)} - 1 \right)^{2} e_{t} P_{t}^{x} Q_{t}^{x},$$

(59)

$$AC_{t}^{m}(s) = \frac{\phi}{2} \left( \frac{P_{t}^{m,PCP}(s)}{\Pi P_{t-1}^{m,PCP}(s)} - 1 \right)^{2} P_{t}^{m} Q_{t}^{m},$$

(60)

The menu cost function for adjusting $P_{t}^{d}(s)$ is the same for the case of PCP and LCP.

Domestic intermediate good firm $s$ maximizes discounted intertemporal profits given by equation (55) through choosing $P_{t}^{d}(s)$, $P_{t}^{x,PCP}(s)$ and $N_{t}^{dx}(s)$ subject to the constraints of equations (4), (14), (18), (20), (52), (57) and (59). The first-order conditions are given by equations (22), (23), (27).

Imported goods firm $s$ maximizes equation (56) by choosing $P_{t}^{m,PCP}(s)$ and $N_{t}^{m}(s)$ subject to the constraints of equations (5), (19), (21), (58) and (60). The first-order conditions are given by equations (24) and (28).