Trade, Urbanization and Capital Accumulation in a Labor Surplus Economy

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Abstract

In many developing countries there are abundant supplies of “surplus labor” from rural areas. We examine in a simple, dynamic framework how the existence of this large supply of rural unskilled labor affects trade, urbanization, capital accumulation, factor returns, as well as sectoral and aggregate output and social welfare. We find that since the migration equilibrium pins down the real unskilled wage, many comparative-static results with respect to changes in trade-related policies in our surplus-labor economy are different from the conventional model without surplus labor. While these commonly adopted trade protection or investment subsidy policies in a surplus-labor economy need not expedite urbanization as is usually expected, they may even retard capital accumulation and reduce aggregate output. On the contrary, under reasonable factor intensity assumptions, a reduction in migration barriers may likely help immiserization by enhancing capital accumulation, inducing urbanization and enhancing aggregate output. Our numerical results indicate that trade protection in forms of import tariffs can generate strong intertemporal distortion and may cause severe under-urbanization and economy-wide miserization.

Keywords: Surplus Labor and Migration, Capital Barriers and Accumulation, Trade and Urban Development.

JEL Classification Numbers: O41, F16, R23.

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1 Introduction

In many developing countries such as China and India, there are abundant supplies of “surplus labor” from rural areas. For example, the following table shows noticeable under-urbanization where rural population accounted for as high as 90% and no less than half of the total population in all economies in 1950. Even by 2000, rural population in China, Egypt, India and Thailand still exceeded 50%. 1

<table>
<thead>
<tr>
<th>Year</th>
<th>Brazil</th>
<th>China</th>
<th>Egypt</th>
<th>India</th>
<th>Mexico</th>
<th>Korea</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1950</td>
<td>75</td>
<td>87</td>
<td>68</td>
<td>84</td>
<td>58</td>
<td>79</td>
<td>90</td>
</tr>
<tr>
<td>2000</td>
<td>19</td>
<td>64</td>
<td>55</td>
<td>72</td>
<td>26</td>
<td>18</td>
<td>78</td>
</tr>
</tbody>
</table>

In various country studies, rural surplus labor has been found to account for 13 – 28% of the total population surplus labor in China, India and Thailand.2 For the case of China, such surplus labor has been estimated in the range of 150 – 250 millions.3 In the presence of a large stock of rural surplus labor and on-going rural-urban migration, not only there exists a significant urban-rural wage gap (often in the order of 2 : 1), but there is also little growth in real effective wage rate of the unskilled even in urban areas (such a growth rate has been less than 1% in China over 1992 – 2005 for workers with 9 years of schooling or less).

Over four decades ago, several development economists led by Lewis (1954), Fei and Ranis (1961, 1964) and Sen (1966) have studied its implication for labor-market performance and economic advancement. In the present paper, we extend this tradition to a dynamic open-economy setting, to examine how the existence of this large supply of rural unskilled labor affects trade, urbanization, capital accumulation, as well as sectoral and aggregate output and social welfare. This is important because international trade is important for many of these surplus labor economies and capital accumulation is essential for economic takeoff in their development processes.

In this paper, we construct a simple, dynamic framework exhibiting the following specific features:

(i) A small open economy with an abundant supply of surplus labor in rural areas at which a nontraded good (backyard subsistence farming) is produced using unskilled labor with a Ricardian technology.

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1 In contrast, the rural population is 25% or less for most of the OECD countries.
2 See, for example, Bowlus and Sicular (2003) for the case of China, Hanson (1971) for Egypt, Horowitz (1974) for India, and Bertrand and Squire (1980) for Thailand.
(ii) There are two types of traded goods, an exportable (processed food, for example), produced with capital and unskilled labor, and an import substitutable (laptop computer, for example), produced with capital, unskilled labor as well as skilled labor.

(iii) Unskilled labor is paid a market wage rate that makes potential migrants from rural areas indifferent between migrating and not migrating.

(iv) There are barriers to domestic capital accumulation.

(v) In addition to examining the effects of a migration-related institutional factor that reflects rural-urban migration barriers, we evaluate the long-term consequences of four commonly adopted trade-related policies: import tariffs, direct export subsidies, and sector-specific investment subsidies.

We find that since the migration equilibrium pins down the (real) unskilled wage, several comparative-static results with respect to changes in trade-related policies in our surplus-labor economy are different from the conventional model without surplus labor.

In conventional models, all trade-related policies generate qualitatively identical effects on factor prices, capital-labor ratios, and factor and output supplies; moreover, the responses of non-specific factor prices (unskilled wage and capital rental rates), capital-labor ratios and capital supply all depend crucially on the factor intensity rankings.

In our labor surplus economy, not only most of the trade policy effects depends on the factor intensity rankings, but investment subsidies and trade protections also yield different steady-state effects on capital rental, capital accumulation, and aggregate skilled labor in the urban areas. Perhaps most surprising is that these commonly adopted trade-related policies in a surplus-labor economy need not encourage capital accumulation nor induce rural-urban migration and urbanization as is usually expected. Such policies may even retard capital accumulation and reduce aggregate output if the production of the import-competing good uses capital more intensively. We also find that, when the import-competing sector is more capital intensive (both in quantity and cost sense), a reduction in migration barriers lowers both unskilled and skilled wage rates, raises capital rental and hence encourages capital accumulation, induces rural-urban migration and thus expedites urbanization, and enhances aggregate output. In our calibrated economy, a moderate increase in trade protection from the benchmark value of 10% to 12% import tariff can delay the urbanization process by 36 years. Our findings are useful for assessing the long-run macroeconomic
and welfare consequences of the some commonly adopted trade-related and migration-related policies in developing countries.

Related Literature

Since the seminal works by Lewis (1954), Fei and Ranis (1964) and Sen (1966), hereinafter referred to as LFRS, the implications of rural surplus in a dual economy have been widely studied by development economists. The celebrated studies by Todaro (1969) and Harris and Todaro (1970, hereinafter referred to as HT) are perhaps most influential applications of LFRS, where urban unemployment and labor policies are examined under an institutionally fixed minimum wage (above its equilibrium level) in the urban areas.

Among many extensions to HT, we highlight only one strands more relevant to our study that examines trade and migration. In a seminal paper, Khan (1980) reexamines generalized HT models from the perspective of Heckscher-Ohlin-Samuelson and finds that a uniform subsidy to labor with a differential subsidy to capital is optimal (in the sense of second-best). In Batra and Naqvi (1987), gains from trade are evaluated in the presence of urban unemployment and the optimal policy established is a uniform subsidy to labor together with free trade (no tax levied on goods). In Beladi and Marjit (1996), the rural sector employs labor and an intermediate good, while both the intermediate good and the final good sectors in the urban area employ capital and labor. A reduction in tariff on the urban final good is shown to lower capital rental and to raise urban employment as long as the urban final good sector is capital intensive. Recently, Chang, Kaltanic and Loayza (2009) argue that a reduction in tariff improves production efficiency from the goods market perspective but increases labor market distortion in a simple HT setting.

The closely related literature is on surplus labor, migration and growth, initiated by Drazen and Eckstein (1988) and Glomm (1992). Drazen and Eckstein propose a 2-period overlapping generations framework with land as a specific factor in the rural sector and capital as a specific factor in the urban sector, while Glomm (1992) designs an infinite lifetime model allowing for rural-urban migration where higher urban productivity is a result of lower communication costs associated with higher population density. While the decentralized equilibrium in Drazen and Eckstein is suboptimal, that in Glomm is Pareto optimal. Rural-urban migration has also been used to explain equilibrium low-growth trap under informational asymmetry. For example, Bencivenga and Smith (1997) illustrate such a possibility due to the adverse selection of workers into urban areas, while Banerjee and Newman (1998) achieve a similar goal by considering the urban modern sector to have lower credit availability due to higher agency costs.
Lucas (2004) addresses the issue of rural/urban migration as a transition from a no-growth agricultural sector using traditional technology to an urban sector where there is persistent growth due to human capital accumulation. Workers in the urban sector must choose their time allocation between human capital accumulation and work. His model explains the existence of a persistent wage differential between the urban and rural sectors as reflecting the return to human capital accumulation that workers must engage in when they migrate to the city. This contrasts with the Harris-Todaro model, where the differential reflects the possibility of spending time in unemployment due to a rigid urban wage.

Notably, while the trade and migration literature is all static, the migration and growth literature is largely within the closed economy framework. Our model features two distinct urban sectors – an exportable and an import-competing product – which are, in-line with the conventional wisdom in the trade and development literature, more suitable for examining intersectoral distortion and allocation. Moreover, we allow for employment of both unskilled and skilled labor in one of our urban sectors, which enables us to study an important issue of development concerning wage inequality within the urban area. We also allow for barriers to capital investment, which have been argued to play an essential role in driving different development paths in various developing economies. Furthermore, we incorporate an array of trade and migration-related policies that enable us to evaluate their effects on factor allocation, urban development and aggregate output.

2 The Model

We examine a model of a small open economy in which there is a rural sector and an urban sector. In the rural sector, unskilled workers are engaged in subsistence agriculture whose output is consumed only in the rural area. In the urban sector, two tradeable goods are produced using inputs of unskilled labor, skilled labor, and capital. The exportable good, \( X \), requires inputs of unskilled labor and is not consumed domestically. The importable good, \( Y \), can be either consumed or used for investment purposes, and is produced using inputs of unskilled labor, skilled labor, and capital. The economy’s supply of skilled labor and unskilled labor is taken as exogenously given, although the allocation of unskilled labor between the rural and urban sectors is determined by household migration decisions. The supply of capital to the urban sector is assumed to be endogenously determined by the savings decisions of households that own skilled labor. In this section we derive the conditions determining the allocation of unskilled labor between the rural and urban areas,
the allocation of income between consumption and investment, and the allocation of capital and unskilled labor between the $X$ and $Y$ sectors in the urban areas.

2.1 The Rural/Urban Migration Decision

The exogenously given stock of unskilled labor, denoted $N > 0$, consists of a group of size $N$ residing in the rural area and a group of size $L$ located in the urban area. By staying in the rural area, an unskilled worker receives a subsistence utility of $U$ per period, which represents the payoff received from subsistence farming. By migrating to the city/township area, an unskilled worker earns the unskilled wage $w$ and consumes instead the traded good $Y$. We will choose the world price of good $Y$ as numeraire, so the domestic price of good $Y$ will be $T_Y = (1 + \tau)$, where $\tau$ is the ad valorem tariff on imports of $Y$. The consumption of an unskilled worker located in the urban sector at time $t$ will be

$$c_{Y,t}^u = \frac{w_t}{T_Y} \tag{1}$$

An unskilled worker receives utility of $\psi U(c_{Y,t}^u)$ from locating in the urban sector, where $\psi \in (0,1]$ and $U$ is strictly increasing and strictly concave, and satisfies the Inada conditions. One can think of $\psi$ as a “migration discounting factor.” That is, rural workers discount the welfare from earning the urban wage because they are away from home and family. This discount could also reflect training costs are skill accumulation required in order to work in the urban area.

Using (1), migration equilibrium implies:

$$\psi U\left(\frac{w_t}{T_Y}\right) = U \tag{2}$$

which pins down the equilibrium real unskilled wage rate, $\frac{w_t}{T_Y}$.\(^4\) This reservation real wage for unskilled labor is driven crucially by the underlying institutional factor regarding rural-urban migration as captured by $\psi$. We will allow for the possibility that $\psi$ can be influenced by government policy, which might influence the amount of rural/urban migration (as in the case of the household registration system in China). When we conduct comparative-static exercises, we will take a constant-elasticity-of-intertemporal-substitution utility form $U = \frac{1}{\nu} (c_Y)^{\frac{\nu}{\nu-1}}$, with $\nu > 1$ and the elasticity of intertemporal substitution being measured by $\frac{\nu}{\nu-1}$. Under this specification, the migration condition will be given by

$$w = T_Y \left(\frac{U}{\psi^{\frac{\nu}{\nu-1}}}\right)^{\frac{\nu}{\nu-1}} \tag{3}$$

\(^4\)Since the unskilled wage will turn out to be constant along the path to the steady state in our formulation, this condition is equivalent to what would be obtained if workers compared the discounted future returns in each location.
Figure 1 illustrates the determination of the unskilled wage rate in the migration equilibrium.

(Insert Figure 1 here.)

2.2 Capital Accumulation

It will be assumed that all capital is owned by households that own skilled labor. The stock of skilled labor is assumed to be exogenously given, which we will normalize to unity for simplicity, $H = 1$. The wage rate earned by skilled workers is denoted $s_t$, so the budget constraint at time $t$ for a household owning a unit of skilled labor can be written as

$$c_{y,t}^s + I_t = \frac{s_t + r_t K_t}{T_Y} + T_t$$

where $K$ is the stock of capital goods owned by the household, $I$ is investment, $r$ is the return on existing capital, and $T$ is the net transfer from the government in terms of good $Y$. In addition, we assume that capital investment is subject to barriers. Thus, the (beginning-of-period) household capital stock is accumulated according to:

$$K_{t+1} = (1 - \delta)K_t + \frac{I_t}{\mu(I_t)}$$

Here $\mu(I_t) > 1$ captures the severity of capital investment barriers, with $\epsilon = I\mu'(I)/\mu(I) \in (0, 1)$ denoting the (constant) elasticity of the capital investment barrier with respect to the level of investment. The marginal cost of investment will be increasing for $\epsilon > 0$, and $\epsilon < 1$ is required for investment to be productive.\(^5\) The capital barrier may be viewed as a variant of the conventional investment adjustment cost or the Parente-Prescott (1994) technology adoption barriers.

The saving decision for a skilled worker household can be derived from the Bellman equation:

$$V(K_t) = \max_{c_{y,t}^s, I_t} \left[ U(c_{y,t}^s) \right] + \frac{1}{1 + \rho} V(K_{t+1})$$

s.t. (4) and (5)

where $\rho$ is the rate of time preference. The solution to this optimization problem yields the Euler equation governing intertemporal consumption:

$$(1 + \rho) \frac{U(c_{y,t}^s)}{U(c_{y,t+1}^s)} \left( \frac{\mu(I_t)}{\mu(I_{t+1})} \right) = \left[ 1 - \delta + \frac{r (1 - \epsilon)}{\mu(I_{t+1}) (T_Y)} \right]$$

\(^5\) An alternative setup is to assume that capital investment barriers depend on the aggregate level of investment $\bar{I}$ where $\bar{I} = I$ in equilibrium. Mathematically, this is equivalent to setting the elasticity $\epsilon = 0$ in the optimization conditions (MRS) and (MG) below, under which all our main findings remain unchanged.
Equation (7) shows that the marginal rate of intertemporal substitution will be equated to the intertemporal relative price on the optimal path. The intertemporal relative price of future consumption is determined by the rental rate on capital, $r$, divided by the marginal cost of a unit of the investment good, $\mu(I_{t+1})T_Y / (1 - \epsilon)$. Greater investment barriers and larger marginal costs of adjustment will reduce the return to postponing consumption, because they reduce the real return to a unit of capital.

In a steady state, (7) yields the modified Golden Rule:

$$r = \frac{(\rho + \delta) \mu(I_t)T_Y}{1 - \epsilon}$$  

(8)

Note that the cost of the investment good on the right hand side of (8) is increasing in $I$, because the investment barriers/adjustment costs are increasing $I$. This yields a positive relationship between the steady state return on capital and the steady state investment level, with the elasticity of the steady state investment level with respect to the return on capital given by $1/\epsilon$.

### 2.3 Factor Allocation in the Urban Sector

We now turn to the determination of factor prices and outputs in the urban sector, which will determine the allocation of factors between the $X$ and $Y$ sectors. Production in each sector is assumed to take place under conditions of constant returns to scale in production and perfect competition.

The exportable is produced using capital and unskilled labor with a Cobb-Douglas technology, $X = BK_X^\theta_X L_X^{\theta_X}$ where $B > 0$, $\theta_jX$ is the cost share of factor $j \in \{K, L\}$ in production of good $X$, and $\theta_KX + \theta_LX = 1$. This production technology has the associated unit cost function

$$C^X(w, r_X) = \frac{1}{B} \left( \frac{r_X}{\theta_KX} \right)^{\theta_KX} \left( \frac{w}{\theta_LX} \right)^{\theta_LX}$$  

(9)

where $r_X$ is the rental on capital paid by firms in the $X$ sector. We allow for the possibility of a government subsidy to the use of capital at rate $\sigma_{KX}$, which results in a cost of capital in the $X$ sector of $r_X = r/S_{KX}$ where $S_{KX} = 1 + \sigma_{KX}$. The producer price of good $X$ is $pT_X$, where $p$ is the world price of the exportable, $T_X = 1 + \eta$, and $\eta$ is the ad valorem export subsidy rate (applied to the world price). Assuming that the world price of the exportable and the government policy wedges are constant over time, the zero profit condition for the $Y$ sector at time $t$ will be

$$C^X(w_t, \frac{r_t}{S_{KX}}) = pT_X$$  

(10)
This condition must hold with equality if good X is produced at \( t \).

The import-competing good is produced with capital, unskilled labor and skilled labor under the following constant-returns technology:

\[
Y = A [F(K_Y, H)]^{1-\theta_{LY}} Y^{\theta_{LY}}
\]

where \( A > 0, \theta_{LY} \in (0,1) \) is labor’s share in unit costs and \( F \) is a well-behaved constant-returns-to-scale function satisfying \( F_K > 0, F_H > 0, F_{KK} < 0, F_{HH} < 0 \) and \( F_{KH} > 0 \). The last property ensures capital-skill complementarity in the Pareto sense (i.e., more skilled labor raises the marginal product of capital), which has strong empirical support (e.g., see Griliches 1969 and Krusell et al 2000). The unit cost function for the \( Y \) sector is

\[
C_Y(w,Y,s) = \frac{1}{A} \left( \frac{G(r_Y,s)}{1-\theta_{LY}} \right)^{1-\theta_{LY}} \left( \frac{w}{\theta_{LY}} \right)^{\theta_{LY}}
\]

Note that the cost function for the \( Y \) sector can be written as \( C_Y(w,Y,s) = \phi(w,G(r_Y,s)) \), where \( G(.) \) is the unit cost function corresponding to the “sub-production function” \( F(.) \) and \( \phi(.) \) is the unit cost function for the Cobb Douglas function with inputs of unskilled labor and the composite input \( F \). The functions \( \phi \) and \( G \) will be increasing and homogeneous of degree one in their respective input prices, and with negative own and positive cross price effects. With a subsidy to capital in the \( Y \) sector at rate \( \sigma_{KY} \), \( r_Y = r/S_{KY} \equiv r/(1 + \sigma_{KY}) \). Assuming \( Y \) sector tax and tariff policies are constant over time, the zero profit condition will be

\[
C_Y(w_t, \frac{r_t}{S_{KY}}, s_t) = Y_t
\]

In this formulation we have also assumed that skilled labor is used as a specific factor only in the import-competing sector. This has been done for analytical simplicity.\(^6\)

Equilibrium in the urban factor markets and goods markets will require that firms earn zero profits in each sector, and that the demand for each of the productive factors equal their supply in the urban sector. Using the unit cost functions (9) and (11), the full employment conditions require for skilled labor, capital, and unskilled labor can be expressed as:

\[
C_s^Y(w_t, r_t/S_{KY}, s_t)Y_t = H = 1
\]

\[
C_r^X(w_t, r_t/S_{KY})X_t + C_r^Y(w_t, r_t/S_{KY}, s_t)Y_t = K_t
\]

\[
C_w^X(w_t, r_t/S_{KY})X_t + C_w^Y(w_t, r_t/S_{KY}, s_t)Y = L = \bar{N} - N_t
\]

For a given level of factor supplies \( \{H, K_t, \bar{N}\} \), the competitive profit conditions ((9) and (11)), the

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\(^6\)We could allow skilled labor to be used in the exportable sector as long as it is used less intensively relative to unskilled labor than in the import sector. In this case, the results remain quantitatively unchanged.
migration condition (3), and the full employment conditions ((13)-(15)) will determine the factor prices \(\{w_t, r_t, s_t\}\), output levels, \(\{X, Y\}\) and allocation of unskilled workers to the urban sector, \(L\).

### 2.4 Budget Balance and Goods Market Equilibrium

The demand for \(Y\) sector output for domestic consumption will be \(c_{Y,t} = L_t c^Y_{Y,t} + c^s_{Y,t}\). Using the household budget constraints (1) and (4) and letting \(M_t\) denote the level of imports, we can express the goods market equilibrium condition as

\[
M_t = \frac{s_t + r_t K_t}{T_Y} + T_t + \frac{w_t L_t}{T_Y} - Y_t
\]

(16)

The level of government transfers is determined by the government budget constraint, which requires that the transfer equal the difference between trade tax revenue and the cost of subsidies to the usage of capital in the urban sectors. Using our definitions of the wedges in factor markets and product markets and letting \(K_{j,t}\) be the quantity of capital allocated to product of good \(j\) at time \(t\), the level of transfers will be

\[
T_t = (T_Y - 1) M_t - \frac{r_t}{T_Y} \left[ \left( \frac{S_{KY} - 1}{S_{KY}} \right) K_{Y,t} + \left( \frac{S_{KX} - 1}{S_{KX}} \right) K_{X,t} \right] - \frac{p_t (T_X - 1)}{T_Y} X_t
\]

(17)

For a given set of policies government policies and and factor endowments, the outputs and factor prices in (16) and (17) are determined by the zero profit and full employment conditions in the urban sector production activities. Therefore, these two equations can be solve for the levels of imports and transfers government budget constraint pins down the (real) lump-sum transfer, \(T\).

### 3 Steady State Equilibrium and Comparative Statics

We now turn to an analysis of the steady state equilibrium for the surplus labor economy. The analysis of the competitive equilibrium is greatly simplified by the fact that the model has a block recursive structure in the surplus labor case, which allows us to solve for the factor prices independently of the factor supplies. As a result, factor prices will be constant along the transition path to the steady state. We can then use the steady state factor prices to solve for the outputs of the traded goods and the quantity of unskilled labor in the urban sector. This allows us to obtain results on the effect of government policy changes on factor prices, output levels, and the degree of urbanization.
3.1 Factor Prices and Capital Accumulation

The competitive profit conditions, (10) and (12), require that price equal unit cost in a sector at each point in time where it is in operation. Combining these equations with the rural/urban migration condition, (3), yields a recursive system of three equations in the three factor prices \((w, r, s)\). That is, the unskilled wage \(w\) is pinned down by the world price of importable and the migration-related institutional factor given in (3). Given the equilibrium value of \(w\), (10) determines the rental \(r\) and then (12) determines the skilled wage \(s\).

The effect of changes in the policy parameters \((S_{KX}, S_{KY}, T_X, T_Y, \psi)\) on factor prices can be determined by total differentiation of the system of equations (10), (12), and (3). It is convenient to express these comparative statics results using the “hat algebra” where \(\hat{Z} \equiv \frac{\partial}{\partial Z}\):

\[
\hat{w} = \hat{T}_Y - \nu \hat{\psi} \\
\hat{r} = \frac{1}{\theta_{KX}} \left[ \hat{T}_X - \theta_{LX} \left( \hat{T}_Y - \nu \hat{\psi} \right) \right] + \hat{S}_{KX} \\
\hat{s} = \frac{(1-\theta_{LY}) \theta_{KX} + \theta_{LX} \theta_{KY}}{\theta_{KX} \theta_{HY}} \hat{T}_Y + \frac{\theta_{KY}}{\theta_{HY}} \left[ \left( \hat{S}_{KY} - \hat{S}_{KX} \right) \hat{T}_X - \frac{\theta_{LY} \theta_{KY} - \theta_{LY} \theta_{KX}}{\theta_{KX} \theta_{HY}} \nu \hat{\psi} \right]
\]

The existence of surplus labor locks the real wage of unskilled labor into the utility level available in the rural sector, so it will only be affected by policies that affect the cost of migration to urban areas. With a fixed real wage (in terms of good \(Y\)), the return to capital is increasing in policies that favor the exportable sector and the return to skilled labor is decreasing in policies that favor the exportable sector.

When the institutional factor changes in favor of rural-urban migration by mitigating the associated barriers \((\hat{\psi} > 0)\), the unskilled wage is lower and the capital rental higher. Its effect on the skilled wage is negative if the import-competing sector is more capital intensive in the cost sense relative to the exporting sector (i.e., \(\theta_{LX} \theta_{KY} > \theta_{LY} \theta_{KX}\)). Combining (18) and (20), we obtain:

\[
\hat{s} - \hat{w} = \frac{1}{\theta_{KX} \theta_{HY}} \left[ \theta_{KY} \left( \hat{T}_Y - \hat{T}_X \right) - (\theta_{KY} - \theta_{KX}) \nu \hat{\psi} \right]
\]

An increase in the relative protection offered to the \(Y\) sector relative to the \(X\) sector raises the skill premium. Moreover, if the capital cost share in the \(Y\) sector exceeds one in the \(X\) sector \((\theta_{KY} > \theta_{KX}\)) then a change in the institutional factor in favor of rural-urban migration reduces the skill premium.\(^7\)

\(^7\)Notice that, even when \(Y\) is more capital intensive in the cost sense, the capital cost share in the \(Y\) sector need not exceed one in the \(X\) sector.
The following result summarizes the effects of parameter changes on factor prices:

**Proposition 1: (Factor Prices)** \textit{In a labor surplus economy,}

(i) \textit{the wage of skilled labor is increasing in the import tariff and the subsidy to Y sector investment, but decreasing in the export subsidy and the subsidy to X sector investment. The real unskilled wage is unaffected by any of these policy changes;}

(ii) \textit{the rental on capital is increasing in the export subsidy and the investment subsidy to the exportable sector, decreasing in the tariff, and unaffected by an investment subsidy to the exportable sector;}

(iii) \textit{a uniform investment subsidy to both sectors, }$\hat{S}_{KX} = \hat{S}_{KY} > 0$, \textit{does not affect either wage rate, but has a positive effect on the rental rate on capital.}

(iv) \textit{while a reduction in migration barriers lowers the unskilled wage and raises the capital rate, it suppresses the skilled wage rate if the importable sector is capital intensive in the cost sense;}

(v) \textit{none of the factor prices respond to changes in the stock of skilled labor or capital.}

With constant policy parameters, factor returns will be constant during the transition to the steady state as well as in the steady state itself because factor prices are insensitive to changes in the factor supplies. A similar insensitivity of factor prices to factor supplies arises in the two factor, two good version of the Heckscher-Ohlin model in the small country case, although in the present context the result requires the existence of surplus labor as well as the exogeneity of goods prices.

The steady state investment level is positively related to the return to capital from (8), so we can invert that relationship and use the fact that the steady state capital stock will equal $K = I/(\delta \mu (I))$ from (5) to solve for the steady state capital stock,

$$K = \kappa \left( \frac{r}{T_Y} \right)$$

where $\kappa$ is an increasing function of the real return on capital. Totally differentiating this condition and substituting from (19):

$$\dot{K} = \gamma \left[ \frac{1}{\theta_{KX}} \left( \dot{T}_X - \dot{T}_Y \right) + \hat{S}_{KX} + \frac{\theta_{LX}}{\theta_{KX}} \nu \hat{\psi} \right]$$

where $\gamma \equiv \frac{1 - \epsilon}{\epsilon} > 0$. The steady state supply of capital is more elastic the lower is $\epsilon$, where $\epsilon$ is the rate at which the cost of investment increases with the level of investment.
Applying the results of Proposition 1 to (22) yields the effects of parameter changes on the steady state capital stock.

**Proposition 2:** (Capital Accumulation) In a labor surplus economy,

(i) an increase in the export subsidy or the investment subsidy to the exportable sector, or a reduction in the import tariff increases the steady state capital stock. A change in the investment subsidy to the importable sector has no effect on aggregate investment or aggregate capital;

(ii) while a reduction in migration barriers enhances capital accumulation, the stock of skilled labor does not affect aggregate investment or aggregate capital.

Changes in the stock of skilled labor have no effect on factor prices, and hence have no effect on the steady state capital stock.

### 3.2 Sectoral Outputs and Urban Employment

Given the solution for factor prices, the equilibrium levels of \( \{X_t, Y_t, L_t\} \) can be solved from the full employment conditions for the three urban sector factor markets, (13)-(15). Since skilled labor is used only in the production of good \( Y \), the factor prices are determined independently of factor supplies, the full employment conditions can be solved recursively for the output levels and the employment of unskilled labor in the urban sector.

With skilled labor being used only in the import-competing sector, the equilibrium output of good \( Y \) is determined by (13),

\[
Y = \frac{H}{C_s^Y(w, r/S_{KY}, s)} \tag{23}
\]

With constant factor prices, output of \( Y \) will be constant along the path to the steady state. Substituting this solution into (14), we obtain the solution for output in the \( X \) as a function of the stock of capital, \( K_t \). In the steady state, the capital stock will be given by (21), so that the level of \( X \) sector output will be given by,

\[
X = \kappa \left( \frac{r}{T_Y} \right) - \frac{C_t^Y(w, r/S_{KY}, s)H}{C_s^X(w, r/S_{KY})C_s^Y(w, r/S_{KY}, s)} \tag{24}
\]

In order to derive the effect of policy changes on the steady state output levels, we will find it useful to derive some preliminary results on the effect of policy changes on the unit factor requirement in each of the traded good sectors. Under the assumption of Cobb-Douglas production
function for the $X$ sector, the own price elasticities of the factor demands are given by $rC_r^X/C_r^X = -\theta_{LX}$ and $wC_w^X/C_w^X = -\theta_{KX}$. Utilizing (18) and (19), we have:

$$
\begin{align*}
\dot{C}_w^X &= \left(\dot{T}_X - \dot{T}_Y\right) + \nu \dot{\psi}
\dot{C}_r^X &= \frac{\theta_{LX}}{\theta_{KX}} \left[\dot{T}_Y - \dot{T}_X\right] - \nu \dot{\psi}
\end{align*}
$$

(25)

A higher relative price of the exportable good will result in substitution of unskilled labor for capital in the $X$ sector, because the cost of capital is positively related to the relative price of good $X$. Similarly, a reduction in the migration barrier will reduce $w/r$, resulting in substitution of unskilled labor for capital.

The unit cost function for the $Y$ sector takes the form $C_Y^Y = \phi(w, G(r/S_{KY}, s))$, where $\phi$ is Cobb-Douglas and $G$ is constant returns to scale. Due to the Cobb Douglas formulation for $\phi$, the unit labor requirement will $C_w^Y = \theta_{LY} T_Y/w$. Since the real wage is locked in place by the migration condition, (3), the unit labor requirement in $Y$ will respond only to changes in the migration costs, $\psi$. For the input requirements of of capital and skilled labor, we can define the elasticity of demand associated with the sub-production function $F$ as $\eta_{sr} = r G_{sr}/(S_{KY} G_s) > 0$ and $\eta_{rs} = s G_{rs}/G_r > 0$. The constant returns to scale assumption for $G$ then implies $\eta_{ss} = -\eta_{sr}$ and $\eta_{rr} = -\eta_{rs}$. Using these properties, we further obtain the change in unit labor requirements in the $Y$ sector to be:

$$
\begin{align*}
\dot{C}_w^Y &= \nu \dot{\psi}
\dot{C}_r^Y &= \eta_{rs} \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \left[\frac{1}{\theta_{KX}} \left(\dot{T}_Y - \dot{T}_X\right) + \left(\dot{S}_{KY} - \dot{S}_{KK}\right) - \frac{1}{\theta_{KX}} \frac{\theta_{LX} - \theta_{LY}}{\theta_{KY} + \theta_{HY}} \nu \dot{\psi}\right]
\dot{C}_s^Y &= \eta_{sr} \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \left[\frac{1}{\theta_{KX}} \left(\dot{T}_X - \dot{T}_Y\right) + \left(\dot{S}_{KK} - \dot{S}_{KY}\right) + \frac{1}{\theta_{KX}} \frac{\theta_{LX} - \theta_{LY}}{\theta_{KY} + \theta_{HY}} \nu \dot{\psi}\right]
\end{align*}
$$

(26)

Comparing (25) and (26), it can be seen that an increase in the relative price of $X$ will result of substitution away from capital in both sectors, and substitution toward the use of skilled (unskilled) labor in the $Y$ ($X$) sector. A reduction in migration costs will reduce the price of unskilled labor, which increases the demand for skilled labor and reduces the demand for capital in the $X$ sector if $X$ has a larger unskilled labor cost share than does $Y$. Also note that subsidizing investment in the $Y$ sector relative to the $X$ sector results in substitution toward capital and away from skilled labor in the $Y$ sector.

We are now ready to analyze the effects of the parameters on sectoral outputs and unskilled labor in urban areas. Since $\dot{Y} = \dot{H} - \dot{C}_s^Y$ from (13), we can totally differentiate (23) and substitute

8If the function $G$ is Cobb-Douglas, $\eta_{sr} = \theta_{KY}/(\theta_{KY} + \theta_{HY}) = 1 - \eta_{rs}$. 

13
from (26) to obtain:
\[
\dot{Y} = \dot{H} + \eta_{sr} \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right) \left[ \frac{1}{\theta_{KY}} \left( \dot{T}_Y - \dot{T}_X \right) + \left( \dot{S}_{KY} - \dot{S}_{KX} \right) - \frac{1}{\theta_{KY}} \frac{\theta_{LY}}{\theta_{KY} + \theta_{HY}} \dot{\psi} \right]
\] (27)

The output in the Y sector is increasing in the protection offered to the Y sector relative to the X sector, \(T_Y/T_X\). This generates a Lerner-symmetry type result for the Y sector, since protection of the Y sector is equivalent in effect to an export tax of equal proportion on the X sector. The impact of a reduction in migration costs and capital market subsidies on output of Y reflect the impact of these changes on the demand for skilled labor per unit of output noted above.

Totally differentiating the steady state capital market equilibrium condition (24) and substituting from (25), (26), and (18)-(20) yields,
\[
\dot{X} = \frac{\Delta + \gamma + \lambda_{KX} \theta_{LY}}{\lambda_{KX} \theta_{RX}} \left( \dot{T}_X - \dot{T}_Y \right) + \frac{\Delta}{\lambda_{KX}} \left( \dot{S}_{KX} - \dot{S}_{KY} \right) + \frac{\gamma}{\lambda_{KX}} \dot{S}_{KX} + \frac{\left( \theta_{LY} - \theta_{LY} \right) \Delta + (\gamma + \lambda_{KX}) \theta_{LY}}{\lambda_{KX} \theta_{RX}} \frac{\dot{\psi}}{\lambda_{KX}} - \frac{1}{\lambda_{KX}} \dot{H}
\] (28)

where \(\lambda_{KX} \equiv XC_{\eta} / K\) denotes the fraction of all capital employed in the X sector and \(\Delta \equiv (1 - \lambda_{KX}) (\eta_{rs} + \eta_{sr}) \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right) > 0\) captures the degree of substitutability between skilled labor and capital in the Y sector. Thus, the output of good X is increasing in \(T_X\) and \(S_{KX}\), and decreasing in \(T_Y\) and \(S_{KY}\). A reduction in migration barriers will increase the output of exportables because it lowers the real cost of unskilled labor. Note also that the short run effects of policy changes (i.e. with a fixed \(K_t\)) can be obtained by setting \(\gamma = 0\). A uniform subsidy to capital in both sectors will have no effect on the output of X in the short run, but will raise output in the long run due to its positive effect on the steady state capital stock.

The impacts of parameter changes on outputs from (28) and (27) can be summarized as:

**Proposition 3:** (Sectoral Outputs)  *In a labor surplus economy,*

(i) **Policies that favor the Y sector relative to the X sector (i.e. increase in \(T_Y\) or \(S_{KY}\), decrease in \(T_X\) and \(S_{KX}\)) or an increase in the stock of skilled labor will reduce the output of \(X\) raise the output \(Y\);**

(ii) **while a uniform investment subsidy to both sectors reduces exportable output without affecting the production of the importable, a reduction in migration barriers raises exportable output and lowers importable production.**

The following result, which is established in the Appendix, summarizes the effect of the parameter changes on the allocation of factors between sectors.
Proposition 4: (Factor Allocations) In a labor surplus economy,

(i) Policies that favor the $Y$ sector relative to the $X$ sector or an increase in the stock of skilled labor will raise employment of capital and unskilled labor in the $Y$ sector and reduce it in the $X$ sector;

(ii) a higher import tariff or a lower export subsidy increases the capital-labor ratios in both sectors;

(iii) a higher investment subsidy to the importable sector leaves the exportable sector’s capital-labor ratio unchanged, but raises the importable sector’s capital-labor ratio by an equal proportion;

(iv) while a larger stock of skilled labor has no effect on capital-labor ratios, a reduction in migration barriers lowers the exportable sector’s capital-labor ratio unambiguously and suppresses the importable sector’s capital-labor ratio if the importable sector is capital intensive in the cost sense.

Policies that raise the return to producing the importable will encourage both factors to be reallocated from the exportable to the importable sector. Turning now to the effects of a higher investment subsidy to the importable sector on the capital-labor ratios, the exportable sector’s capital-labor ratio remains unchanged because the wage-rental ratio ($w/r$) is unchanged. In the importable sector, however, the subsidy results in a lower real rental rate which leads to a higher capital-labor ratio.

In response to a higher import tariff/export subsidy, the real unskilled wage remains unchanged while the real the real rental rate falls. This leads to higher capital-labor ratios in both sectors. Proposition 4 shows that while the results on factor input demands resemble those in the standard model with no surplus labor, the effects on the capital-labor ratios are very different. In the presence of surplus labor, an import tariff/export subsidy yields different steady-state effects than an investment subsidy to the importable sector, and such effects are independent of the factor intensity ranking.

3.3 Urbanization

The effect of parameter changes on migration of unskilled workers can be obtained by totally differentiate the unskilled labor market clearing condition (15) and apply (25) and (26), arriving
\[
\hat{L} = \lambda_{LX} \left( \dot{X} + \dot{C}_w^X \right) + (1 - \lambda_{LX}) \left( \dot{Y} + \dot{C}_w^Y \right)
= \frac{\Gamma + \lambda_{LX} (\gamma + \lambda_{KKX}) \left( \dot{T}_X - \dot{T}_Y \right) + \Gamma \lambda_{KKX} \left( \dot{S}_{KX} - \dot{S}_{KY} \right) + \frac{\lambda_{LX} \gamma \dot{S}_{KX}}{\lambda_{KKX}}}{1 - (1 - \lambda_{KKX}) \eta_{sr} \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right)}
+ \frac{\left( 1 - \lambda_{KKX} \right) \frac{\theta_{LX} \theta_{LY}}{\theta_{HY} + \theta_{KY}} \left[ \Gamma + \lambda_{LX} \lambda_{KKX} \eta_{rs} \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right) \right]}{\lambda_{KKX} \theta_{KKX}}
+ \frac{\lambda_{LX} \eta_{Ly} \dot{S}_{KX}}{\lambda_{KKX}}.
\]

(29)

where \( \lambda_{LX} = \frac{XC^X_w}{L} \) is the share of urban unskilled labor employed in the \( X \) sector and \( \Gamma \equiv \left( (\lambda_{LX} - \lambda_{KKX}) \eta_{sr} + \lambda_{LX} (1 - \lambda_{KKX}) \eta_{rs} \right) \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right) \). To see the interpretation of \( \Gamma \), consider the effect of a subsidy to capital in the \( X \) sector. This subsidy has no effect on the unit labor requirements in either sector, but will result in an expansion of output in the \( X \) sector and a contraction in the \( Y \) sector from Proposition 3(i). The effect of this change in outputs on the demand for unskilled labor, holding the capital stock constant, is given by \( \Gamma \). A sufficient condition for this change to raise the demand for unskilled labor is that the expanding \( X \) sector be unskilled labor intensive relative to \( Y \) in the quantity sense (i.e., \( \lambda_{LX} > \lambda_{KKX} \), which ensures \( \Gamma > 0 \)) is sufficient for a subsidy to capital in the \( Y \) sector (i.e., \( \dot{T}_{KY} < 0 \)) to result in a decrease in employment of unskilled labor in urban areas. This factor intensity condition is also sufficient for a subsidy to capital in the \( X \) sector to raise urban employment of the unskilled. However, \( \Gamma > 0 \) is not necessary for this result because the subsidy to capital in the \( X \) sector also has a positive impact on the steady state capital (as long as \( \gamma > 0 \)), and a higher steady state capital stock will raise the demand for unskilled labor. Note also that that a uniform subsidy to capital in both sectors will raise urban employment of unskilled labor because it raises the capital stock (as long as \( \gamma > 0 \)) but leaves factor prices unaffected. The results for changes in the price of traded goods are similar, although they have an additional term resulting from the fact that an increase in the relative price of the exportable will raise employment of unskilled labor in the \( X \) sector (via a multiplier, \( \lambda_{LX} / \theta_{KKX} \)). The factor intensity condition \( \lambda_{LX} > \lambda_{KKX} \) is sufficient for an increase in the price of the export (import) good to raise (reduce) employment of the unskilled in urban areas. Furthermore, a sufficient condition for a change in the institutional factor in favor of rural-urban migration to raise urban employment and induce urbanization is that the import-competing sector is capital intensive in both the cost and the quantity sense. Interestingly, should the import-competing sector be capital intensive in the quantity sense but sufficiently labor intensive in the cost sense such that \( \theta_{LX} < \theta_{LY} \), it is possible that a rural-urban migration policy may result less migration flows. Additionally, that the import-competing sector is capital intensive in the quantity sense is necessary and sufficient for a
rising skilled labor stock to reduce urban employment of unskilled workers.

Summarizing these results, we have:

**Proposition 5:** (Urbanization) *In a labor surplus economy,*

(i) *when the import-competing sector more capital intensive in the quantity sense, an increase in the import tariff or the investment subsidy to the importable sector, a decrease in the export subsidy or the investment subsidy to the exportable sector, retards the urbanization process;*

(ii) *when the import-competing sector is more capital intensive in the quantity sense, a larger stock of skilled labor slows down the urbanization process;*

(iii) *when the import-competing sector is more capital intensive in both quantity and cost sense, a reduction in migration barriers promotes urbanization.*

Some of the comparative-static results concerning outputs and urban employment can be better understood using diagrammatic analysis with which we proceed. For illustrative purposes, we only provide such an analysis for two comparative-static exercises: an increase in $S_{KY}$ and an increase in $T_Y$.

Consider first an increase in the capital subsidy to the import sector $S_{KY}$. Figure 2a illustrates the effect of an increase in $S_{KY}$ when the import sector is more capital intensive than the export sector. $E_1$ is the initial endowment point and given input prices, outputs are $(X_1, Y_1)$. Once the capital subsidy is imposed (or increased) the endowment shifts from $E_1$ to $E_2$ as $L$ falls. The subsidy leads to an increase in $k_Y$ and a consequent increase in $Y$ and a decrease in $X$ to $(X_2, Y_2)$. The case in which the export sector is more capital intensive than the import sector (Figure 2b) is a bit more complicated. In this case $L$ can increase or decrease. Assume that initially that the endowment is at $E_1$ and initial production is at $(X_1, Y_1)$. In the case in which $L$ falls the endowment moves to $E_2$ and $k_Y$ increases. Production of $X$ decreases to $X_2$ and production of $Y$ increases to $Y_2$. If $L$ increases the endowment moves to $E_3$, $k_Y$ increases and production shifts to $(X_3, Y_3)$. As in the previous case, $X$ decreases and $Y$ increases.

(Insert Figures 2a and 2b here.)

Consider next an increase in the tariff $T_Y$. Figure 3 illustrates changes to outputs when a higher tariff is levied. Initially suppose that the endowment is at $E_1$ and initial production is at...
(X_1, Y_1). The tariff will reduce K. The amount of urban labor L could increase or decrease (we have shown the case in which L decreases). The endowment point shifts to E_2 and k_X and k_Y increase. Production of X falls to X_2 and production of Y rises to Y_2.

(Insert Figure 3 here.)

### 3.4 Consumption and Aggregate Output

We conclude our comparative static analysis by examining the effect of parameter changes on the steady state consumption of skilled workers and on aggregate output. Define the skilled worker’s shares of income from wage, net rental and net government transfer as \( \zeta_H, \zeta_K \) and \( \zeta_T \), respectively \( (\zeta_H + \zeta_K + \zeta_T = 1) \). Totally differentiating (4) and conveniently evaluating the results at \( T = 0 \) (and hence \( \zeta_T = 0 \) and \( \zeta_K = 1 - \zeta_H \)), we obtain:

\[
\hat{c}_Y^s = \zeta_H \left( \hat{s} - \hat{T}_Y \right) + \zeta_K \left[ \Lambda \left( \hat{r} - \hat{T}_X \right) + \hat{K} \right] + \zeta_T \hat{T} \\
= \frac{1}{\theta_{KX}} \left[ \zeta_H \frac{\theta_{KY}}{\theta_{HY}} - (1 - \zeta_H)(\Lambda + \gamma) \right] \left( \hat{T}_Y - \hat{T}_X \right) + \zeta_H \frac{\theta_{KY}}{\theta_{HY}} \left( \hat{S}_{KY} - \hat{S}_{KX} \right) \\
+ (1 - \zeta_H)(\Lambda + \gamma) \hat{S}_{KX} - \left[ \zeta_H \frac{\theta_{LX} \theta_{KY} - \theta_{LY} \theta_{KX}}{\theta_{KX} \theta_{HY}} - (1 - \zeta_H) \frac{\theta_{LX}}{\theta_{KX}}(\Lambda + \gamma) \right]\hat{\psi}
\]

where \( \Lambda \equiv \frac{\hat{r}_X - \epsilon \hat{\delta}_\mu}{\pi - \hat{\delta}_\mu} > 0 \). Thus, the effect of a lower capital subsidy to the X sector or a higher protection offered to the Y sector relative to the X sector on the consumption of a skilled worker depends positively on the wage income share but negatively on the degree of capital barriers. A capital subsidy to the Y sector always raises skilled worker’s consumption unambiguously, whereas mitigating migration barriers may hurt the welfare of skilled workers in an economy with labor-intensive exporting if the wage income share is high and the degree of capital barriers is low. While a uniform trade protection to both sectors leaves skilled worker’s consumption unaffected, a uniform subsidy to capital in both sectors lowers skilled worker’s consumption.

Turning now to aggregate output, we define the output share of the X sector as \( \chi \) (and hence that of the Y sector as \( 1 - \chi \)) and then totally differentiate (??) to yield:

\[
\hat{R} = \chi \left[ \hat{X} + (\hat{T}_X - \hat{T}_Y) \right] + (1 - \chi) \hat{Y} \\
= \frac{1}{\theta_{KX}} \left( \gamma + \frac{\lambda_{KX}}{\lambda_{KX}} \right) \left( \hat{T}_X - \hat{T}_Y \right) + \frac{1}{\theta_{KX}} \left( \gamma + \frac{\lambda_{KX} \theta_{KY} + \theta_{HY}}{\lambda_{KX}} \right) \hat{S}_{KX} \\
+ \frac{1}{\theta_{KX} \theta_{KY} + \theta_{HY}} \left[ \gamma + \frac{(\gamma + \lambda_{KX}) \left( \theta_{LX} \theta_{KY} + \theta_{LY} \right)}{\lambda_{KX}} \\lambda_{KX} \right] \hat{\psi} + \frac{\lambda_{KX} - \chi}{\lambda_{KX}} \hat{H}
\]
where $Y \equiv \chi_{XKX} \Delta + (1 - \chi) \eta_{\frac{\theta_{KY} + \theta_{HY}}{b_{HY}}} > 0$ if the import-competing sector is capital intensive in the quantity sense to which we restrict our attention. Then, a lower capital subsidy to the $X$ sector, a higher capital subsidy to the $Y$ sector, or a greater trade protection offered to the $Y$ sector relative to the $X$ all leads to lower aggregate output, as any such change turns out to hurt the exporting sector more than benefiting the import-competing sector. While a uniform trade protection to both sectors has no effect on aggregate output, a uniform subsidy to capital in both sectors enhances aggregate output unambiguously. Moreover, the long-run effect of mitigating migration barriers raises aggregate output if the import-competing sector is also capital intensive in the cost sense. Finally, whether an increase in the stock of skilled labor advances aggregate output depends exclusively on whether the labor share exceeds the output share in the $X$ sector. Under normal circumstances (as often experienced in developing countries), the traditional sector ($X$) often employs a larger share of factor inputs than its output share due to lower factor productivity in the absence of skilled worker’s contributions. In this case, the stock of skilled labor has a positive long-run effect on aggregate output.

Summarizing the above findings yields:

**Proposition 6:** (Skilled Worker’s Consumption, Aggregate Output and Welfare) In a labor surplus economy with skilled workers receiving essentially zero net real transfer from the government,

(i) an increase in the subsidy to investment in the importtable sector reduces skilled worker’s consumption and social welfare unambiguously;

(ii) when the wage income share is sufficiently high and the degree of capital barriers is sufficiently low, an increase in the import tariff or a decrease in the export subsidy or the investment subsidy to the exportable sector raises skilled worker’s consumption and social welfare;

(iii) when the import-competing sector more capital intensive in the quantity sense, an increase in the import tariff or the investment subsidy to the importtable sector, or a decrease in the export subsidy or the investment subsidy to the exportable sector, lowers aggregate output;

(iv) when the import-competing sector is more capital intensive in both quantity and cost sense, a reduction in migration barriers raises skilled worker’s consumption, aggregate output and social welfare;

(v) while a higher stock of skilled labor has no effect on skilled worker’s consumption or social
welfare, it raises aggregate output if the capital share exceeds the output share in the exportable sector.

Recall that the welfare is driven exclusively by skilled worker’s consumption of the importable. From the budget constraint (4), we learn that the skilled worker’s consumption of the importable depends on three factors: (i) real skilled wage income \((\frac{s}{T})\), (ii) investment cost adjusted net real rental income \((\frac{r - \delta u}{T}) K\), and (iii) net real transfer from the government \((T)\). In response to an increase in any trade-related policy, real skilled wage income rises. While an investment subsidy to the importable sector does not affect net real rental income so that skilled worker’s consumption rises unambiguously in response, an increase in the import tariff or a decrease in the export subsidy or the investment subsidy to the exportable sector tends to lower net real rental income, leading to ambiguous effects on skilled worker’s consumption.

Any trade-related policies delineated above raises the output of the importable and reduces the output of the exportable as well as the domestic relative price of the exportable \((\frac{p^X}{T})\). While the former effect raises aggregate output, the latter two effects reduces it. When the import-competing sector is capital intensive in the quantity sense, the former effect is outweighed by the latter, thereby leading to a lower level of aggregate output.

3.5 Comparison with Conventional Models

For comparison purposes, let us begin by summarizing briefly the comparative static results of trade-related policies in conventional models without surplus labor. In this case, the total labor endowment is fixed and there is no migration equilibrium relationship. In this case, the system of equilibrium conditions is no longer recursive. From the the two competitive profit conditions, (10) and (12), one may express the unskilled and skilled wages as functions of capital rental. With these functional relationships, we can use the skilled labor market equilibrium condition, (13), to write the output of the import-competing good as a function of capital rental, which can then be substituted into the unskilled labor market equilibrium condition, (15), to express the output of the exportable as a function of capital rental. These together with the steady-state capital stock, (21), and the capital market equilibrium condition, (14), subsequently yield a fixed point relationship to determine equilibrium rental. Due to the algebraic complexity, we will relegate the detailed derivations to the Appendix. The comparative-static analysis suggests that, if the import-competing sector is strongly capital-intensive (see the Appendix for a formal definition), then an
increase in trade protection/capital subsidy to the import-competing sector or a decrease in trade protection/capital subsidy to the exportable sector will reduce the unskilled wage, raise the skilled wage, suppresses the capital-labor ratio in the exportable sector and encourages capital accumulation. The effects of these trade-related policies on other endogenous variables are all ambiguous.

The following table reports findings obtained in the conventional model without surplus labor with respect to higher \( \tau \), lower \( \eta \), lower \( \sigma_X \) and higher \( \sigma_Y \), as well as those obtained in our labor surplus economy.

<table>
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<th>( w )</th>
<th>( r )</th>
<th>( s )</th>
<th>( X )</th>
<th>( Y )</th>
<th>( K_X )</th>
<th>( K_Y )</th>
<th>( k_X )</th>
<th>( k_Y )</th>
<th>( K )</th>
<th>( L )</th>
<th>( c_Y^* )</th>
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<td>+( \uparrow )</td>
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<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>-( \downarrow )</td>
<td>?</td>
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<td>0</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
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<td>( \uparrow )</td>
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<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>higher ( \psi )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>higher ( H )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

Note: \(^{\dagger}\) if \( \lambda_{LX} > \lambda_{KX} \) (Y capital-intensive in the quantity sense)
\(^{\ddagger}\) if \( \theta_{LX}\theta_{KY} > \theta_{LY}\theta_{KX} \) (Y capital-intensive in the cost sense)
\(^{\star}\) if \( \zeta_H \) (wage income share) is high and \( \gamma \) (capital barriers) is low
\(^{\star}\) if \( \lambda_{KX} \) (capital share in \( X \)) exceeds \( \chi \) (output share in \( X \))
\( \downarrow \) if \( Y \) capital-intensive in both the quantity and the cost sense

Three important findings concerning trade-related policy changes are reached, which are all contrasting sharply with those obtained in conventional models without surplus labor. First, trade-related policy effects are largely independent of factor intensity; in the case of a uniform capital subsidy, the factor intensity ranking becomes completely irrelevant even with respect to changes in employment of unskilled labor in urban areas. Second, the effects of trade protection and capital subsidy are very different, particularly in factor prices, capital-labor ratios and capital accumulation. Third, trade protections or capital subsidies toward the import-competing sector discourage
Concerning the migration-related institutional factor and the skilled labor stock, an institutional change in favor of rural-urban migration enhances labor-intensive exporting activities and promotes capital accumulation and urbanization. An increase in the stock of skilled labor used as a specific factor in the import-competing sector shifts resources from the exporting to the importable sector without affecting factor prices, factor proportions or aggregate capital. When unskilled workers are used more intensively in the exporting sector, their employment in the urban area decreases.

**Remark:** It may be of interest to note that, when the government cannot do better in mitigating capital or migration barriers, the optimal policy will feature free trade (see the proof in the Appendix). Of course, this result assumes that the private investment barriers also represent the social barriers to investment. If these barriers are not social costs, then there will be an argument for a subsidy to investment. Similarly, the result also requires that the reservation wage for unskilled labor represents the true social cost. If these costs differ, there will be an argument for a subsidy/tax on location-specific employment.

### 4 Numerical Analysis

In the numerical analysis, the utility function is assumed to be log linear, \( \ln (c) \), and the capital barrier function \( \mu(I) \) takes an iso-elastic form, \( \mu(I) = \mu_0 I^{0.5} \) (hence, \( \epsilon = 0.5 \)). Further assume the production function of the importable to be Cobb-Douglas with \( \alpha_K = 0.25 \) and \( \alpha_H = 0.15 \) (so the unskilled labor share coefficient is 0.6). Set the capital share coefficient of the production function of the exportable as \( \beta = 0.15 \) (so the unskilled labor share coefficient is 0.85). Under these technology parameters, the import-competing sector is capital intensive in both quantity and cost sense. We further choose the time discount rate as \( \rho = 0.05 \) and the capital depreciation rate as \( \delta = 0.1 \), as commonly used in the growth and development literature. In addition, we normalize \( B = 1 \) and \( \bar{U} = 1 \) and select an initial set of trade-related policy parameters as: \( \tau = 0.1 \), \( \eta = 0.1 \), \( \sigma_X = 0.0 \), and \( \sigma_Y = 0.1 \) (that is, the investment subsidy is only to the “modern” import-competing sector).

We then calibrated three parameters and two exogenous variables to fit real world observations: (i) \( \mu_0 = 0.09225 \) for the capital-output ratio to be about 2.5, (ii) \( A = 5.5971 \) for the skilled to unskilled wage ratio to be about 2.5, (iii) \( \psi = 0.5906 \) for the urban-rural wage ratio to be about 2, (iv) \( p = 4.5 \) for the capital rental rate \( r \) to be about 10%, and (v) \( \bar{N} = 100 \) for a reasonable rural-urban unskilled worker proportion falling in the range of 15 – 20%. 

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Under this parametrization, both net trade and lump-sum transfer are about zero. Moreover, about 5% of the urban population are skilled. About 18% of the unskilled reside in urban areas, with 45% in the exportable sector and 55% in the importable sector. Concerning the allocation of capital $K$, about 24% is in the exportable sector and 76% in the importable sector. In this calibrated economy, trade dependence (the ratio of imports plus exports to aggregate output) is about 74%, whereas about 37% of importables are imported. All these figures are realistic in developing economies with a large fraction of surplus labor.

We conduct a trade policy experiment, raising $\tau$ and $\sigma_Y$ one at a time by 1% to compute the resulting percentage changes in the following endogenous variables (noting that both policy changes would not alter the real unskilled wage rate):

<table>
<thead>
<tr>
<th>$r$</th>
<th>$s$</th>
<th>$X$</th>
<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$c^s$</th>
<th>$V$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>-0.51</td>
<td>0.50</td>
<td>-5.64</td>
<td>0.50</td>
<td>-0.47</td>
<td>-2.38</td>
<td>-0.56</td>
<td>-0.16</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>0</td>
<td>0.18</td>
<td>-0.86</td>
<td>0.16</td>
<td>0</td>
<td>-0.29</td>
<td>-0.028</td>
<td>-0.0081</td>
</tr>
</tbody>
</table>

The numerical results indicate that, in the presence of surplus labor with capital and migration barriers, trade protection to the import-competing sector has strong intertemporal distortion and may cause severe under-urbanization and reduce aggregate output and social welfare significantly.

We further conduct the same experiment with greater capital barriers ($\gamma$ raises from its benchmark value of 1.0 to 0.9). We then obtain the following percentage changes.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$s$</th>
<th>$X$</th>
<th>$Y$</th>
<th>$K$</th>
<th>$L$</th>
<th>$c^s$</th>
<th>$V$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>-0.51</td>
<td>0.50</td>
<td>-14.1</td>
<td>0.50</td>
<td>-0.42</td>
<td>-2.81</td>
<td>-14.1</td>
<td>-0.17</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>0</td>
<td>0.19</td>
<td>-2.38</td>
<td>0.16</td>
<td>0</td>
<td>-0.41</td>
<td>-2.28</td>
<td>-0.0088</td>
</tr>
</tbody>
</table>

Thus, with greater capital barriers, trade protection has strong intertemporal distortion and may cause severe under-urbanization.

Finally, we look into the dynamic process through which the calibrated economy becomes urbanized. To facilitate this analysis, we normalize the units to maintain constant populations of the skilled and the unskilled, but allow the two production technologies to grow over time according to:

$$A(t) = A \left[ 0.8 + 0.55 \left( 1 - e^{-0.001t} \right) \right]$$

$$B(t) = B \left[ 0.95 + 0.4 \left( 1 - e^{-0.0015t} \right) \right]$$
Then, under the benchmark parametrization with $K_0 = 0.8 \cdot K$ and $\tau = 0.1$, the urbanization process starts in year 70 and the process completes in year 108 (see the top panel of Figure 4). Keeping all other parameters but raising trade protection moderately by 20% with $\tau = 0.12$, we find that the urbanization process is delayed by 36 years, not starting until year 106 (see the bottom panel of Figure 4).

5 Concluding Remarks

In this paper, we have constructed a dynamic open-economy model to examine how the existence of a large supply of rural unskilled labor affects trade, capital accumulation, labor allocation and output. We have found that several comparative-static results with respect to changes in trade-related policies are different from the conventional model without surplus labor. All three policies we examine raise the real skilled wage rate and leave the real unskilled wage rate unchanged. A higher investment subsidy in the import sector leaves the rental rate unchanged whereas trade policies reduce the rental rate.

All three policies reallocate resources away from the export sector towards the import sector. They also increase the capital-labor ratio in the importable sector regardless of which sector is capital intensive. Trade policies increase the capital-labor ratio in the export sector and investment subsidies leave it unchanged. Higher investment subsidies do not increase investment or capital stock and trade policies lower both. Finally, the supply of unskilled labor in the urban areas falls if the import sector is capital intensive.

We have constructed a dynamic open-economy model to examine how the existence of a large supply of rural unskilled labor affects trade, capital accumulation, labor allocation and output. Along these lines, there are at least two interest avenues for future work. One is to calibrate the model to a particular economy that features substantial surplus labor, such as China, India and Thailand, to explain the speed as well as the pattern of transition along the urbanization process. Another is to calculate all trade-related distortion wedges to fit observed data in the above-mentioned economies, following the methodology developed by Hsieh-Klenow (2009), to evaluate how much the productivity an economy may gain by removing each and all of the trade barriers.
References


Appendix

Proof of Proposition 4: Utilizing (25) and (26), the effects of the parameters on the capital-labor ratios immediately follow:

\[ \hat{k}_X = \hat{C}_r^X - \hat{C}_w^X = \frac{1}{\theta_{KK}} \left[ \left( \hat{T}_Y - \hat{T}_X \right) - \nu \hat{\psi} \right] \]  
(A1)

\[ \hat{k}_Y = \hat{C}_r^Y - \hat{C}_w^Y = \eta_{rs} - \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \left[ \frac{1}{\theta_{KK}} \left( \hat{T}_Y - \hat{T}_X \right) + \left( \hat{S}_{KY} - \hat{S}_{KK} \right) - \frac{1}{\theta_{KK}} \frac{\theta_{LY} - \theta_{LY}}{\theta_{KY} + \theta_{HY}} \nu \hat{\psi} \right] \]  
(A2)

That is, the capital-labor ratio in the X sector is independent of capital tax/subsidy and increasing in the relative protection offered to the Y sector relative to the X sector, \( T_Y/T_X \). While the capital-labor ratio in the Y sector is also increasing in the relative protection measured by \( T_Y/T_X \), it depends positively on the relative subsidy to capital in the Y sector, \( T_{KY}/T_{XY} \), as well. The import-competing sector being capital intensive in the cost sense is sufficient for a change in the institutional factor in favor of rural-urban migration to reduce capital-labor ratios in both sectors.

Using (25)- (28), we can derive the comparative statics results for the usage of capital and unskilled labor in each sector.

\[ \hat{K}_X = \hat{X} + \hat{C}_r^X = \frac{\Delta + \gamma}{\lambda_{KK} \theta_{KK}} \left( \hat{T}_X - \hat{T}_Y \right) + \frac{\Delta}{\lambda_{KK}} \left( \hat{S}_{KK} - \hat{S}_{KY} \right) + \frac{\gamma}{\lambda_{KK}} \hat{S}_{KK} 
+ \frac{\theta_{LY} - \theta_{LY}}{\theta_{KY} + \theta_{HY}} \Delta + \gamma \frac{\lambda_{XX}}{\lambda_{KK} \theta_{KK}} \nu \hat{\psi} - \frac{1 - \lambda_{KK} \theta_{KK}}{\lambda_{KK} \theta_{KK}} \hat{H} \]  
(A3)

\[ \hat{K}_Y = \hat{Y} + \hat{C}_r^Y = \frac{\Delta}{1 - \lambda_{KK}} \left[ \frac{1}{\theta_{KK}} \left( \hat{T}_Y - \hat{T}_X \right) + \left( \hat{S}_{KY} - \hat{S}_{KK} \right) - \frac{1}{\theta_{KK} \theta_{HY} + \theta_{HY} \theta_{HY}} \nu \hat{\psi} \right] + \hat{H} \]  
(A4)

\[ \hat{L}_X = \hat{X} + \hat{C}_w^X = \frac{\Delta + \gamma + \lambda_{KK}}{\lambda_{KK} \theta_{KK}} \left( \hat{T}_X - \hat{T}_Y \right) + \frac{\Delta}{\lambda_{KK}} \left( \hat{S}_{KK} - \hat{S}_{KY} \right) + \frac{\gamma}{\lambda_{KK}} \hat{S}_{KK} 
+ \frac{\theta_{LY} - \theta_{LY}}{\theta_{KY} + \theta_{HY}} \Delta + \gamma \frac{\lambda_{XX}}{\lambda_{KK} \theta_{KK}} \nu \hat{\psi} - \frac{1 - \lambda_{KK} \theta_{KK}}{\lambda_{KK} \theta_{KK}} \hat{H} \]  
(A5)

\[ \hat{L}_Y = \hat{Y} + \hat{C}_w^Y = \eta_{sr} \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \left[ \frac{1}{\theta_{KK}} \left( \hat{T}_Y - \hat{T}_X \right) + \left( \hat{S}_{KY} - \hat{S}_{KK} \right) - \frac{1}{\theta_{KK} \theta_{HY} + \theta_{HY} \theta_{HY}} \nu \hat{\psi} \right] + \hat{H} \]  
(A6)

where \( \Delta \equiv (1 - \lambda_{KK}) (\eta_{rs} + \eta_{sr}) \left( \frac{\theta_{KY} + \theta_{HY}}{\theta_{HY}} \right) > 0. \)

Derivation of the Comparative Statics under the Conventional Framework in the Absence of Surplus Labor: Consider the conventional framework with no surplus labor where the unskilled labor is fixed at \( L = \tilde{L} = \tilde{N} \). For simplicity, we assume Cobb-Douglas technologies, implying the following unit cost functions:

\[ C^X(w,r_X) = \frac{1}{B} \left( \frac{r_X}{\theta_{KK}} \right)^{\theta_{KK}} \left( \frac{w}{\theta_{LY}} \right)^{\theta_{LY}} \]  
\[ C^Y(w,r_Y,s) = \frac{1}{A} \left( \frac{r_Y}{\theta_{KY}} \right)^{\theta_{KY}} \left( \frac{w}{\theta_{LY}} \right)^{\theta_{LY}} \left( \frac{s}{\theta_{HY}} \right)^{\theta_{HY}} \]
where $\theta_{KX} + \theta_{LY} = 1$ and $\theta_{KY} + \theta_{LY} + \theta_{HY} = 1$. Totally differentiating the above unit cost functions, we have:

\begin{align*}
\hat{C}_w^X &= \theta_{KX} \left( \hat{\tau} - \hat{S}_{KX} \right) - (1 - \theta_{LY}) \hat{w} \\
\hat{C}_r^X &= -(1 - \theta_{KX}) \left( \hat{\tau} - \hat{S}_{KX} \right) + \theta_{LY} \hat{w} \\
\hat{C}_w^Y &= \theta_{KY} \left( \hat{\tau} - \hat{S}_{KY} \right) - (1 - \theta_{LY}) \hat{w} + \theta_{HY} \hat{s} \\
\hat{C}_r^Y &= -(1 - \theta_{KY}) \left( \hat{\tau} - \hat{S}_{KY} \right) + \theta_{LY} \hat{w} + \theta_{HY} \hat{s} \\
\hat{C}_s^Y &= \theta_{KY} \left( \hat{\tau} - \hat{S}_{KY} \right) + \theta_{LY} \hat{w} - (1 - \theta_{HY}) \hat{s}
\end{align*}

(A7) (A8)

These together imply:

\begin{align*}
\hat{k}_X &= \hat{C}_r^X - \hat{C}_w^X = \hat{w} - \hat{r} + \hat{S}_{KX} \\
\hat{k}_Y &= \hat{C}_r^Y - \hat{C}_w^Y = \hat{w} - \hat{r} + \hat{S}_{KY}
\end{align*}

These, together imply:

\begin{align*}
\hat{k}_X &= \hat{C}_r^X - \hat{C}_w^X = \hat{w} - \hat{r} + \hat{S}_{KX} \\
\hat{k}_Y &= \hat{C}_r^Y - \hat{C}_w^Y = \hat{w} - \hat{r} + \hat{S}_{KY}
\end{align*}

(A9)

In the conventional setup, the modified golden rule, (8), steady-state capital, (21), as well as the competitive profit conditions, (10) and (12), and the full employment conditions, (13)-(15), continue to hold. Moreover, the change in steady-state capital can be written as:

$$\bar{K} = \gamma \left( \hat{\tau} - \hat{T}_Y \right)$$

Denote $\Delta_{\theta} \equiv \theta_{LY} \theta_{KX} - \theta_{LY} \theta_{KY}$ and $\Delta_{\lambda} \equiv \lambda_{LY} - \lambda_{KX}$ which are, respectively, positive if sector $Y$ is capital-intensive in the cost and quantity sense. Totally differentiating competitive profit conditions yields:

\begin{align*}
\hat{w} &= \frac{1}{\theta_{LY}} \left( \hat{T}_X + \theta_{KX} \hat{S}_{KX} - \theta_{KX} \hat{\tau} \right) \\
\hat{s} &= \frac{1}{\theta_{LY} \theta_{HY}} \left( \theta_{LY} \hat{T}_Y - \theta_{LY} \hat{T}_X + \theta_{KY} \hat{S}_{KY} - \theta_{LY} \theta_{KX} \hat{S}_{KX} - \Delta_{\theta} \hat{\tau} \right)
\end{align*}

These, together imply:

\begin{align*}
\hat{k}_X &= \frac{1}{\theta_{LY}} \left( \hat{T}_X + \hat{S}_{KX} - \hat{\tau} \right) \\
\hat{k}_Y &= \frac{1}{\theta_{LY}} \left( \hat{T}_X + \theta_{KX} \hat{S}_{KX} - \hat{\tau} \right) + \hat{S}_{KY}
\end{align*}

Thus, the direct effects of more protection/subsidy to $X$ (ignoring $\hat{\tau}$ for the moments) are to raise unskilled wage and lower skilled wage. The direct effects of greater protection/subsidy to $Y$ (ignoring $\hat{\tau}$ for the moments) are to raise skilled wage (no direct effect on unskilled wage). Rental has negative effects on unskilled wage and on skilled wage if sector $Y$ is capital-intensive in the cost sense.

It is thus immediate to pin down the changes in capital-labor ratios:

\begin{align*}
\hat{\lambda}_X &= \frac{1}{\theta_{LY}} \left( \hat{T}_X + \hat{S}_{KX} - \hat{\tau} \right) \\
\hat{\lambda}_Y &= \frac{1}{\theta_{LY}} \left( \hat{T}_X + \theta_{KX} \hat{S}_{KX} - \hat{\tau} \right) + \hat{S}_{KY}
\end{align*}

Totally differentiating the full employment conditions and utilizing (A7), (A8) and (A9), we obtain:

\begin{align*}
\hat{Y} &= -\hat{C}_s^Y = \hat{s} - \hat{T}_Y \\
&= \frac{1}{\theta_{LY} \theta_{HY}} \left[ \theta_{LY} \left( 1 - \theta_{HY} \right) \hat{T}_Y - \theta_{LY} \hat{T}_X + \theta_{KY} \hat{S}_{KY} - \theta_{LY} \theta_{KX} \hat{S}_{KX} - \Delta_{\theta} \hat{\tau} \right]
\end{align*}
\[
\dot{X} = \hat{C}_w - \frac{1}{\lambda} \frac{\lambda}{\lambda} \left[ \dot{Y} + \hat{C}_w \right]
\]
\[
= \left( 1 - \frac{\lambda}{\lambda} \right) \frac{\lambda}{\lambda} \left[ \theta_{LY} + \left( 1 - \frac{\lambda}{\lambda} \right) \theta_{LY} \right] \frac{\lambda}{\lambda} \dot{T}_X - \left( 1 - \frac{\lambda}{\lambda} \right) \dot{Y}
\]
\[
+ \frac{\theta_{LY} + \left( 1 - \frac{\lambda}{\lambda} \right) \theta_{LY}}{\lambda} \dot{S}_{LY} - \left( 1 - \frac{\lambda}{\lambda} \right) \theta_{LY} \dot{S}_{LY} - \frac{\lambda}{\lambda} \dot{\theta}_{LY} \dot{r}
\]
\[
\dot{K} = \lambda \dot{X} \left( \dot{X} + \hat{C}_X \right) + \left( 1 - \frac{\lambda}{\lambda} \right) \left( \dot{Y} + \hat{C}_Y \right) = \gamma \left( \dot{r} - \dot{Y} \right)
\]
\[
= \frac{1}{\lambda} \left[ \lambda \dot{X} \dot{X} - \lambda \dot{X} \left( 1 - \frac{\lambda}{\lambda} \right) \dot{S}_{LY} + (\lambda - \lambda) \dot{S}_{LY} \right]
\]

or, by straightforward but tedious manipulations,
\[
\dot{r} = \frac{\theta_{LY} \dot{S}_{LY}}{\left( 1 + \frac{\gamma}{\lambda} \right) \lambda \dot{X} \dot{X} + \lambda \dot{X} \left( \dot{X} + \hat{C}_X \right) - \lambda \dot{X} \left( \dot{X} + \hat{C}_X \right) \dot{S}_{LY} - \Delta_\lambda \dot{S}_{LY}} (A10)
\]

where \( \Delta_\lambda \equiv \lambda \dot{X} \dot{X} - \lambda \dot{X} \left( \dot{X} + \hat{C}_X \right) \) is positive if sector \( Y \) is sufficiently capital-intensive in the quantity sense (which implies \( \Delta_\lambda > 0 \) but not vice versa). Notice that with \( \Delta_\lambda > 0 \), \( \frac{\dot{K}}{\dot{Y}} > 0 \) and \( \frac{\dot{r}}{\dot{Y}} > 0 \). Thus, the above results indicate that the direct effects of greater protection/subsidy to \( Y \) or less protection/subsidy to \( X \) are to raise \( Y \) and lower \( X \). Moreover, rental has negative effect on \( Y \) if \( Y \) is capital-intensive in the cost sense (\( \Delta_\theta > 0 \)) and negative effect on \( X \) if \( \theta_{LY} > \lambda \dot{X} \dot{X} \). Furthermore, if \( Y \) is sufficiently capital-intensive in the quantity sense (\( \Delta_\lambda > 0 \)), then \( r \) is increasing in the protection/subsidy to \( Y \) and decreasing in the protection/subsidy to \( X \). For the purpose of illustration, we call sector \( Y \) “strongly capital-intensive” if \( 0 < \Delta_\theta < \theta_{LY} / \lambda \dot{X} \) and \( \Delta_\lambda > 0 \).

Sectoral capital allocation gives:
\[
\dot{K}_Y = \dot{X} + \hat{C}_r = \left( \dot{X} - \dot{S}_{LY} \right)
\]
\[
= \frac{1}{\lambda} \left( \dot{X} - \dot{S}_{LY} \right)
\]
\[
\dot{K}_X = \dot{X} + \hat{C}_r = \frac{1}{\lambda} \left[ \dot{X} - \dot{S}_{LY} \right]
\]

Substituting (A10) into other expressions above and assuming a strongly capital-intensive
import-competing sector, we then arrive at: 

\[ \frac{\partial w}{\partial \tau} < 0, \frac{\partial r}{\partial \tau} > 0, \frac{\partial s}{\partial \tau} > 0, \frac{\partial X}{\partial \tau} > 0, \frac{\partial Y}{\partial \tau} > 0, \frac{\partial K_X}{\partial \tau} > 0, \frac{\partial K_Y}{\partial \tau} > 0, \frac{\partial k_X}{\partial \tau} > 0, \frac{\partial k_Y}{\partial \tau} > 0, \frac{\partial K_X}{\partial \eta} > 0, \frac{\partial K_Y}{\partial \eta} > 0, \frac{\partial k_X}{\partial \eta} > 0, \frac{\partial k_Y}{\partial \eta} > 0, \frac{\partial w}{\partial \sigma_Y} < 0, \frac{\partial r}{\partial \sigma_Y} > 0, \frac{\partial s}{\partial \sigma_Y} > 0, \frac{\partial X}{\partial \sigma_Y} > 0, \frac{\partial Y}{\partial \sigma_Y} > 0, \frac{\partial K_X}{\partial \sigma_Y} > 0, \frac{\partial K_Y}{\partial \sigma_Y} > 0, \frac{\partial k_X}{\partial \sigma_Y} > 0, \frac{\partial k_Y}{\partial \sigma_Y} > 0, \frac{\partial w}{\partial \sigma_X} > 0, \frac{\partial r}{\partial \sigma_X} < 0, \frac{\partial s}{\partial \sigma_X} < 0, \frac{\partial X}{\partial \sigma_X} < 0, \frac{\partial Y}{\partial \sigma_X} < 0, \frac{\partial K_X}{\partial \sigma_X} < 0, \frac{\partial K_Y}{\partial \sigma_X} < 0, \frac{\partial k_X}{\partial \sigma_X} < 0, \frac{\partial k_Y}{\partial \sigma_X} < 0 \]

It is important to note that, from the discussion above, these comparative-static results depend crucially on factor intensity rankings.

Social Planner’s Problem: The social planner’s problem can be expressed as maximizing the payoff of a skilled worker, subject to the resource constraint and the requirement that surplus labor receive its reservation value in the traded goods sector:

\[ \Omega(K_t) = \max_{c^s_{Y,t}, I_t, K_{Y,t}, L_{Y,t}} \left[ U \left( c^s_{Y,t} \right) \right] + \frac{1}{1+\rho} \Omega(K_{t+1}) \]

s.t. \[ c^s_{Y,t} + I_t + c^u_{Y,t} L_t = A \left[ F(K_{Y,t}, H) \right]^{1-\theta_L} L_{Y,t}^{\theta_L} + pB(K_t-K_{Y,t})^{\theta_K} (L_t-L_{Y,t})^{1-\theta_L} \]

\[ K_{t+1} = (1-\delta) K_t + \frac{L_t}{\mu(I)} \]

\[ c^u_{Y,t} = \bar{c} \]

This formulation assumes that the social costs of adjusting the capital stock to the optimal level equal the private costs from the household optimization problem, and that the social cost of labor to the traded goods sectors equals the rural wage rate. Under these assumptions, the optimal policy will involve free trade without factor market interventions (i.e., marginal revenue products of factors evaluated at world prices are equated across sectors, the marginal revenue product value of labor equals its reservation value in the rural sector, and the intertemporal marginal rate of substitution in consumption is equated to the cost of capital).

---

\[ ^9 \text{We could sign } \frac{\partial K_X}{\partial \sigma_Y} < 0, \frac{\partial K_X}{\partial \sigma_Y} > 0, \frac{\partial K_X}{\partial \sigma_Y} < 0, \text{ and } \frac{\partial K_X}{\partial \sigma_Y} > 0, \text{ if we further assume } \Delta_\theta < \frac{\theta_K \phi \mu}{1-\lambda_{LY}}. \]
Figure 1: Rural-Urban Migration Equilibrium

\[ \psi U(w/(1+\tau)) \]
Figure 2: Effects of an Increase in the Capital Subsidy to the Import Sector on Outputs and Urban Employment

(a) The Import Sector is More Capital Intensive

(b) The Export Sector is More Capital Intensive
Figure 3: Effects of an Increase in the Tariff on Outputs and Urban Employment

Figure 4: Raising Tariff from 10 to 20% Delay the Urbanization Process by 36 Years