Schumpeterian Growth, Trade, and Dynamic Comparative Advantage

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Abstract

We study the effects of trade on economic growth in a Schumpeterian framework. The model excludes scale effects and technology transfer, the two usual channels in the literature through which trade affects growth, leaving only comparative advantage. Comparative advantage and the trading pattern are determined endogenously. Endogeneity of production and trading patterns leads to results quite different from those found in most of the related literature. Trade need not increase initial output of either country because of an externality absent from static models. Irrespective of what happens to initial output, trade may increase the balanced growth rate but also may decrease it because of a trade-off between productive efficiency and R&D efficiency. Our model has tractable transition dynamics, which we describe completely. We show that trade leads to a stable world income distribution in some cases, but in other cases leads to an unstable and perhaps even degenerate distribution. In some cases, trade’s effect on a country’s growth rate is the same as if that country had adopted its trading partner’s R&D technology, even though no technology transfer ever occurs.
1 Introduction

In classic static models of international trade, trade increases the level of income by allowing countries to exploit their comparative advantages. It does not work through aggregate scale effects or technology transfer. The situation is quite the reverse in most of the endogenous growth literature, where, with few exceptions, trade affects growth through aggregate scale effects and technology transfer but not through comparative advantage. The situation is somewhat strange. It is well known that aggregate scale effects are rejected by the data, so they are an inappropriate channel for trade to affect growth. Technology transfer is a legitimate channel and even seems to be important, but it is not trade. It may be facilitated by trade, and indeed the evidence suggests that it is, but it is not trade. There is almost no discussion of the relation between the classic trade mechanism of comparative advantage and economic growth, and what little there is takes place in the context of first-generation growth models where the aggregate scale effect is present and mediates the effects of trade and comparative advantage. In this paper, we study the effects of trade on growth in a second-generation growth model whose structure excludes the aggregate scale effect and in which we have ruled out technology transfer by assumption. We find that trade affects growth solely through comparative advantage but in ways different from the previous small literature on this topic.

The existing literature links international trade to the growth rate primarily through two channels: the aggregate scale effect and technology transfer. In models with aggregate scale effects (which includes all first generation endogenous growth models), opening countries to trade increases the scale of the economies. The details of the scale increase vary with the specific structure of the model, but one way or another trade makes more productive resources available to every economy, effectively increasing broadly-defined total factor productivity and so raising the growth rate. See, for example, Barro and Sala-I-Martin (1997), Connolly (2000), and all other first-generation growth models with trade. In those models, it is not trade \textit{per se} that matters, only the size of the economy. Anything that increases that size raises the growth rate. Trade does that, so it also increases the growth rate. It is well known, however, that aggregate scale effects are inconsistent with the data. Backus, Kehoe, and Kehoe (1992) is the classic reference. The aggregate scale effect apparently is an inappropriate channel for trade to affect growth. In models of technology transfer, trade is a vehicle for the exchange of technological know-how. Trade opens countries to each other’s knowledge, helping each country learn and adopt the production techniques of its trading partners and so increasing its stock of knowledge. Because the rate of knowledge accumulation is treated as proportional to the stock of existing knowledge, the technology transfer enabled by trade raises the growth rates of all trading partners. Rivera-Batiz and Romer (1991), Howitt (2000) and Peretto(2003) present this mechanism, and Coe and Helpman (1995, 2008) provide evidence that it is a significant phenomenon. However, trade is not a necessary element in this channel. We can always have technology transfer without trade, as shown by Rivera-Batiz and Romer (1991). Technology transfer can foster growth, trade may facilitate technology transfer, but trade is not a necessary element.

A small literature uses a first-generation growth framework to examine comparative advantage as a channel through which trade can affect economic growth. The main result is that trade may raise or lower the world growth rate depending on the reallocation of resources that trade induces. Suppose, as in Grossman and Helpman (1990), that each country $i$ produces a set of goods $G_i$ comprising a single consumption good $Y_i$ unique to country $i$ and an infinite variety of intermediate goods $X_{1i}, X_{2i}, \ldots$, also unique to country $i$: $G_i = \{Y_i, X_{1i}, X_{2i}, \ldots\}$. The intersection of any two sets $G_i$ and $G_j$ is empty. At any moment, country $i$ knows how to produce $n_i$ of the intermediate goods in $G_i$. Country $i$ does R&D to expand the number of varieties of intermediate goods that it can produce. Intermediate goods are inputs only for the production of final goods. Labor is an input in all sectors and is the only input in the production of intermediate goods and R&D. Intermediate good $X_{ji}$ requires $a_{X_{ji}}$ units of labor per unit of $X_{ji}$ produced. R&D requires $a_{Ri}$ units of labor per unit of R&D undertaken. The ratio $a_{Ri}/a_{X_{ji}} = b_i$ is central to the effects of trade
on growth. Trade is introduced by allowing the two countries to trade consumption goods and intermediate goods. R&D output is not tradeable. Varieties of goods discovered by a country can be produced only in that country. Define comparative advantage in R&D as the relation between the two countries’ values of $b$: country 1 has a comparative advantage in R&D if and only if $b_1 > b_2$. Trade raises the world growth rate if it induces the country with the comparative advantage in R&D to move resources from the final goods and/or intermediate goods sectors into R&D. That need not be the effect of trade, however. It also is possible for trade to raise the demand of a country’s final goods or intermediate goods and thereby induce it to move resources out of R&D and into manufacturing. If that country happens to be the one with the comparative advantage in R&D, the world growth rate falls. See Grossman and Helpman (1991, 1995), Young (1991), and Galor and Mountford (2006) for extensions and variations. These results are interesting, but they suffer from three limitations. First, the patterns of production and trade do not depend on comparative advantage. Each country is endowed with a set of goods that it may develop and produce. No country may develop and produce goods that have been endowed upon another country. Every country trades every type of good it produces, irrespective of its comparative advantage relative to any other country. The meaning, then, of the term "comparative advantage" is very different from the standard usage. In fact, an exogenous pattern of production and trade is common to almost all the literature on trade and growth. Second, in this literature comparative advantage is defined in terms of goods that are assumed to be non-tradeable: the fruit of R&D, which is an increase in the number of varieties that a country has learned to produce. Third, all comparative advantage effects work through the same channel as the scale effect. Consequently, changing the model to eliminate the scale effect also may eliminate or at least alter the effects of trade on growth. That is what happens with taxation, so there is good reason to believe it may happen with trade.¹ In fact, as we shall see, it does happen in some cases.

Seater(2007), Arabshahi(2008), and Yenokyan(2009) provide a partial resolution of the problems with the original contributions on trade and growth by endogenizing the pattern of trade and making it a function of comparative advantage. They use a standard first-generation two-sector model (Barro and Sala-I-Martin, 2004, chapter 5) extended to two countries. Two goods are produced by two types of reproducible factors in Cobb-Douglas production functions. One sector produces a good $Y$ that can be used as consumption $C$ or as an investment good $K$ and the other sector produces another type of capital good $H$ that augments labor. Both $Y$ and $H$ are tradable. Comparative advantage is defined in terms of the relative prices of $Y$ and $H$, and comparative advantage determines the pattern of trade, that is, which goods a country exports and which it imports. Trade never reduces growth rates and in most cases raises them. The model can be reduced to one with only one reproducible factor, $H$, by dropping $K$ and making $Y$ be only consumption. It then can be shown that the effect of trade on a country’s growth rate depends on the type of good the country imports. If a country imports the consumption good $C$, its growth rate is the same under trade and autarky. In contrast, if a country imports the factor of production $H$, then its growth rate is higher with trade. When a pair of trading partners exchange two factors of production, such as $K$ and $H$, both have higher growth rates than under autarky. The result is intuitively sensible. Endogenous growth is driven by increasing the amounts of reproducible factors of production. If a country is less efficient at producing a factor of production than another country, then the first country can raise its growth rate by reducing its production of that factor and trading for it on the world market. Note that in this approach $H$ is not treated as human capital, which is not tradeable, but rather as a type of physical capital.²

¹Stokey and Rebelo (1995) criticize first-generation growth models because they predict large negative effects of income taxes that are not observed in the data. Peretto (2007) shows that a second-generation growth model, by eliminating the scale effect, also drastically changes the predictions concerning taxes, with some types of taxes predicted to increase the growth rate, some to have ambiguous effects, and some to reduce the growth rate.

²See Bond and Trask (1997) and Bond, Trask, and Wang (2003) for important early contributions that use the two-sector model to analyze trade and growth but that treat $H$ as non-tradeable human capital. They consequently obtain the same kind of one-sided effects of trade on growth as in Seater (2007), Arabshahi (2008), and Yenokyan (2009) for the case where only $C$ and $H$ are tradeable. Also, Bond and Trask (1997) and Bond, Trask, and Wang
The idea is that many kinds of machines have embodied in them characteristics that replace skill. The original textile machines of the Industrial Revolution, for example, apparently were like that:

[W]ith the marvelously perfect and self-acting machinery of today no special skill is required on the part of the attendant. *The machinery itself supplies the intelligence.*

Quoted by Clark (2007), emphasis added.

This argument is reasonable, but the mathematical treatment is a something of a shortcut and a weakness of the analysis. It would be desirable to distinguish between the physical machine itself and the qualities embodied in it. One would expect the quantity of capital to enter the production function in the usual way and the amount of embodied quality to enter separately as a factor augmenting labor in exactly the same way as the human capital it replaces. The other weakness of the analysis is that the model has an aggregate scale effect. Even though it is not the scale effect that causes the effects of trade on growth, mathematically trade is working through the same part of the model structure that gives rise to the scale effect. Consequently, reformulating the model to eliminate the scale effect may well change the way that trade affects growth.

We retain the ideas proposed by Seater(2007), Arabshahi(2008), and Yenokyan(2009), but we embed them in a second-generation endogenous growth model, thus eliminating the scale effect. We also distinguish between physical intermediate goods and the quality embodied in them. The goods are tradeable, and buying one of them also buys its labor-augmenting quality. Countries differ in their efficiency at producing goods and at doing R&D. The trade pattern is decided endogenously by the quality-adjusted price ratio of the tradeable goods. Opening a closed economy to trade can have surprising results. Trade always has a direct effect of increasing the output levels of the trading partners at the moment that trade opens. However, there is an indirect effect arising from an externality that in principle could be strong enough that trade actually reduces the output level of one or both partners. Irrespective of trade’s effect on current output levels, it may increase or reduce the growth rate of output, a result reminiscent of Grossman and Helpman (1990) but arising through a completely different mechanism. The effect of trade on the home country’s growth rate depends on its trading partner’s R&D efficiency in quality improvement of the good that the home country imports. The home country’s R&D efficiency in the good it exports is irrelevant to its growth rate.

In some cases, trade guarantees world balanced growth and hence a stable world income distribution. In other cases, however, growth rates differ across countries at least temporarily and possibly permanently. In those cases, the world income distribution is unstable. It is even possible for the world income distribution to degenerate because one country always has a higher growth rate than the other and the difference between the growth rates is asymptotically constant. In that case, the faster growing country’s share of world income goes to 1 asymptotically, not because the other country is shrinking but rather because the other country grows at a slower rate forever. Many of the results on world income distribution differ markedly from those presented by Acemoglu and Ventura (2002). We show that Acemoglu and Ventura’s analysis amounts to a special restricted case of ours.

We also show that trade may mimic technology transfer in its effect on growth. A country that imports an intermediate good that has high quality embodied in it may appear to be acquiring the technology of the country that manufactured the imported good, but in fact all the recipient country is acquiring is the quality embodied in the good. That distinction has not been made

(2003) restrict attention to a small open economy that takes world prices as given, whereas Seater (2007), Arabshahi (2008), and Yenokyan (2009) generalize the analysis to countries that may be large relative to each other and thus may affect world prices.
in the empirical literature, which as a result suffers from an "omitted variables" problem and attributes too much effect to technology transfer.

In the remainder of the paper, we first analyze the closed economy and then extend the analysis to an open economy to derive the effects of trade on growth.

2 The Closed Economy
The closed economy produces three types of goods: final, processed, and intermediate. Intermediate goods are combined with labor to produce processed goods. Processed goods are used to produce final goods. Final goods are used for consumption, as an input into the production of intermediate goods, and for research to improve the quality of the intermediate goods. Endogenous growth models usually have only final goods and intermediate goods. Adding the third type of good facilitates the analysis of trade, as we discuss later.

2.1 Final Goods
Identical competitive firms produce a single homogeneous final good $Y$ using two non-durable processed goods $X_1$ and $X_2$ as inputs. The production function for the representative firm is Cobb-Douglas:

$$Y = X_1^\epsilon X_2^{1-\epsilon}$$  \hspace{1cm} (1)

We take the final good as the numeraire, so $P_Y = 1$. The representative firm’s profit is

$$\pi_Y = Y - P_{X_1}X_1 - P_{X_2}X_2$$  \hspace{1cm} (2)

from which we obtain the indirect demand functions

$$P_{X_1} = \epsilon (X_2/X_1)^{1-\epsilon}$$  \hspace{1cm} (3)

$$P_{X_2} = \epsilon (X_1/X_2)^\epsilon$$  \hspace{1cm} (4)

where $P_{X_1}$ and $P_{X_2}$ are the prices of $X_1$ and $X_2$. See the Appendix for the complete derivation.

2.2 Processed Goods
The processed goods sector comprises two industries, each producing a single homogeneous good. Both industries are competitive in all markets. The representative firms in the two industries use non-durable intermediate goods $G$ and labor $l$ to produce their respective processed goods. Their production functions are:

$$X_1 = \int_0^{N_1} G_{1j}^\lambda \left(Z_{1j}^\delta Z_1^{1-(\delta+\gamma)} l_{1j} \right)^{1-\lambda} dj, \hspace{1cm} 0 < \lambda, \gamma, \delta < 1$$  \hspace{1cm} (5)

$$X_2 = \int_0^{N_2} G_{2j}^\lambda \left(Z_{2j}^\delta Z_2^{1-(\delta+\gamma)} l_{2j} \right)^{1-\lambda} dj, \hspace{1cm} 0 < \lambda, \gamma, \delta < 1$$  \hspace{1cm} (6)

where $G_{ij}$ is the amount of the intermediate good of type $(i,j)$ used in industry $i$, $Z_{ij}$ is the quality of good $G_{ij}$, $Z_i \equiv (1/N_i) \int_0^{N_i} Z_{ij} dj$ is the average quality of class-$i$ intermediate goods (explained momentarily), $l_{ij}$ is the amount of labor (number of workers) working with intermediate good $G_{ij}$, and $N_i$ is the number of varieties of intermediate goods used in each industry. There are two classes of intermediate goods, $(G_{1j})_{j=0}^{N_1}$ and $(G_{2j})_{j=0}^{N_2}$, with one class providing inputs for the $X_1$ industry and the other class providing inputs for the $X_2$ industry. Each intermediate good is
in one and only one class, so the sets of intermediate goods used by the \( X_1 \) and \( X_2 \) industries are disjoint and generally have different numbers of elements (i.e., in general \( N_1 \neq N_2 \)). Each intermediate good \( G_{ij} \) has its own quality \( Z_{ij} \), determined by the \( \text{R&D} \) that has been done by the firm that produces \( G_{ij} \). We discuss the industrial structure and \( \text{R&D} \) of the intermediate goods sector in the next section. Labor productivity depends on the quality of the intermediate good it works with. To allow for knowledge spillovers, we let labor productivity in industry \( X_1 \) depend on both the average quality \( Z_1 \) of the \( \{G_{1j}\} \) goods used in industry \( X_1 \) and the average quality \( Z_2 \) of the \( \{G_{2j}\} \) goods used in industry \( X_2 \). Industry \( X_2 \)'s situation is symmetric. The importance of knowledge spillovers is governed by the parameters \( \delta \) and \( \lambda \). Setting \( \delta + \lambda = 1 \) would exclude knowledge spillovers across industries. As discussed in the Introduction, the quality \( Z_{ij} \) of intermediate good \( G_{ij} \) is embodied in the good itself but augments the workers that use that good.

An example of this production system is the Shanghai Metro. Metro services are the final good \( Y \), produced by combining a control system \( X_1 \) and an operation system \( X_2 \). The control system \( (X_1) \) comprises many different types of computers \( (G_{1j}) \) that are used by different workers \( l_{1j} \). The workers’ productivity depends on quality \( Z_{ij} \) of the computer he uses, a spillover from other qualities in the control system, \( Z_{ik} \) with \( k \neq j \), and also possibly a spillover \( Z_2 \) from the operation system. The structure of the operation system \( X_2 \) is similar.

Processed goods firms choose quantities of intermediate goods and labor to maximize their profit:

\[
\max_{\{G_{ij}, l_{ij}\}} \pi_{X_i} = P_{X_i}X_i - \int_0^{N_i} P_{G_{ij}}G_{ij}dj - \int_0^{N_i} w_l dj
\]

(7)

where \( P_{G_{ij}} \) is the price of \( G_{ij} \), \( w \) is the wage rate, and the firm takes all prices as given. The demand functions for intermediate goods and labor are

\[
G_{1j} = \left( \frac{\lambda P_{X_1}}{P_{G_{1j}}} \right)^{\frac{1}{\gamma + 1}} Z_{1j}^{\delta} Z_2^{1-(\delta+\gamma)} l_{1j}
\]

(8)

\[
G_{2j} = \left( \frac{\lambda P_{X_2}}{P_{G_{2j}}} \right)^{\frac{1}{\gamma + 1}} Z_{2j}^{\delta} Z_1^{1-(\delta+\gamma)} l_{2j}
\]

(9)

\[
l_{1j} = \left( \frac{P_{X_1}}{w} \right)^{\frac{1}{\gamma + 1}} G_{1j} \left( Z_{1j}^{\gamma} Z_2^{1-(\delta+\gamma)} \right)^{\frac{1}{\gamma + 1}}
\]

(10)

\[
l_{2j} = \left( \frac{P_{X_2}}{w} \right)^{\frac{1}{\gamma + 1}} G_{2j} \left( Z_{2j}^{\gamma} Z_1^{1-(\delta+\gamma)} \right)^{\frac{1}{\gamma + 1}}
\]

(11)

2.3 Intermediate Goods

The intermediate goods sector, like the processed goods sector, comprises two industries distinguished by which processed goods industry buys their products, as explained above. We divide the discussion of the intermediate goods industries into two parts, the first describing the behavior of incumbents and the second the behavior of entrants.

2.3.1 Incumbents

Each intermediate goods industry comprises a continuum of monopolistically competitive firms. A firm produces a single intermediate good \( G_{ij} \) unique to that firm and also undertakes research and development (R&D) to improve the quality \( Z_{ij} \) of the good it produces. An increase in quality raises the demand for the good as shown above and thus raises profit.
Production, technologies, R&D technologies, and costs are the same for all firms within a given industry but differ across industries. The industrial structure thus is one of symmetry within each intermediate goods industry but asymmetry across the two industries. The element of asymmetry is unusual in Schumpeterian growth models, and we regard it as one of the contributions of our analysis. Asymmetry is useful for our discussion of international trade because it provides a natural division among goods along which comparative advantage may operate. Beyond that, however, it is a step toward a more realistic analysis than the usual framework of complete symmetry provides. Symmetry is analytically convenient, but it also is unrealistic. Industries typically do not all make the same decisions at the same time, so our industrial structure is more realistic than the standard imposition of universal symmetry. We would have liked to eliminate symmetry entirely, but analytical tractability required that we maintain it at the level of firms within each industry. Perhaps future researchers will succeed in eliminating the symmetry restriction altogether.

All firms in industry $i$ have a linear technology that converts $A_i$ units of the final good into one unit of intermediate good $G_{ij}$:

$$A_i G_{ij} = Y_{ij}$$

where $Y_{ij}$ is the amount of the final good used by firm $j$ in industry $i$. Similarly, the R&D production functions are the same within an industry but differ across the industries. Spending one unit of the final good on R&D in industry $i$ yields $\alpha_i$ units of quality improvement:

$$Z_{ij} = \alpha_i R_{ij}$$

where $R_{ij}$ is amount of the final good $Y$ spent on R&D. The firm obtains the resources for $R$ from retained earnings.$^3$ Firms face a fixed operating cost $\phi_{ij}$ that depends on the average quality of the firm’s own industry, $Z_i$, and of other industry $Z_k$. There are two channels of influence. First, the operating cost depends positively on own industry quality. A more sophisticated industry is more complex and requires more sophisticated inputs, so the demand for operating cost inputs is increasing in industry quality. On the reasonable assumption, commonly made in the literature, that the cost of producing those inputs rises with their sophistication, higher industry quality then implies a higher price for the factors that are used to run the firms’ operations. We borrow a page from the adjustment cost literature and assume that fixed operating costs are convex in the level of industry sophistication. Second, operating costs are reduced by advances in knowledge, which in our model is captured by quality. We suppose that all knowledge is useful in reducing operating costs, that is, that knowledge spillovers from both an intermediate goods firm’s own industry and also from the other industry lower operating costs. Thus both own industry knowledge $Z_i$ and other industry knowledge $Z_j$ help reduce costs. The general form of the operating cost function is thus $\phi_{ij} = \Phi_{ij} (Z_i; Z_i, Z_k)$ with $\Phi'_1 > 0, \Phi'_{11} > 0, \Phi'_2 < 0$, and $\Phi'_3 < 0$. To keep the analysis tractable, we assume that all firms in a given industry have the same cost function, which takes the analytically convenient form

$$\Phi_{ij} (Z_i; Z_i, Z_k) = \theta_1 \frac{Z_i^3}{Z_i Z_k} = \theta_2 \frac{Z_i^2}{Z_k}$$

The cubic term in the numerator captures the convexity of cost in complexity, and the two terms in the denominator capture the effect of knowledge in reducing costs. Thus we have

$$\phi_{1j} = \theta_1 \frac{Z_i^3}{Z_k} \equiv \phi_1; \quad \phi_{2j} = \theta_2 \frac{Z_i^2}{Z_i} \equiv \phi_2$$

(14)

Dependence of cost on only industry averages and not the firm’s own quality level is not restrictive because as we show later firms within a given industry behave symmetrically, so that each firm’s quality equals the average quality of the industry.

$^3$It would be slightly more precise to distinguish between investment $I$ and retained earnings $R$ because in principle the two need not be the same. However, the requirements of general equilibrium will make them the same, so we keep the notation simple by imposing $I = R$. 

7
The intermediate goods firm’s profit is revenue less production costs:

\[ F_{ij} = P_{ij} G_{ij} - A_i G_{ij} - \phi_i \]  

The firm retains some amount \( R_{ij} \) of its profit to invest in R&D and distributes the rest to its owners. Profit net of retained earnings is

\[ \Pi_{ij} = F_{ij} - R_{ij} \]  

The present discounted value \( V_{ij}(t) \) of this net profit is

\[
V_{ij}(t) = \int_t^{\infty} \Pi_{ij}(\tau) e^{-r(t) \tau} d\tau \\
= \int_t^{\infty} \left[ G_{ij} (P_{ij} - A_i) - \phi_i - R_{ij} \right] e^{-r(t) \tau} d\tau
\]  

The firm chooses the paths of its product price \( P_{ij} \) and its R&D expenditure \( R_{ij} \) to maximize (17) subject to the demand function (8), the R&D production function (13), and the average qualities, \( Z_1 \) and \( Z_2 \), which the firm takes as given.

Differentiating Eq.17 with respect to time gives the firm’s rate of return to equity (i.e., entry):

\[
r_{ij}^E = \frac{\Pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}}
\]  

which is the usual profit rate plus the capital gain rate.

### 2.3.2 Entrants

We assume that entry and exit are costless. For simplicity, we refer only to entry, even though exit also always is possible. We explored an extension of the model with costly entry, but we were unable to obtain closed-form solutions.\(^4\) Costless entry implies that \( N_i \) is a jumping variable. Whenever the net present value of a new firm \( V \) differs from the entry cost of zero, new firms jump in or out to restore equality between the value of the firm and the entry cost. We thus have at all times

\[ V_{ij} = 0 \]  

As a result, we also have \( \dot{V} = 0 \). Multiplying both sides of eq. (18) by \( V \) and imposing \( V = 0 \) and \( \dot{V} = 0 \) implies that

\[ \Pi_{ij} = 0 \]  

as in Peretto (Oct 1999). We make the usual assumption that new entrants arrive with the average level of quality in their industry. That assumption is not restrictive because, as we show below, the firms within each industry behave symmetrically and always have the same level of quality.

### 2.4 Households

The economy is populated by a representative household that supplies labor inelastically in a perfectly competitive market and purchases assets (corporate equity). We assume for simplicity that there is no population growth.\(^5\) The utility function of the representative household is

\[
U(t) = \int_t^{\infty} log(c) e^{-\rho t} d\tau
\]  

\(^4\)See Peretto (2007) for discussions of costly entry in a framework similar to ours.

\(^5\)Positive population growth would induce perpetual entry at the rate of population growth. Extra population would give rise to incipient profit from entry and so induce the entry necessary to keep the rate of return to entry at zero. No important results obtained below would change. In particular, there would be no scale effect because our model’s second-generation structure (simultaneous quality-improvement and variety expansion) nullifies it. See, for example, Peretto (2007).
where \( c \) is consumption per capita and \( \rho \) is the rate of time preference.

The only assets that the household can accumulate are firms that it owns. The household’s lifetime budget constraint therefore is

\[
0 = \int_0^\infty \left( \int_0^{N_1} \Pi_1 dj + \int_0^{N_2} \Pi_2 dj + wL - C \right) e^{-\int^t \not r(s) ds} dt
\]

(22)

where \( C \) is aggregate consumption and \( L \) is population. The intertemporal consumption plan that maximizes discounted utility (21) is given by the consumption Euler equation, which as usual can be written as

\[
r = \rho + \frac{\dot{C}}{C}
\]

(23)

### 2.5 General Equilibrium

We start with the intermediate goods firm. Condition (20), that instantaneous profit is zero, implies that firms pay no dividends but instead retain all earnings for investment in R&D. The household owners of the firm reap their return in the form of increasing consumption as R&D delivers higher quality and raises output. The optimal values for the prices \( P_{G_{ij}} \) and retained earnings \( R_{ij} \) are slightly different for the two intermediated goods industries. The derivations are straightforward manipulations of the first-order conditions for the firm’s maximization problem and so are relegated to the Appendix. The solutions for the prices are mark-ups over variable cost:

\[
P_{G_{1j}} = \frac{A_1}{\lambda} \equiv P_{G_1}
\]

(24)

\[
P_{G_{2j}} = \frac{A_2}{\lambda} \equiv P_{G_2}
\]

(25)

The intermediate goods producers in a given industry all charge the same price, so we can drop the firm subscript \( j \) and write the price for all intermediate goods in industry \( i \) as \( P_{Gi} \). Equality of prices implies from the demand function (8) that the quantities sold are a linear function (which is the same for all firms in industry \( i \)) of the amount of labor \( l_{ij} \) that the processed goods firms allocate to work with intermediate good \( G_{ij} \). The firm’s current-value Hamiltonian is

\[
CVH_{ij} = G_i(P_{Gi} - A_i) - \phi_i - R_{ij} + q_{ij}(\alpha_i R_{ij})
\]

where \( q_{ij} \) is the co-state variable. The Hamiltonian is linear in R&D expenditure, so the solution for investment expenditure \( R_{ij} \) is bang-bang:

\[
R_{ij} = \begin{cases} 
\infty & \text{if } 1/\alpha > q_{ij} \\
> 0 & \text{if } 1/\alpha = q_{ij} \\
= 0 & \text{if } 1/\alpha < q_{ij}
\end{cases}
\]

We rule out the first possibility of \( R_{ij} = \infty \) because it is inconsistent with market equilibrium. We also rule out the other corner solution, \( 1/\alpha < q_{ij} \), because it implies no economic growth, and we are interested here in the case where perpetual growth occurs. We thus have the interior solution

\[
\frac{1}{\alpha_i} = q_{ij}
\]

(26)

The left side of eq. (26) is the same for all \( j \), so all firms in industry \( i \) choose the same level of R&D, which we denote \( R_i \). That in turn implies from (15) that all firms in the industry have the same profit.
The Maximum Principle gives the necessary condition for the evolution of the co-state variable \( q_1 \), which we can rearrange as

\[
  r_{ij} = \frac{\partial F_{ij}}{\partial Z_{ij}} \frac{1}{q_{ij}} + \frac{\dot{q}_{ij}}{q_{ij}}
\]

This equation defines the rate of return to R&D (i.e., to quality innovation) \( r_{ij} \) as the percentage marginal revenue from R&D plus the capital gain (percentage change in the shadow price). Because \( \frac{1}{\alpha_i} = q_{ij} \), we also have \( \dot{q}_{ij}/q_{ij} = 0 \). As with intermediate goods prices, the expressions for the rates of return differ across the two industries. The rate of return for industry 1 is obtained by substituting (8), (15), (24), and (26) into (27):

\[
  r_{1j} = \delta \alpha_1 A_1 \frac{1 - \frac{1}{\lambda}}{\lambda} \left( \frac{\lambda P_{X_1}}{A_1 / \lambda} \right)^{\frac{1}{\gamma}} \left( \frac{Z_{1j}}{Z_2} \right)^{(\delta + \gamma) - 1} l_{1j}
\]

Following the same steps as in industry 1, we get the rate of return to R&D in industry 2:

\[
  r_{2j} = \delta \alpha_2 A_2 \frac{1 - \frac{1}{\lambda}}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2 / \lambda} \right)^{\frac{1}{\gamma}} \left( \frac{Z_{2j}}{Z_1} \right)^{(\delta + \gamma) - 1} l_{2j}
\]

At this point, we follow Peretto (Oct. 1999) and impose a simplification that avoids technical complications that have no effect on the analysis or results. We assume that (1) at the initial time all firms in industry \( i \) have the same level of quality \( Z_{ij} = Z_i \) and (2) new firms enter with the average quality level \( Z_i \) of the industry. These two assumptions lead directly to an equilibrium that is symmetric within each industry, with all firms in an industry always making the same decisions on pricing, R&D expenditures, and market size. All firms in industry \( i \) choose the same prices and sell the same quantity of goods, all of which have the same quality. Because the intermediate goods \( G_{ij} \) have the same price and quality, the processed goods industry allocates the same amount of labor to each of them. We henceforth drop the firm subscript except where clarity demands otherwise. As noted above, our model is not entirely symmetric. Firms in industry 1 are all alike, firms in industry 2 are all alike, but firms in industry 1 differ from firms in industry 2. This element of asymmetry increases the realism of the model and also plays an important role in determining the effect of international trade on economic growth.

The internal symmetry of each industry leads to simplified expressions for the rates of return:

\[
  r_{1j} = r_1 = \delta \alpha_1 A_1 \frac{1 - \frac{1}{\lambda}}{\lambda} \left( \frac{\lambda P_{X_1}}{A_1 / \lambda} \right)^{\frac{1}{\gamma}} \left( \frac{Z_1}{Z_2} \right)^{(\delta + \gamma) - 1} l_1
\]

\[
  r_{2j} = r_2 = \delta \alpha_2 A_2 \frac{1 - \frac{1}{\lambda}}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2 / \lambda} \right)^{\frac{1}{\gamma}} \left( \frac{Z_2}{Z_1} \right)^{(\delta + \gamma) - 1} l_2
\]

To determine the value of \( l_i \), we use the facts that the final goods and processed goods sectors both are competitive in all markets and have Cobb-Douglas production. The final goods sector pays a total compensation of \( \epsilon Y \) to the \( X_1 \) sector and \( (1 - \epsilon) Y \) to the \( X_2 \) sector. The \( X_1 \) and \( X_2 \) sectors in turn pay their workers \( (1 - \lambda) \) of their product, that is, \( (1 - \lambda) \epsilon Y \) and \( (1 - \lambda) (1 - \epsilon) Y \), respectively. However, we also can write total labor compensation in the industries as

\[
  \int_0^{N_i} w l_{ij} dj = w L_i
\]

where

\[
  L_i = \int_0^{N_i} l_{ij} dj
\]

See Appendix (2.1).
We then take the ratio of total compensation to labor in the two industries:

\[
\frac{wL_1}{wL_2} = \frac{L_1}{L_2} = \frac{(1 - \lambda)\epsilon Y}{(1 - \lambda)(1 - \epsilon)Y} = \frac{\epsilon}{1 - \epsilon}
\]  

(32)

Substituting \(L_2 = L - L_1\) and solving for \(L_1\) gives

\[
L_1 = \epsilon L
\]

\[
L_2 = (1 - \epsilon)L
\]

Finally, symmetry of employment implies that

\[
l_1 = L_1/N_1 = \epsilon L/N_1
\]

(33)

\[
l_2 = L_2/N_2 = (1 - \epsilon)L/N_2
\]

(34)

As noted above, entry drives instantaneous profit to zero and then keeps it there by forcing intermediate goods firms to retain all earnings and sink them into R&D. As a result, entry lasts only an instant and does not continue through time. Variety expansion thus is not a source of persistent growth in this model. In that regard, the model is similar to Peretto (Oct 1999).

The growth rates of quality innovation are

\[
g_1 = \frac{\dot{Z}_1}{Z_1} = \frac{\alpha_1 R_1}{Z_1} = \frac{\alpha_1 F_1}{Z_1} = \alpha_1 \left[ A_1 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_1}}{A_1/\lambda} \right)^{\frac{1}{1-\lambda}} \frac{(Z_1)}{(Z_2)}^{(\delta+\gamma)-1} l_{1j} - \theta_1 \frac{Z_1}{Z_2} \right]
\]

(35)

\[
g_2 = \frac{\dot{Z}_2}{Z_2} = \frac{\alpha_2 R_2}{Z_2} = \frac{\alpha_2 F_2}{Z_2} = \alpha_2 \left[ A_2 \frac{1 - \lambda}{\lambda} \left( \frac{\lambda P_{X_2}}{A_2/\lambda} \right)^{\frac{1}{1-\lambda}} \frac{(Z_2)}{(Z_1)}^{(\delta+\gamma)-1} l_{2} - \theta_2 \frac{Z_2}{Z_1} \right]
\]

(36)

These growth rates depend positively on \(L_i/N_i\), which is individual firm size, not with \(L_i\) itself, which is related to population size. That distinction is one of the main differences between second-generation growth models like ours and first-generation models. In our model, an increase in \(L_i\) raises demand by the processed goods sector for intermediate goods and thereby raises profit of the existing intermediate goods firms. The increase in profit induces entry of new firms and raises \(N_i\) to keep \(L_i/N_i\) constant. An increase in population will not cause an increase in growth rate. The scale effect of first-generation models is absent here. That is why we could make population growth positive without affecting anything important.

With the solutions for the intermediate goods sector in hand, we can solve the rest of the model. As noted earlier, the prices \(P_{Gi}\) determine the quantities \(G_i\) from the demand equations (8) and (9). The values of the \(Z_i\) are the solutions to the differential equations (35) and (36) subject to the initial values of the \(Z_i\). We describe those solutions below. We also have derived the labor allocations \(l_{ij} = L_i/N_i\). We use those solutions to solve the processed goods sector’s production functions (5) and (6) for \(X_1\) and \(X_2\). We then substitute the solutions for \(X_1\) and \(X_2\) into the final goods sector’s production function (1) to get \(Y\) and into the indirect demand functions for processed goods (3) and (4) to get the prices \(P_{X_1}\) and \(P_{X_2}\). Using the solutions for \(P_{X_1}, P_{X_2}, X_1, \) and \(X_2\), we can write the rates of return in (30), (31), (35) and (36) entirely as functions of parameters, the state variables \(Z_1\) and \(Z_2\), and the number of firms in each intermediate goods industry \(N_1\) and \(N_2\).
We have two remaining unknowns: \( N_1 \) and \( N_2 \). We get one equation for determining them by imposing the no arbitrage condition that rates of return must be equal, which allows us to set the two expressions on the right sides of (30) and (31) equal to each other and using our previous results to substitute functions of \( N_1 \) and \( N_2 \) for \( P_{X_1}, P_{X_2}, l_1, \) and \( l_2 \). The remaining equation is the Euler equation, but to use it we first must look at the amounts of income earned in each sector.

Competitive firms and Cobb-Douglas production together imply that processed goods industries 1 and 2 receive payments of \( \epsilon Y \) and \((1 - \epsilon) Y\), respectively, so \( P_{X_1} X_1 = \epsilon Y \) and \( P_{X_2} X_2 = (1 - \epsilon) Y\), where \( P_{Y_i} \) is the price of good \( Y_i \) in terms of the final good. The processed goods industries in turn are competitive with Cobb-Douglas production. The intermediate goods firms in class 1 (i.e., those in the set \( \{G_{ij}\} \)) together receive a total payment of \( \lambda P_{X_1} X_1 = \lambda Y \), and the workers in processed goods industry 1 receive total compensation of \( w L_1 = (1 - \lambda) P_{X_1} X_1 = (1 - \lambda) \epsilon Y \). Similarly, the payments to intermediates and workers employed in processed goods industry 2 are \( \lambda P_{X_2} X_2 = \lambda (1 - \epsilon) Y \) and \( w L_2 = (1 - \lambda) P_{X_2} X_2 = (1 - \lambda) (1 - \epsilon) Y \). Total compensation paid to intermediate good producers and labor is \( N_1 G_1 P_{G_1} + N_2 G_2 P_{G_2} = \lambda Y \) and \( w (L_1 + L_2) = (1 - \lambda) Y \). Quality \( Z_{ij} \) does not get paid directly from the final good sector. The return to \( Z_i \) is generated indirectly by increasing the demand for the intermediate \( G_{ij} \) in which it is embodied, as shown in equations (8) and (9).

The household budget constraint (22) together with the zero profit condition (20) \( \Pi_i = 0 \) implies that \( w L = C \). Thus households consume all their wage income and save (through retained earnings) all their dividend income. We have just shown that \( w L = (1 - \lambda) Y \), so the ratio between consumption and output is constant at the value \( C/Y = 1 - \lambda \). That means that consumption \( C \) always grows at the same rate as income \( Y \). Combining the demand functions of intermediate goods (8) and (9 in the final good production (1) to eliminate \( G_i \), we get

\[
Y = \kappa Z_1^G Z_2^{1-G} L
\]

where \( \kappa = \lambda^{\lambda/(1-\lambda)}(1-\epsilon)^{(1-\epsilon)/(1-\lambda)} e^{(1-\lambda)} P^{-\lambda e/(1-\lambda)} P_{G_1}^{-\lambda/(1-\lambda)} P_{G_2}^{-\lambda/(1-\lambda)} e^\epsilon (1-\epsilon)^{(1-\epsilon)} \), and \( \Gamma = 2(\delta + \gamma) \epsilon - (\delta + \gamma) - \epsilon + 1 \in (0,1) \). The growth rate of \( Y \) is a weighted average of the growth rates of the \( Z_i \):

\[
\dot{Y} \over Y = \Gamma g_1 + (1 - \Gamma) g_2
\]

The growth rates \( g_1 \) and \( g_2 \) are given by equations (35) and (36) and are functions of model parameters, the current values of the state variables \( Z_1 \) and \( Z_2 \), and of \( N_1 \) and \( N_2 \). Thus the Euler equation also provides an equation in the two unknowns \( N_1 \) and \( N_2 \), giving us the second equation that we need to solve for \( N_1 \) and \( N_2 \).

### 2.6 Balanced Growth Path

On the balanced growth path, the growth rates of \( Z_1 \) and \( Z_2 \) are equal, and the ratio \( Z_1/Z_2 \) is constant. Then the following growth rate are equal:

\[
g^* = \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} = \frac{\dot{Y}}{\overline{Y}} = \frac{\hat{C}}{\overline{C}} = \frac{\dot{X}_1}{X_1} = \frac{\dot{X}_2}{X_2} = \frac{\dot{G}_1}{G_1} = \frac{\dot{G}_2}{G_2} = \frac{\dot{w}}{\overline{w}}
\]

We get the quality ratio \( (Z_1/Z_2)^* \) on the balanced growth path by noting that \( g_1 = g_2 = g^*, \)

\[
r_1 = r_2 \equiv r, \text{ and, from the Euler equation, } r = g^* + \rho.
\]

From those relations, we obtain (see the Appendix) the following quadratic form:

\[
\alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right)^2 - \alpha_2 \theta_2 = 0
\]

\(^7\text{Note that, in contrast to the Solow growth model, constancy of the consumption ratio is an endogenous outcome rather than a restriction imposed \textit{a priori}.}\)
The two roots are

\[
\begin{align*}
\left( \frac{Z_1}{Z_2} \right)^* &= \sqrt{\frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}} > 0 \\
\left( \frac{Z_1}{Z_2} \right)^* &= -\sqrt{\frac{\alpha_2 \theta_2}{\alpha_1 \theta_1}} < 0
\end{align*}
\]

We discard the negative solution because it is economically meaningless, so the balanced growth rate is

\[
g^* = \frac{\delta}{1 - \delta} \sqrt{\alpha_1 \theta_1 \alpha_2 \theta_2} - \frac{1}{1 - \delta} \rho
\]  \hspace{1cm} (41)

The growth rate depends positively on the R&D productivities \(\alpha_1\) and \(\alpha_2\) and on the fixed operating cost parameters \(\theta_1\) and \(\theta_2\). The higher the productivity of R&D, the higher the return to R&D, which implies a higher growth rate. The higher the fixed operating cost, the lower the profit for incumbents and thus the smaller the number of firms in the market, which drives up the market size of firms \(L_i/N_i\). From eqs. (28) and (29) we see that the larger the market size, the higher return in R&D. Consequently, the growth rate is positively related to fixed operating cost. Peretto (Oct 1999) obtains the same result for the same reason. Define \(\alpha_1 \theta_1\) and \(\alpha_2 \theta_2\) as “R&D ability” for industries 1 and 2, respectively. Then we see from (41) that the economy’s growth rate is positively related to the R&D abilities of the two industries.

The growth rate is unrelated to unit costs of production, \(A_1\) and \(A_2\). A change in unit costs has two opposite effects that exactly cancel. One effect is a positive “direct effect”: eq (28) shows that a decrease in unit costs directly causes an increase in the return of R&D and hence also in the growth rate. The other effect is a negative “indirect effect”: a decrease in unit cost causes a higher incipient profit and induces entry, which reduces firm size \(L_i/N_i\) and hence reduces the return to R&D. These two effects cancel, so the growth rate is not affected by unit costs. The fact that unit costs do not affect growth rates is important later in understanding why trade does not guarantee a higher growth rate. The trade pattern is determined by the quality-adjusted price ratio, which does depend on unit costs, but the growth rate depends only on R&D ability, which does not depend on unit costs. A country may end up importing a good with a very low unit cost but also with a low associated R&D ability, thus leading to a decrease in the country’s growth rate.

2.7 Transition Dynamics

Our model permits a full characterization of the economy’s transition dynamics. The no-arbitrage condition requires the jumping variables \(N_1\) and \(N_2\) to adjust instantly to equalize the returns to R&D across the two processed goods industries. Thus the right sides of (30) and (31) are always equal. Combining the growth rates of qualities (35) and (36) gives

\[
\begin{align*}
\frac{Z_1}{Z_2} &= \frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2} = g_1 - g_2 = \frac{r_1}{\delta} - \alpha_1 \theta_1 \left( \frac{Z_1}{Z_2} \right) - \left[ \frac{r_2}{\delta} - \alpha_2 \theta_2 \left( \frac{Z_2}{Z_1} \right) \right]
\end{align*}
\]  \hspace{1cm} (42)

Imposing equality of rates of return and multiplying through by \(Z_1/Z_2\) gives

\[
(Z_1/Z_2) = -\alpha_1 \theta_1 (Z_1/Z_2)^2 + \alpha_2 \theta_2
\]  \hspace{1cm} (43)

The steady state is the value of \(Z_1/Z_2\) that makes \((Z_1/Z_2) = 0\), which is equivalent to making \((Z_1/Z_2)^2 = (Z_1/Z_2) = 0\), which in turn is equivalent to making \(\dot{Z}_1/Z_1 = \dot{Z}_2/Z_2\). Setting \((Z_1/Z_2) = 0\) in (43) and rearranging gives the quadratic form

\[
(Z_1/Z_2)^2 = \alpha_2 \theta_2 / \alpha_1 \theta_1
\]
One root is $\sqrt{\alpha_2 \theta_2 / \alpha_1 \theta_1} > 0$, which is stable. When $Z_1/Z_2 > \sqrt{\alpha_2 \theta_2 / \alpha_1 \theta_1}$, eq. (43) implies $(Z_1/Z_2) < 0$, so $Z_1$ decreases relative to $Z_2$, and $Z_1/Z_2$ returns to $\sqrt{\alpha_2 \theta_2 / \alpha_1 \theta_1}$. When $0 < Z_1/Z_2 < \alpha_2 \theta_2 / \alpha_1 \theta_1$, eq. (43) implies $(Z_1/Z_2) > 0$ so that $Z_1$ rises relative to $Z_2$ and again $Z_1/Z_2$ returns to $\sqrt{\alpha_2 \theta_2 / \alpha_1 \theta_1}$. See Figure 1. The other root is $-\sqrt{\alpha_2 \theta_2 / \alpha_1 \theta_1} < 0$ and is unstable. Being negative, it is of no direct economic interest because returns to equilibrium point where $Z_1/Z_2$ cannot be negative. However, the instability of the negative root does imply that at points above it (i.e., less negative) the economy moves even farther above, that is, toward the positive quadrant. Again, see Figure 1. Thus the positive root is the unique attractor for all points in the positive quadrant and so is the unique globally stable equilibrium value for the quality ratio $Z_1/Z_2$. At that equilibrium ratio, $Z_1$ and $Z_2$ grow at the same rate, so $Z_1/Z_1 = Z_2/Z_2 = g^*$ in equilibrium. From (38), $Y/Y = g^*$. One can work through the rest of the model’s equations to establish that (41) holds, so the stable equilibrium point where $Z_1/Z_1 = Z_2/Z_2$ is indeed the balanced growth path (steady state) of the economy is stable.

3 The Open Economy

We now introduce international trade. There are two countries, home and foreign. They have the same production functions for the final good $Y$ and for the processed goods $X_1$ and $X_2$. They also have the same utility functions. The countries differ in their intermediate goods sectors, having different production productivities $A_1$ and $A_2$, R&D productivities $\alpha_1$ and $\alpha_2$, and fixed operating costs $\phi_1$ and $\phi_2$. We assume that only intermediate goods $G_{ij}$ are tradeable. The two countries are “large” in the sense that their actions affect world prices. The more common small open economy assumption then becomes a special case in which certain effects disappear because prices do not respond to a country’s actions. To avoid complications arising from strategic behavior, we suppose that neither economy is able to exercise monopoly power. That assumption is consistent with a model in which each country comprises a multitude of agents who are not able to form a cartel to act as monopolists or monopsonists. There is no technology transfer, and for simplicity there also is no foreign investment.

As before, we have two classes of intermediate goods, $\{G_{1j}\}$ and $\{G_{2j}\}$. To keep the model tractable, we assume that a country either can produce all of one class of intermediates or can import all of that class. It cannot produce some varieties within a class and import others. However, which if either class it chooses to import and which to export will be determined by comparative advantage, not imposed a priori. In that regard, the model differs from most of the small literature on trade and growth in which comparative advantage plays a role. In the rest of literature, countries always trade all varieties they produce and where the allocation of the sets of varieties that each country produces is exogenous. See Grossman and Helpman (1990) or Acemoglu and Ventura (2002), for example.

In our model, growth is driven by quality improvement, so for trade to affect growth, it must affect a country’s rate of improvement in quality. Such a possibility arises in our model because of the effect that trade has on the goods that a country produces. Comparative advantage pushes countries in the direction of specializing in producing a subset of goods and trading with other countries for the goods in which those countries have specialized. Once a country stops producing a type of good, it also stops doing R&D to improve that good’s quality and instead accepts whatever quality improvements are made in the country that does produce the good. Quality improvement of the good in question shifts from the home country to its trading partner, which has a different efficiency of R&D than the home country. As we will see, under trade, the home country’s growth rate is an average of the domestic and foreign rates of technical progress rather than being an average of just domestic technical progress, so in general trade alters growth rates. The change in the growth rate can be positive or negative. The formal derivation of that result follows in
the sections below, but the intuitive argument is that productive efficiency determines the pattern of trade and production, whereas the growth rate is determined by R&D efficiency, which has no necessary relation to production efficiency. Thus it may be optimal to cease domestic production of a good because a trading partner can produce it more cheaply (where cheapness will be defined precisely below) even though that same trading partner is less efficient at improving the quality of the good. Thus trade may reduce growth rates. The reduction may be optimal because everyone may see an increase in current output with trade but a slower growth rate of future output. Furthermore, because of an externality that quality has on industry production functions, it even is possible that opening a country to trade lowers that country’s current output. Finally, even though there is no technology transfer in the model, trade in goods leads to outcomes that may be identical to those that would emerge from technology transfer.

3.1 Trade Patterns

When the home country is open, processed goods firms have more options for obtaining intermediate goods. They can buy domestic goods, foreign goods, or both. Consider the production function for processed good $X_1$ produced at home:

$$X_{H1} = \int_0^{N_{H1}} (G_{H1j} - G_{H1j}^E) \lambda \left[ Z_{H1j}^{\delta} Z_{H1}^{\gamma} (\overbrace{Z_{H2}}^{1-(\delta+\gamma)} l_{H1H}) \right]^{1-\lambda} dj$$

$$+ \int_0^{N_{F1}} (G_{F1k}^I) \lambda \left[ Z_{F1k}^{\delta} Z_{F1}^{\eta} (\overbrace{Z_{H2}}^{1-(\delta+\gamma)} l_{H1F}) \right]^{1-\lambda} dk$$

The subscripts $H$ and $F$ denote the home and foreign countries, respectively. $X_{H1}$ is output of type-1 processed goods by the home country. $G_{H1j}$ is the quantity of intermediate good 1j produced in the home country, $G_{H1j}^E$ is the amount of $G_{H1j}$ exported, and $G_{H1j} - G_{H1j}^E$ is the amount of $G_{H1j}$ not exported and used by the home country processed goods firms. $G_{F1k}^I$ is the amount of intermediate good 1k imported from the foreign country. $Z_{H1}$ is the average quality of domestic intermediate goods in class 1. As in the closed economy, symmetry within the industry ensures that all individual quality levels are the same and so equal to the average level. $Z_{F1}$ is the quality of foreign in class 2. $N_{H1}$ and $N_{F1}$ are the numbers of firms in industry 1 of home and foreign countries. Finally, $l_{H1H}$ and $l_{H1F}$ are the quantities of labor assigned to work with the domestic and foreign intermediate goods. Note that only one of $G_{H1j}^E$ and $G_{F1k}^I$ can be positive at any time because we are restricting a trading country either to export all of one class of intermediate goods or to import it. If, for example, $G_{H1j}^E$ is positive, then $G_{F1k}^I$ is zero, $G_{H2j}^E$ also is zero, and $G_{F2k}^I$ is positive.

As before, $\overbrace{Z_{H2}}$ is the knowledge spillover from the class-2 intermediates. The difference now is that the class-2 goods $G_2$ may be of either domestic or foreign origin, so we must keep track of which quality is in play. We assume that $\overbrace{Z_{H2}}$ equals the quality $Z_{H2}$ of home-produced $G_2$ goods if the home country’s $X_2$ industry uses only domestically produced types of $G_2$, that $\overbrace{Z_{H2}}$ equals the quality $Z_{F2}$ of foreign-produced $G_2$ goods if the home country’s $X_2$ industry uses only foreign produced types of $G_2$, and that $\overbrace{Z_{H2}}$ equals a geometric weighted average of domestic and foreign qualities $Z_{H2}$ and $Z_{F2}$ if the home country’s $X_2$ industry uses both domestically produced and foreign produced types of $G_2$:

$$\overbrace{Z_{H2}} = \begin{cases} 
Z_{H2} & \text{only home-produced } G_2 \text{ used} \\
Z_{H2}^{\eta} Z_{F2}^{1-\eta} & \text{both home- and foreign-produced } G_2 \text{ used} \\
Z_{F2} & \text{only foreign-produced } G_2 \text{ used}
\end{cases}$$
where $0 < \eta < 1$.

Processed goods firms in industry 1 in the home country choose the combination of intermediate good 1 to buy from domestic firms and foreign firms to maximize profit:

$$
\max \pi_{X_1} = P_{XH1}X_{H1} - \int_0^{N_1} P_{G_{H1j}}(G_{H1j} - G_{H1j}^E) dj - \int_0^{N_F1} P_{G_{H1Fj}}G_{F1k}^I dk
- \int_0^{N_1} w_l H_1 dj - \int_0^{N_1} w_l H_1 F dk
$$

where the choice variables are $G_{H1j} - G_{H1j}^E$, $G_{F1k}^I$, $l_{H1H}$, and $l_{H1F}$, and choice is subject to the trade balance constraint mentioned above. Solving for labor demand and plugging into the first-order conditions for $G_{H1j} - G_{H1j}^E$ and $G_{F1k}^I$ yields a bang-bang solution in which processed goods firms buy only $G_{H1j} - G_{H1j}^E$ or only $G_{F1k}^I$. The processed good firm buys $G_{H1j} - G_{H1j}^E$ or $G_{F1k}^I$ according to which has the lower quality adjusted price: $P_{G_{H1}}/Z_{H1}^{(\delta+\gamma)(1-\lambda)/\lambda}$ and $P_{G_{F1}}/Z_{F1}^{(\delta+\gamma)(1-\lambda)/\lambda}$, respectively. The situation for producers of $X_2$ is similar and need not be discussed.

International trade takes place if each country has a comparative advantage in selling a good. In our model, comparative advantage means that each country has a lower quality-adjusted price for one class of intermediate goods:

$$
\frac{P_{G_{H1}}}{Z_{H1}^{(\delta+\gamma)(1-\lambda)/\lambda}} \leq \frac{P_{G_{F1}}}{Z_{F1}^{(\delta+\gamma)(1-\lambda)/\lambda}}, \quad \text{and} \quad \frac{P_{G_{H2}}}{Z_{H2}^{(\delta+\gamma)(1-\lambda)/\lambda}} \geq \frac{P_{G_{F2}}}{Z_{F2}^{(\delta+\gamma)(1-\lambda)/\lambda}} \tag{44}
$$

or the reverse. The direction of the inequalities determines which goods are exported and imported. That direction is inconsequential to our results, so we suppose hereafter that the equalities are in the direction shown in (44). Because the quality adjusted price does not depend on the quantity bought, a country will buy all of an intermediate good from whoever has the lower quality adjusted price. That means that if the home country decides to buy a class of intermediate goods from the foreign country, the home country will stop producing that class of intermediate goods and so will specialize in production. Countries specialize completely if strict inequality holds in (44), which means under our imposed direction of the inequalities that the home country specializes in intermediate good 1 and the foreign country specializes in good 2. Each country stops producing one class of intermediate goods and devotes all its energy to producing the other class. With weak inequality, a country may not fully specialize, meaning that it may import a good but also continue to make it at home. We discuss complete and incomplete specialization in more detail below.

Let the final good in the home country be the numeraire: $P_{Y_H} = 1$. The price of the final good in the foreign country is $P_{Y_F}$. Recall that the price of the intermediate good equals the monopolistic markup times unit cost, so $P_{G_{H1}} = A_{H1}/\lambda$, $P_{G_{H2}} = A_{H2}/\lambda$, $P_{G_{F1}} = P_{Y_F} A_{F1}/\lambda$, and $P_{G_{F2}} = P_{Y_F} A_{F2}/\lambda$. Using these facts, we see that the comparative advantage condition (44) is equivalent to

$$
\frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^{(\delta+\gamma)(1-\lambda)/\lambda} \geq P_{Y_F} \geq \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^{(\delta+\gamma)(1-\lambda)/\lambda} \tag{45}
$$

The price $P_{Y_F}$ must be in this closed interval because otherwise condition (44) would be violated and both countries would try to export the same good, implying a market disequilibrium. If we ignore the quality ratios and look at the unit cost ratio only, then inequality (45) is a typical trade condition for Ricardian-type model. In basic static Ricardian model, labor is the only factor of

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See the Appendix for details.
production for tradeable goods, and the relative wage across countries must be inside an interval defined by the unit cost ratios. In our model, it is not final goods but rather intermediate goods that are traded, which are produced from the final goods. So the relevant interval is defined by the productivity ratios. The difference between (45) and the standard expression is that, in traditional static Ricardian model, the productivity ratios are in terms of the unit cost ratios only, which are constant, whereas the productivity ratios in our model include not only the unit cost ratios but also the quality ratios.

We can arrange (45) into another typical version of the statement of comparative advantage:

$$A H_2 Z_{H_2}^{(b+\gamma)(1-\lambda)} \leq \frac{A H_2}{Z_{H_2}} \leq \frac{A F_1}{Z_{F_1}} \leq A F_1 Z_{F_1}^{(b+\gamma)(1-\lambda)}$$

(46)

The home country has a lower quality-adjusted production cost ratio (intermediate good 1 over good 2), so it specializes in intermediate good 1.

Notice that our model differs from most others that have studied the effect of trade on growth through the channel of comparative advantage, such as Grossman and Helpman (1990) and Acemoglu and Ventura (2002), in that the pattern of trade - that is, which goods a country imports and exports - is determined endogenously rather than being imposed a priori.

3.2 Complete Specialization

When condition (45) holds with strict inequality, the home country has a strictly lower quality-adjusted price in intermediate good 1, and foreign country has a strictly lower price in good 2. Both countries completely specialize in the class of goods in which they have a comparative advantage. The final goods industry is competitive and has a Cobb-Douglas production function, so the final good producer in the home country pays compensation \((1 - \epsilon)\lambda Y_H\) to the producers of intermediate industry 2, which are foreign firms. Similarly, the final good industry in the foreign country pays compensation \(\epsilon L_P Y_F\), measured with the final good price from the home country, to the intermediate producers of industry 1, which are firms in the home country. Trade balance thus requires \(1 \cdot Y_H (1 - \epsilon) \lambda = P_{Y_F} \cdot Y_F \epsilon \lambda\). This condition can be rewritten (see Appendix 5.3) as

\[P_{Y_F} = \left[\frac{(1 - \epsilon) L_H / \epsilon L_F}{1 - \lambda}\right]^{1 - \lambda}\]

which indicates the relative value of all goods imported over the value of those exported by the home country. We therefore can write the condition for complete specialization as

$$\frac{A H_2}{A F_2} \left(\frac{Z_{F_2}^{(b+\gamma)(1-\lambda)}}{Z_{H_2}}\right) > \left[\frac{(1 - \epsilon) L_H}{\epsilon L_F}\right]^{1 - \lambda} > \frac{A H_1}{A F_1} \left(\frac{Z_{F_1}^{(b+\gamma)(1-\lambda)}}{Z_{H_1}}\right)$$

(47)

This expression means the relative population size must be inside a certain interval, which depends on the initial quality ratios at the moment of opening to trade, and the unit costs. If the relative population size is too high or too low, then complete specialization does not occur. Intuitively, one country is too small to service all of the other country’s needs. Note that it is not just the population sizes that matter but rather the population sizes weighted by the elasticities \(\epsilon\) and \((1 - \epsilon)\) of final good output with respect to the two processed goods inputs and by the elasticity \((1 - \lambda)\) of processed goods with respect to augmented labor. We discuss the interpretation of the expression \(\left[\frac{(1 - \epsilon) L_H / \epsilon L_F}{1 - \lambda}\right]^{1 - \lambda}\) below when we turn to incomplete specialization.

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9 Appendix section (5.7) gives a detailed comparison between our measure of comparative advantage and that in Dornbush, Fisher and Samuelson (1977).
3.2.1 Level effect

Trade affects both the level and growth rate of final output. We discuss the level effect first and then turn to the growth rate in the next section. Under complete specialization, the home country abandons industry 2 of the intermediate goods sector, and the foreign country abandons industry 1 of that sector. Once production of a line of intermediate goods has been stopped, there is no value in doing R&D to improve its quality, so R&D in that line of goods stops when production stops. As a result, \(Z_{H2}\) and \(Z_{F1}\) stop growing, but \(Z_{H1}\) and \(Z_{F2}\) continue to grow. That widens the price interval within which complete specialization occurs, by (47). So, if the world economy starts in a position of complete specialization, it stays there forever.

Complete specialization is equivalent to an integrated economy with \(G_1\) produced by the technologies from the home country and \(G_2\) produced by the technologies from foreign country. The home country produces only intermediate good 1 and sells it at the price \(P_{GH1} = A_{H1}/\lambda\), and it imports intermediate good 2 at the price \(P_{YF}A_{HF2}/\lambda\).\(^{10}\)

Following similar steps as in the closed economy, we get the home country’s final output under autarky and trade (see the Appendix):

\[
Y_{H}^{\text{Autarky}} = \kappa_H^{'} \left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{GH1}} \right) Z_{H2}^{1-(\delta+\gamma)} \epsilon L_H \right]^{\epsilon} \left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{GH2}} \right) Z_{H1}^{1-(\delta+\gamma)} (1 - \epsilon) L_H \right]^{1-\epsilon} \quad (48)
\]

\[
Y_{H}^{\text{Trade}} = \kappa_H^{'} \left[ \left( \frac{Z_{F1}^{\delta+\gamma}}{P_{GF1}} \right) Z_{F2}^{1-(\delta+\gamma)} \epsilon L_H \right]^{\epsilon} \left[ \left( \frac{Z_{F2}^{\delta+\gamma}}{P_{GF2}} \right) Z_{H1}^{1-(\delta+\gamma)} (1 - \epsilon) L_H \right]^{1-\epsilon} \quad (49)
\]

where \(\kappa_H^{'} = \lambda^{\lambda/(1-\lambda)}(1-\epsilon)^{(1-\epsilon)/(1-\lambda)}\epsilon^{\lambda\epsilon/(1-\lambda)}\). From the derivation of \(Y\) in the closed economy, we can see that

\[
\left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{GH1}} \right) Z_{H2}^{1-(\delta+\gamma)} \epsilon L_H \right]^{\epsilon}
\]

is the contribution from \(X_{H1}\) to \(Y_H\), in which \(Z_{H2}\) is the spillover from \(X_{H2}\) and \(\epsilon L_H\) is labor employed in industry 1, and

\[
\left[ \left( \frac{Z_{H1}^{\delta+\gamma}}{P_{GH2}} \right) Z_{H1}^{1-(\delta+\gamma)} (1 - \epsilon) L_H \right]^{1-\epsilon}
\]

is the contribution from \(X_{H2}\) to \(Y_H\) in autarky, in which \(Z_{H1}\) is the spillover from \(X_{H1}\) and \((1 - \epsilon) L_H\) is labor employed in industry 2.

Upon comparing (48) and (49), we see that trade affects final output through two channels. The first channel is the quality-adjusted price \(P_{GH2}^{\lambda/(1-\lambda)} Z_{F2}^{\delta+\gamma}\). Imports have a lower quality-adjusted price, so substituting the imported \(G_{F2}\) for the domestically produced \(G_{H2}\) yields higher quality for the same expenditure and thus greater output of \(Y\). Trade has a positive effect on output through this channel. The second channel is the spillover to industry 1 through \(Z_{F2}^{1-(\delta+\gamma)}\), which is an externality. Firms in a given processed goods industry base their decision on which intermediate goods to buy on the quality-adjusted prices. What they ignore is that their decision of which good to buy determines the knowledge spillover in the other processed goods industry. Consider what happens when a firm in the \(X_2\) industry decides to use foreign intermediate goods because they have a lower quality adjusted price than domestic goods. The knowledge spillover to the \(X_1\) industry is \(\hat{Z}_{H2} = Z_{F2}\). That spillover is an externality, and it can be either positive or negative, depending on whether \(Z_{F2} > Z_{H2}\) or \(Z_{F2} < Z_{H2}\). The fact that the foreign goods have a

\(^{10}\)See derivations in Appendix section (5.2).
lower quality adjusted price does not imply that they have a higher quality level than the domestic good. The quality level $Z_{F2}$ of the foreign good can be below the quality level $Z_{H2}$ of the domestic good, and the quality adjusted price of the foreign good can still be less than the quality adjusted price of the domestic good if the foreign good’s unadjusted price $P_{G,F}$ is sufficiently below the unadjusted price $P_{G,H}$ of the domestic good. Trade does not change the intermediate goods prices, which still are given by the mark-up rules $P_{G,F1} = A_{F1}/\lambda$ and $P_{G,H1} = A_{H1}/\lambda$. Thus, a sufficiently large difference between production efficiency in the two countries (i.e., between $A_{H1}$ and $A_{F1}$) can make the quality adjusted price for foreign goods lower than the quality adjusted price for domestic goods even if the absolute quality level of the foreign goods is lower than the absolute quality level of the domestic goods. Thus the externality can be either positive or negative depending on whether $Z_{F2} > Z_{H2}$ or $Z_{F2} < Z_{H2}$. Note that the externality operates across different industries in a given country. There is no externality across countries: if one processed good industry in the home country imports a good with a lower quality level than the domestic alternative, the home country’s other processed good industry suffers a negative externality, but there is no externality in the foreign country.

This externality leads to the surprising and apparently novel result opening trade between two countries immediately may reduce the level of income in one or both countries. This possibility is not the result of any effect of trade on growth, which we have not discussed yet, but rather is purely a possible impact effect of opening two countries to trade. The externality comes into play the moment that trade starts. If the externality is negative, output in the processed goods industry affected by it will fall in response to the economy opening to trade. If the externality is sufficiently strong, output of final goods $Y$ will fall in response to the opening of trade. There also is nothing to prevent both countries from experiencing a negative externality, so it is possible for trade to reduce output in both countries.

The trade pattern in (44) implies \((P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma)} > Z_{H2}/Z_{F2}\). We have the following results:

1. The necessary and sufficient conditions for trade to decrease initial output level are $Z_{H1} > Z_{F2}$ and

\[
\left(\frac{P_{G,H2}}{P_{G,F2}}\right)^{\lambda/(1-\lambda)(\delta+\gamma)} > Z_{H2}/Z_{F2} > \left(\frac{P_{G,H2}}{P_{G,F2}}\right)^{\lambda/(1-\lambda)(\delta+\gamma) - 2(\delta+\gamma)/2} > 1
\]

This condition means that imports have such a low quality that the externality is high enough to reduce the final output. If $(\delta + \gamma) \to 1$, which means the spillover effect from the other industry, $[1 - (\delta + \gamma)]$, is close to 0, then $(P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma) - 2(\delta+\gamma)/2} > (P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma)}$. Thus the necessary and sufficient condition may be unlikely to hold.

2. The sufficient condition for trade to increase initial output level is $Z_{H2} < Z_{F2}$, which means the quality of imports is higher than quality of the domestic goods.

3. The necessary conditions for trade to increase initial output are

(a) $Z_{H2} < Z_{F2}$

or

(b) $Z_{H2} > Z_{F2}$ and $(P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma)} > (P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma) - 2(\delta+\gamma)/2} > Z_{H2}/Z_{F2} > 1$, where $Z_{F2}$ is not low enough to make the negative externality large enough to dominate the direct positive level effect.

---

\[\text{Note that the externality operates across different industries in a given country. There is no externality across countries: if one processed good industry in the home country imports a good with a lower quality level than the domestic alternative, the home country’s other processed good industry suffers a negative externality, but there is no externality in the foreign country.}

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1. $Z_{H1} > Z_{F2} \Rightarrow Z_{H2}/Z_{F2} > 1$, so according to the trade pattern condition, $(P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma)} > Z_{H2}/Z_{F2} > 1$, and $\lambda/(1-\lambda)(\delta+\gamma) > 0$. Thus $P_{G,H2}/P_{G,F2} > 1$. Appendix (5.4.2) shows $(P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma)} > (P_{G,H2}/P_{G,F2})^{\lambda/(1-\lambda)(\delta+\gamma) - 2(\delta+\gamma)/2}$ given $[1 - (\delta + \gamma)] > 0$ by assumption.
### 3.2.2 Growth Rate Effect

When the two countries completely specialize, their balanced growth rates are the same with exactly the same structure as in the closed economy:

\[
g_H = g_F = \frac{\delta}{1 - \delta} \sqrt{\alpha_{H1}^{\theta_{H1}} \alpha_{F2}^{\theta_{F2}}} - \frac{1}{1 - \delta} \rho \tag{50}
\]

We thus have a world balanced growth rate. The derivation follows the same steps as for the closed economy, so we relegate it to the Appendix. We already know that the growth rate is positively related to R&D abilities, \(\alpha_{11}\theta_{11}\) and \(\alpha_{21}\theta_{21}\), where \(i\) is the country. After complete specialization, the home country abandons production of the whole set of domestic intermediate goods 2 and imports them from the foreign country. Domestic R&D for improving the quality of the \(G_2\) goods ends when production ends, and all quality improvement in the \(G_2\) goods takes place abroad.

After trade, the total market size increases for each intermediate goods industry and unit costs change. However, neither of these changes affects the world balanced growth rate because entry of new firms absorbs them. For example, after trade, the firm size in industry 1 in home country is (see the Appendix):

\[
\left( N_{H1}^* \right)_{\text{Trade}} = \frac{L_H}{\alpha_{H1} \delta A_{H1}^{1 - \lambda} \left( \frac{\lambda^2 \epsilon}{A_{H1}} \right)^\frac{1}{\gamma - x} \left( \frac{\epsilon}{1 - \epsilon} \right)^\frac{1}{1 - \delta} \left( \frac{A_{H1}}{A_{F2}} \right)^{\frac{\lambda(1 - \lambda)}{\gamma - x}} \left( \frac{\sqrt{\alpha_{F2}^{\theta_{F2}}}}{\alpha_{H1}^{\theta_{H1}}} \right)^{\gamma - 1}}
\]

and in autarky it is

\[
\left( N_{H1}^* \right)_{\text{Autarky}} = \frac{\epsilon L_H}{\alpha_{H1} \delta A_{H1}^{1 - \lambda} \left( \frac{\lambda^2 \epsilon}{A_{H1}} \right)^\frac{1}{\gamma - x} \left( \frac{\epsilon}{1 - \epsilon} \right)^\frac{1}{1 - \delta} \left( \frac{A_{H1}}{A_{H2}} \right)^{\frac{\lambda(1 - \lambda)}{\gamma - x}} \left( \frac{\alpha_{H2}^{\theta_{H2}}}{\alpha_{H1}^{\theta_{H1}}} \right)^{\gamma - 1}}
\]

When the home country opens to trade, intermediate good producers in industry 1 face not only the domestic market but also the foreign market. Their total market size increases from \(\epsilon L_H\) to \(L_H\). If the number of firm did not change, then the market size of incumbent firms would increase and so would the incipient profit. Thus zero-cost entry instantaneously causes \(N_{H1}\) to increase to prevent the incipient profit from being realized. The larger total market size does not affect the return to R&D and hence does not affect the growth rate, either. That is how endogenous entry eliminates the scale effect. Similarly, entry absorbs any effect of the change in unit cost induced by trade. The only channel through which trade affects the growth rate is R&D ability.

Comparative advantage, which is determined by the quality-adjusted price ratio, does not guarantee that the home country imports the good with a higher R&D ability. Indeed, condition (47) shows R&D ability is irrelevant to the determination of the trade pattern, which depends only on unit costs and the initial value of qualities at the moment that trade opens. Conversely, the factors that determine the trade pattern have nothing to do with determining the growth rate. Equation (50) shows that the balanced growth rate depends only on R&D abilities and not on either unit costs or initial qualities. It therefore is possible that, when trade opens, the home country imports the good with a lower quality-adjusted price and a higher initial quality but also with a lower R&D ability. In that case, trade increases output’s initial level but decreases its growth rate, which means a decrease in the future output relative to what it would have benn with no trade.

As mentioned above, when trade opens, a country might import a good with a worse quality than the domestic product but also with a much lower unit cost so that the quality-adjusted price of imports still beats that of the domestic product. As a result, trade could decrease the initial
level of output through the externality effect. However, that does not necessarily decrease the balanced growth rate because the "worse quality" is only the initial level at the moment that trade starts. As long as the country is importing a good with a higher R&D ability, it gets a higher balanced growth rate. R&D ability depends only on R&D productivity $\alpha_{ij}$ and the fixed operating cost parameters $\theta_{1j}$ and $\theta_{2j}$, not the initial level of quality.

In light of the foregoing results, it obviously is even possible for trade to make a country worse off in both its initial level and growth rate of output. If the home country imports a good with a sufficiently low initial quality level, then trade decreases initial output through the externality effect. If the home country’s trade partner has a lower R&D ability in producing the good that the home imports, then the home country also suffers a lower growth rate.

### 3.2.3 Transition dynamics

When the two countries satisfy the condition for complete specialization but are not on the balanced growth path, the growth rates of their incomes are equal:

$$g = \frac{\dot{Y}_i}{Y_i} = \Gamma \frac{\dot{Z}_{H1}}{Z_{H1}} + (1 - \Gamma) \frac{\dot{Z}_{F2}}{Z_{F2}}, \text{ where } i = H, F$$

(51)

where the constant $\Gamma = (2\epsilon - 1)(\delta + \gamma) + 1 - \epsilon$. The formal derivation is in the Appendix, but the intuition is straightforward. When the two countries are completely specialized, each does R&D to improve the qualities of one of the two sets of intermediate goods. Each country imports the good that it does not produce. Consequently, each country uses the same sets of intermediate goods, one made at home and one made abroad, and so enjoys the same quality improvements. As a result, their growth rates are the same weighted average of the growth rates of the two qualities $Z_{H1}$ and $Z_{F2}$. We will see later that this property does not hold in the case of incomplete specialization.

We can see how the growth rate of income converges to it balanced growth path by examining time path of the quality ratio $Z_{H1}/Z_{F2}$, whose growth rate is:

$$\frac{(Z_{H1}/Z_{F2})}{Z_{H1}/Z_{F2}} = g_{H1} - g_{F2} = \frac{r_{H1}}{\delta} - \alpha_{H1}\theta_{H2} \frac{Z_{H1}}{Z_{F2}} - \left(\frac{r_{F2}}{\delta} - \alpha_{F2}\theta_{F2} \frac{Z_{F2}}{Z_{H1}}\right)$$

which is similar to the growth rate of the closed economy’s quality ratio $Z_1/Z_2$, given by equation (42). The foregoing expression contains the rates of return $r_{H1}$ and $r_{F2}$. Our model does not include international investment, so the no-arbitrage condition doesn’t hold across countries. Nonetheless, the rate of return to R&D is the same in the two countries. Using the same reasoning as for the closed economy, we can show that each country’s growth rate of consumption equals its growth rate of income. We have just seen that the latter growth rate is the same in the two countries, so the two countries also have the same growth rate of consumption. Then from the Euler equation for each country, we obtain the result that the rates of return $r_{H1}$ and $r_{F2}$ are equal:

$$g = \frac{\dot{c}_i}{c_i} = r_i - \rho, \text{ where } i = H, F$$

The transition dynamics of the qualities under complete specialization thus are similar to the case under the closed economy and we have

$$\frac{(Z_{H1}/Z_{F2})}{Z_{H1}/Z_{F2}} = -\alpha_{H1}\theta_{H1} \left(\frac{Z_{H1}}{Z_{F2}}\right)^2 + \alpha_{F2}\theta_{F2}$$

(52)

The positive root $\sqrt{\alpha_{F2}\theta_{F2}/\alpha_{H1}\theta_{H1}}$ is stable, and the economy converges monotonically to its balanced growth path. Along the transition path, various growth rates are changing but the growth rates of income in the two countries always equal each other.
3.3 Incomplete specialization

Balance of trade requires that the relative price \( P_{Y_F} \) (actually, \( P_{Y_F}/1 \)) to be inside the closed interval given in condition (45) because otherwise both countries would try to export the same good. Condition (47) shows that when the quantity \( [(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda} \) is inside that interval, \( P_{Y_F} \) will be equal to it. However, there is no reason that \( [(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda} \) need be inside the interval. If it is outside the interval, then \( P_{Y_F} \) cannot equal it and will be at whichever boundary is closest to \( [(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda} \). In that case, we have a corner solution. One of the two countries will be completely specialized, producing only one class of intermediates and trading for the other.

In contrast, its trading partner will not be completely specialized but instead will produce both types of intermediate goods. Which country specializes depends on which boundary has been hit. Without loss of generality, we assume that

\[
\left[\frac{(1 - \epsilon) L_H}{\epsilon L_F}\right]^{1-\lambda} > \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda} > \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda}
\]

In that case, \( P_{Y_F} \) "tries" to equal \( [(1 - \epsilon) L_H / \epsilon L_F]^{1-\lambda} \) and so hits the upper bound of the interval:

\[
\left[\frac{(1 - \epsilon) L_H}{\epsilon L_F}\right]^{1-\lambda} > \frac{A_{H2}}{A_{F2}} \left( \frac{Z_{F2}}{Z_{H2}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda} = P_{Y_F} > \frac{A_{H1}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{H1}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda}
\]

We can see what that implies by rearranging the terms in the second part of the inequality as

\[
\frac{A_{H2}}{A_{H1}} \left( \frac{Z_{H1}}{Z_{H2}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda} = P_{Y_F} > \frac{A_{F2}}{A_{F1}} \left( \frac{Z_{F1}}{Z_{F2}} \right)^\frac{(\delta + \gamma)(1-\lambda)}{\lambda}
\]

The expressions at the left and right extremes are the relative prices of the two classes of intermediate goods that would prevail under autarky in the home and foreign country, respectively. When \( P_{Y_F} \) equals the left term, the world price equals the autarkic home price, indicating that the home country derives no price advantage from importing either good from abroad. In contrast, \( P_{Y_F} \) is larger than the autarkic foreign price, so the foreign country still finds it advantageous to specialize in intermediate good 2 and import good 1 from the home country. The equilibrium is one in which the foreign country completely specializes in class-2 intermediates, and the home country produces both goods but also imports class-2 intermediates. In effect, the foreign country is not "big" enough to satisfy the home country’s requirement for class-2 intermediates, i.e., \( (1 - \epsilon) L_H / \epsilon L_F \) is too high relative to \( (A_{H2}/A_{F2})(Z_{F2}/Z_{H2})^{(\delta + \gamma)(1-\lambda)/\lambda} \) at the moment that trade opens. The term \( (1 - \epsilon) L_H / \epsilon L_F \) can be large for two reasons. First, the home country’s population can be large relative to that of the foreign country. This is a straightforward relative size effect. As mentioned earlier, it means that the foreign country is simply too small to meet the demands of the home country, so that the home country finds it worthwhile to continue producing intermediate good 2. Note that this is not a scale effect. Increasing the size of the two countries’ populations equiproportionally leaves everything unchanged, whereas shifting population from one country to another can move the world from the interior of the critical interval to the boundary (or vice versa) even if world population as a whole is unchanged. What matters is the relative size of the home country, not the absolute size. The effect is akin to, though different from, the "market size effect" that Acemoglu and Zilibotti (2001) discuss in relation to cross-country productivity differences. Second, the elasticities \( \epsilon \) and \( 1 - \epsilon \) also play a role in determining whether the world is inside the critical interval or at the boundary. Intuitively, if intermediate good 2 gets a heavy weight (i.e., has a high value of \( 1 - \epsilon \)) in the final good production function, the home country may find it worthwhile to continue producing that good even though the foreign country has specialized in it. The final determination of whether the world is in the interior of the critical region or on its boundary depends on the interaction of the relative population sizes and the production elasticities.
3.3.1 Level Effect

At the moment trade opens, for home country, the output level effect is similar to but not exactly the same as that under complete specialization. The home country imports class-2 intermediates and hence gets $Z_{F_2}$ from the foreign country, which causes an externality to the home country’s processed goods industry $X_1$. Because the home country uses two types of intermediate good 2, the spillover to industry 1 is a combination of the two quality levels $Z_{H_2}$ and $Z_{F_2}$. As mentioned previously, we assume the combination is the weighted average $\hat{Z}_{H_2} = Z_{H_2}^\eta Z_{F_2}^{1-\eta}$. The home country’s final output under autarky and trade are (see the Appendix):

\[
Y_{H}^{\text{Trade}} = \kappa' \left[ \left( \frac{Z_{H_1}^{(\delta+\gamma)}}{P_{G_{H_1}}} \right) \left( \hat{Z}_{H_2} \right)^{1-(\delta+\gamma)} (\epsilon L_H) \right]^{1-\epsilon} \left[ \left( \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}} \right) Z_{H_1}^{1-(\delta+\gamma)} (1 - \epsilon) L_H \right]^{1-\epsilon}
\]

\[
Y_{H}^{\text{Autarky}} = \kappa' \left[ \left( \frac{Z_{H_1}^{(\delta+\gamma)}}{P_{G_{H_1}}} \right) (Z_{H_2})^{1-(\delta+\gamma)} (\epsilon L_H) \right]^{1-\epsilon} \left[ \left( \frac{Z_{F_2}^{(\delta+\gamma)}}{P_{G_{F_2}}} \right) Z_{H_1}^{1-(\delta+\gamma)} (1 - \epsilon) L_H \right]^{1-\epsilon}
\]

where $\kappa' = \lambda^{1/1-\lambda} (1 - \epsilon)^{3/1-\epsilon} \lambda^{\epsilon/1-\lambda}$. Equilibrium requires that the domestic and foreign quality-adjusted prices $Z_{H_2}^{(\delta+\gamma)}/P_{G_{H_2}}$ and $Z_{F_2}^{(\delta+\gamma)}/P_{G_{F_2}}$ be the same. Thus at the moment that trade begins, the second term in the expressions for final output with and without trade are the same for the home country. The difference is the first term. In autarky, the spillover from industry 2 to industry 1 is $Z_{H_2}^{1-(\delta+\gamma)}$, whereas after trade, it is $\left( \hat{Z}_{H_2} \right)^{1-(\delta+\gamma)} = \left( Z_{H_2}^\eta Z_{F_2}^{1-\eta} \right)^{1-(\delta+\gamma)}$. Thus the necessary and sufficient condition for trade to increase initial output level for home is $Z_{F_2} > Z_{H_2}$, and the necessary and sufficient condition for trade to decrease initial output is the reverse, $Z_{F_2} < Z_{H_2}$. The level effect for the home country depends only on the externality because the home imports goods with the same quality adjusted price as the domestic goods. The foreign country specializes in class-2 intermediates and imports good 1 from the home country. Thus the level effect is exactly like the effect under complete specialization. On the one hand, comparative advantage ensures a lower quality-adjusted price for the imports, and on the other hand, the quality of the imports can have a positive or negative externality on the other industry. The necessary and sufficient conditions for positive and negative changes in final output are exactly the same as under complete specialization.

3.3.2 Growth Rate Effect

Following steps similar to those for the case of complete specialization, we obtain the balanced growth rate under incomplete specialization:\(^\text{12}\)

\[
g^{\text{Trade}} = \frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} = \frac{Z_{H_1}}{Z_{H_1}} = \frac{Z_{H_2}}{Z_{H_2}} = \frac{Z_{F_2}}{Z_{F_2}} = \frac{C_H}{C_H} = \frac{C_F}{C_F} = r - \rho
\]

\[
= \frac{\delta}{1-\delta} \sqrt{\alpha_{H_1} \theta_{H_1} (\alpha_{H_2} \theta_{H_2})^\eta (\alpha_{F_2} \theta_{F_2})} \left( \frac{\alpha_{F_2} \theta_{F_2}}{\alpha_{H_2} \theta_{H_2}} \right) - \frac{\rho}{1-\delta}
\]

Along the balanced growth path, the two quality ratios $Z_{H_1}/Z_{H_2}$ and $Z_{H_2}/Z_{F_2}$ are

\[
\left( \frac{Z_{H_1}}{Z_{H_2}} \right)^* = \left( \frac{\alpha_{F_2} \theta_{F_2}}{\alpha_{H_2} \theta_{H_2}} \right) \left( \frac{\alpha_{F_2} \theta_{F_2}}{\alpha_{H_2} \theta_{H_2}} \right)^\eta
\]

\[
\left( \frac{Z_{H_2}}{Z_{F_2}} \right)^* = \frac{\alpha_{F_2} \theta_{F_2}}{\alpha_{H_2} \theta_{H_2}}
\]

\(^\text{12}\)See section (6) of the Appendix for the details.
Recall that $\eta$ and $(1 - \eta)$ are the weights of $Z_{H2}$ and $Z_{F2}$ in the spillover from industry 2 to industry 1, which is $\hat{Z}_{H2} = Z_{H2}^{\eta}Z_{F2}^{1-\eta}$.

Comparing equation (57) with the autarky growth rate given in equation (41), we see the following effects of trade under incomplete specialization:

1. The home country’s growth rate increases if the foreign country has a higher R&D ability in the good that the home country imports, i.e., if $\alpha_{F2}^{1}\theta_{F2} > \alpha_{H2}^{1}\theta_{H2}$. The effect of $\alpha_{F2}^{1}\theta_{F2}$ is small if $(1 - \eta)$ is small.

2. The foreign country’s growth rate increases if the home country has a higher R&D ability in the good that the foreign country imports, i.e., if $\alpha_{H1}^{1}\theta_{H1} > \alpha_{H2}^{1}\theta_{H2}$ or $\alpha_{H2}^{1}\theta_{H2} > \alpha_{F2}^{1}\theta_{F2}$. The home country’s R&D ability in class-2 intermediates enters the growth rate of the foreign country, even though the foreign country does not import that good. The reason is that $Z_{H2}$ affects the accumulation of $Z_{H1}$ which is embodied in the $G_{H1}$ good that the foreign country does import.

### 3.3.3 Transition Dynamics

As in the case of complete specialization, the growth rates of home and foreign income along the transition path are weighted averages of the quality level growth rates:

\[
\frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} = \Gamma \frac{Z_{H1}}{Z_{H1}} + \eta [1 - (\delta + \gamma)] \epsilon + (\delta + \gamma)(1 - \epsilon) \frac{Z_{H2}}{Z_{H2}} + [(1 - \eta)(1 - (\delta + \gamma))] \eta \frac{Z_{F2}}{Z_{F2}}
\]

\[
\frac{Y_{F}^{\text{Trade}}}{Y_{F}^{\text{Trade}}} = \Gamma \frac{Z_{H1}}{Z_{H1}} - (\delta + \gamma) \epsilon \frac{Z_{H2}}{Z_{H2}} + [(1 - (\delta + \gamma))] \epsilon + \delta \frac{Z_{F2}}{Z_{F2}}
\]

where as before $\Gamma = (2\epsilon - 1)(\delta + \gamma) + 1 - \epsilon$. Comparing these growth rates with the corresponding growth rates under complete specialization given in equation (51) reveals two notable differences.

First, the income growth rates under incomplete specialization are weighted averages of the three quality growth rates $Z_{H1}/Z_{H1}$, $Z_{H2}/Z_{H2}$, and $Z_{F2}/Z_{F2}$, whereas under complete specialization they are weighted averages of just two, $Z_{H1}/Z_{H1}$ and $Z_{F2}/Z_{F2}$. Second, under incomplete specialization the income growth rates are different weighted averages of the quality growth rates, whereas under complete specialization they are the same. Consequently, the transitional income growth rates under incomplete specialization generally differ from each other, whereas they are the same under complete specialization. Using (57) and (58), we get the difference between the two income growth rates:

\[
\frac{Y_{H}^{\text{Trade}}}{Y_{H}^{\text{Trade}}} - \frac{Y_{F}^{\text{Trade}}}{Y_{F}^{\text{Trade}}} = [\eta - \eta(\delta + \gamma)](\delta + \gamma) \left( \frac{Z_{H2}}{Z_{H2}} - \frac{Z_{F2}}{Z_{F2}} \right)
\]

where $\eta \epsilon - \eta(\delta + \gamma) + (\delta + \gamma) > 0$. The income growth rates differ whenever the growth rates of $Z_{H2}$ and $Z_{F2}$ differ.

To study the behavior of the economy on the transition path, we need the equations for the growth rates of the quality levels $Z_{H1}$, $Z_{H2}$, and $Z_{F2}$. As before, these can be expressed as functions of the respective rates of return to R&D $r_{H1}$, $r_{H2}$, and $r_{F2}$:

\[
g_{H1}^{1} = \frac{Z_{H1}}{Z_{H1}} = \frac{r_{H1}}{\delta} - \alpha_{H1}^{1}\theta_{H1} Z_{H2}^{\eta} Z_{F2}^{1-\eta}
\]

\[
g_{H2}^{1} = \frac{Z_{H2}}{Z_{H2}} = \frac{r_{H2}}{\delta} - \alpha_{H2}^{1}\theta_{H2} Z_{H2}^{\eta} Z_{H1}^{1-\eta}
\]

\[\text{The derivation is in the Appendix.}\]
Combining (63) with (61) and (62) gives the growth rate of $u$ and $w$

\[ \text{where the steady state time derivatives of } u \text{ and } w \text{ are:} \]

\[ u = 0 \]

\[ w = \_ \]

Given steady state values of $u$ and $w$, so the evolution of the world economy is described by the evolution of $u$ and $w$. Taking time derivatives of $u$ and $w$ and using (64) and (65) gives

\[ \dot{u} = -\alpha_{H1} \theta_{H1} u^2 w^{1-\eta} + \alpha_{H2} \theta_{H2} u \]

\[ \dot{w} = \frac{\delta}{\eta (1-\delta-\gamma) + \gamma} \left( \alpha_{H2} \theta_{H2} \frac{w}{u} - \alpha_{F2} \theta_{F2} \frac{1}{u} \right) \]

These differential equations are non-linear, so we linearize them by Taylor expansion around the steady state values $u^*$ and $w^*$ to obtain

\[ \dot{u} = - \left[ 2 \alpha_{H1} \theta_{H1} u^* (w^*)^{1-\eta} (u - u^*) - \alpha_{H1} \theta_{H1} (1-\eta) (u^*)^2 (w^*)^{-\eta} \right] (w - w^*) \]

\[ \dot{w} = \frac{\delta}{\eta (1-\delta-\gamma) + \gamma} \left( \alpha_{H2} \theta_{H2} w^* - \alpha_{F2} \theta_{F2} (u^*)^{-2} \right) (u - u^*) + \frac{\delta}{\eta (1-\delta-\gamma) + \gamma} \alpha_{H2} \theta_{H2} (u^*)^{-1} (w - w^*) \]

where the steady state $u^*$ and $w^*$ are given by equations (55) and (56). The equilibrium loci $\dot{u} = 0$ and $\dot{w} = 0$ are:

\[ u = - \left( \frac{1-\eta}{2} w^* \right) w + (3 - \eta) u^* \]

\[ w = w^* \]

With these results in hand, we can analyze the stability of the world economy under incomplete specialization. As usual, we relegate the mathematical details to the Appendix. The crucial variable turns out to be $w$. Recall that the trade pattern condition under incomplete specialization is given by condition (53). Given $w \equiv Z_{H2}/Z_{F2}$, under incomplete specialization the initial value of $w$ must satisfy

\[ w > \left\{ \frac{(1-\epsilon) L_Y}{\epsilon L_Y} \right\}^{1-\lambda} \frac{A_{H2}}{A_{F2}} \]
The evolution of the world economy depends on the relation between the initial value of \( w \) and the steady state value \( w^* \).

Suppose for the moment that \( w^* \) also is larger than the right side of (72). Then there are three possible cases.

1. If \( w = w^* \), then \( w \) is on its equilibrium locus and does not change, and \( u \) converges to \( u^* \). The world economy converges to a balanced growth path with perpetual incomplete specialization.

2. If \( w < w^* \), then \( \dot{w} < 0 \). At some finite time, \( w \) falls below the right side of (72). At that point, the economy switches to complete specialization. Its dynamics cease to be governed by (57)-(62) but instead become governed by (51)-(52) discussed earlier. We already have seen that the regime of complete specialization has an asymptotically stable balanced growth path, so once the economy crosses from incomplete to complete specialization, it remains completely specialized.

3. If \( w > w^* \), then \( \dot{w} > 0 \), and the world economy remains incompletely specialized forever. It also diverges from the balanced growth path of the incomplete specialization regime. The difference of the growth rates of two countries converges to the constant

\[
\left( \frac{Y_H}{Y_H} - \frac{Y_F}{Y_F} \right) \rightarrow \left( \delta + \frac{\delta^2}{\eta (1-\delta-\gamma) + \gamma} \right) \alpha_{H2} \theta_{H2} \frac{1}{u^*}
\]

The home country’s growth rate is perpetually above that of the foreign country, and the difference is bounded away from zero. Figure 2 shows the phase diagram for the world economy under incomplete specialization when \( w^* \) is larger than the right side of (72).

Finally, it may be that the steady state \( w^* \) is below the right side of (72). Then, if the world finds itself in a state of incomplete specialization when trade opens, it necessarily will be in case (3) above because incomplete specialization requires that (72) be satisfied.

Figure 3 shows another way to visualize the dynamic behavior of the economy. The horizontal axis is divided into three sections. The middle section is the region of complete specialization, denoted CS in the Figure, in which the home country does R&D on quality \( Z_{H1} \) and the foreign country does R&D on quality \( Z_{F2} \). Outside the CS region are the two regions of incomplete specialization, denoted IS in the Figure. In the IS region to the left of CS the home country completely specializes and does R&D on quality \( Z_{H1} \), whereas the foreign country remains unspecialized and does R&D on both the qualities \( Z_{F1} \) and \( Z_{F2} \). In the IS region to the right of CS, the home country is unspecialized and does R&D on \( Z_{H1} \) and \( Z_{H2} \), whereas the foreign country is specialized and does R&D on \( Z_{F2} \). It is the latter IS region that we have analyzed above. The boundaries of the regions depend on the quality ratios \( Z_{F1}/Z_{H1} \) and \( Z_{F2}/Z_{H2} \). When the quantity \([(1-\eps) L_H/\eps L_F]^{1-\lambda} \) is inside the CS region, as at point 1, the two quality levels \( Z_{H1} \) and \( Z_{F2} \) grow through R&D, and the two quality levels \( Z_{H2} \) and \( Z_{F1} \) are constant (because no R&D is done on them). Consequently, the quality ratio \( Z_{F1}/Z_{H1} \), and the quality ratio \( Z_{F2}/Z_{H2} \) rises, causing the CS boundaries to spread farther apart. As a result, the quantity \([(1-\eps) L_H/\eps L_F]^{1-\lambda} \) remains inside the CS region forever, and the two economies remain completely specialized forever. Behavior is different when \([(1-\eps) L_H/\eps L_F]^{1-\lambda} \) is in one of the IS regions. For example, point 2 corresponds to the case analyzed above, where the home country is not specialized and the foreign country specializes in producing good \( G_{F2} \). In that case, R&D is active for the three quality levels \( Z_{H1} \), \( Z_{H2} \), and \( Z_{F2} \), so they all grow, and no R&D is performed on \( Z_{F1} \), which therefore is constant. The lower (left) boundary of the CS region moves ever lower as time passes, but the movement of the upper (right) boundary depends on whether \( w < w^* \) or \( w > w^* \) (assuming for expository ease that \( w^* \) itself is in the right IS region). If \( w < w^* \), \( Z_{F2} \) grows faster than \( Z_{H2} \), and the upper boundary of the CS region increases over time, eventually passing point 1 and bringing the world into complete specialization. If \( w > w^* \), \( Z_{F2} \) grows more slowly than \( Z_{H2} \), and the upper boundary of the CS region decreases over time, moving away from point 2 and leaving the world farther and farther inside the IS region.
Lastly, recall that the model is completely symmetric. In (53), we assumed for sake of clarity that $P_{Y_F}$ was at the upper boundary of the interval defining the region of complete specialization. Had it been at the lower boundary, the whole discussion would have been in converse, with the home country incompletely specialized and the foreign country incompletely specialized.

3.4 Implications

These results have interesting implications for the behavior of the world income distribution and for the interpretation of tests of the connection between trade and technology transfer.

3.4.1 World Income Distribution

In an important article, Acemoglu and Ventura (2002) argued that trade stabilizes the world income distribution. In their analysis, countries are endowed with non-intersecting sets of intermediate goods that they can produce and sell on the world market. World prices move to equalize rates of return, and the movements of world prices equalize the growth rates of all countries’ incomes. The role of price movements in equalizing growth rates is an important insight. Naturally, as with any model, Acemoglu and Ventura’s model has some limitations, and one would like to know if their major conclusion generalizes to environments that relax their model’s limitations. One limitation of Acemoglu and Ventura’s analysis is that it is carried out in a first-generation endogenous growth model that has a scale effect, and some of their analysis works through the scale effect. Another limitation is that comparative advantage plays no role in determining the patterns of production or trade. Countries are endowed exogenously with non-overlapping sets of goods to produce, and they produce those and only those with no choice in the matter. Finally, production is AK, so there are no transition dynamics. Our analysis relaxes all these restrictions.

Acemoglu and Ventura’s main result on the stabilization of the world income distribution survives some of the generalizations but not others. Elimination of the scale effect in itself does not alter the conclusion that trade can stabilize the world income distribution. There is a world balanced growth path inside the region of complete specialization on which all countries’ incomes grow at the same rate. In fact, Acemoglu and Ventura’s assumption that countries specialize because of their endowment puts all countries in the world’s region of complete specialization by construction, so in a way it is not surprising that the balanced growth behavior for the case of complete specialization in our model resembles that of Acemoglu and Ventura’s analysis. Our model strengthens their conclusion on income distribution stability in one way because it has transition dynamics. We found that all countries grow at the same rate anywhere inside the region of complete specialization, not just on the balanced growth path, so the world income distribution is stable along transition paths, too, provided the world is inside the region of complete specialization. This last proviso leads to the part of our analysis in which Acemoglu and Ventura’s conclusion does not hold, namely, outside the region of complete specialization. We have shown that in the region of incomplete specialization, except at the unstable balanced growth path, countries’ growth rates differ, which means the world income distribution must be changing. Furthermore, in part of the region of incomplete specialization, countries’ growth rates go asymptotically to a constant difference with one country perpetually growing at a faster rate than the other. In that case, the world income distribution degenerates, with the faster growing economy’s share of world income going asymptotically to 1. Note that the slower growing country does not disappear. In fact, it always grows. It just grows more slowly than the other country and so vanishes relative to the faster growing country. This result may correspond to the history of sub-Saharan Africa, whose growth rate usually has been positive but also has lagged behind that of the rest of the world for at least 200 years. It also is interesting to note that very small countries have a tendency to end up with relatively low growth rates. One of the factors determining whether the world is in the region of incomplete specialization, where countries have different growth rates, is population size. If a
country is very small relative to its trading partners, it has a tendency to end up growing more slowly than its partners. The reason Acemoglu and Ventura do not obtain results like these is that their analysis by construction excludes the possibility of incomplete specialization.

### 3.4.2 Technology Transfer

Our analysis also has an implication for econometric analysis of technology transfer. There is a literature, started by Coe and Helpman (1995, 2008), that attempts to measure the amount of technology transfer by matching volume of trade with technical sophistication among trading partners. The general finding is a country that trades with technologically advanced partners has a higher growth rate than a country that trades with less advanced partners. The usual conclusion is that trade facilitates technology transfer. Our analysis suggests that such a conclusion may be unwarranted because trade itself can lead to growth rate results that resemble the effects of technology transfer even when such transfers do not occur.

In our model, trade’s growth rate effect does not work through cross-country technology spillovers or technology transfers, which are ruled out by construction. Countries are not permitted to learn the technology of their trading partners even with trade. In effect, we are assuming that reverse engineering is not possible and that firms are able to keep their production techniques secret. A country can get the “quality” but not the “know-how” from importing a good. Nonetheless, the growth rate can be affected by trade in the same way as if the country actually had adopted the trading partner’s R&D ability for producing the good at home that it imports from the trading partner. If the home country’s trading partner is more efficient at producing a good and also has a higher ability in R&D related to that good, the solution for the growth rate with trade is exactly the same as the solution that would have emerged if the home country had learned the trading partner’s technology and produced the good itself instead of importing it. One would see, then, that countries whose trading partners are sophisticated have higher growth rates that other countries, but the higher growth rates arise from importing sophisticated goods, not from importing the technology to make those goods domestically. This result casts a shadow on the standard interpretations of the data.

It also is interesting that the growth rate under trade need not be the same as if technology transfer occurred. Suppose the foreign country’s quality-adjusted price is lower than the domestic quality-adjusted price for some good. The home country then imports that good. Suppose, however, that the foreign country is relatively inefficient at R&D. When technology is imported in the form of embodied quality, as our model, the result would be a lower growth rate for the home country, as we have discussed above. With technology transfer, in contrast, the home country would merely copy the better production methods of the foreign country and would then pair those with its own relatively efficient R&D. Thus acquiring technology indirectly by buying it as an embodied quality in a traded good is not the same as learning to produce the better quality good yourself.

### 4 Conclusion

We have constructed an asymmetric Schumpeterian growth model without scale effects or technology transfer to discuss the impact of trade on both output levels and growth rates through the channel of comparative advantage alone. The model offers several theoretical advances over the existing literature: the asymmetric element, the absence of a scale effect, tractable transition dynamics, and, most important, the endogeneity of trade patterns. The usual assumption in the pertinent literature is that countries trade everything they produce and the division among countries of what is produced is determined exogenously. In our model, countries may choose to
trade nothing. If they do choose to trade, which goods they export and which they import are determined endogenously.

We have shown that trade can have surprising effects on both the level and growth rate of output. In contrast to the usual result that trade raises incomes of both trading partners, we obtain the result that trade may reduce the initial income of one or both trading partners. The mechanism for this unusual result is an externality that is missing from static models. R&D-driven quality improvement is the engine of growth, but relative cross-country efficiency in doing R&D has nothing to do with the determination of comparative advantage. Comparative advantage in turn is the sole determinant of trade patterns but has nothing to do with the determination of economic growth. It thus is possible for a country to import a good that is lower in quality than the domestic good it replaces because that good is produced so much more cheaply by the trading partner. The lower quality, however, reduces output in other industries through a knowledge spillover effect. We do not by any means expect this mechanism to be the norm, but it is a possibility. In the absence of a large negative externality of this type, trade increases the output of both trading partners.

Irrespective of trade’s effect on the initial level of income, trade may raise or lower growth rates. Again, the issue is that R&D efficiency determines growth rates but is not taken into account when firms decide whether to import a good. What firms care about is the quality-adjusted price that they are offered today. Thus a cheap good, possibly of high current quality, may replace another good that has associated with it a more efficient R&D program. Replacement of the latter good thus has the unintended side effect of lowering the growth rate.

Under complete specialization and in some cases of incomplete specialization, trade equalizes growth rates at least eventually and thus stabilizes the world income distribution. In other cases of incomplete specialization, the growth rate of the country that is not completely specialized is always higher than the growth rate of its completely specialized trading partner, and the difference between them asymptotically converges to a constant.

Finally, we have seen that trade can yield growth outcomes that mimic those arising from technology transfers.
References


Fig. 1: Transition dynamics, closed economy

(IS|Z_{III};Z_{IV};Z_{v2})  CS|Z_{III};Z_{v2}  IS|Z_{III};Z_{IV};Z_{v2}

$$\frac{A_{III}}{A_{III}} \left( \frac{Z_{III}}{Z_{III}} \right)^{\delta}\lambda$$

$$\frac{A_{IV}}{A_{IV}} \left( \frac{Z_{IV}}{Z_{IV}} \right)^{\delta}\lambda$$

Fig. 3: Regions of specialization

Fig. 2: Phase diagram, incomplete specialization